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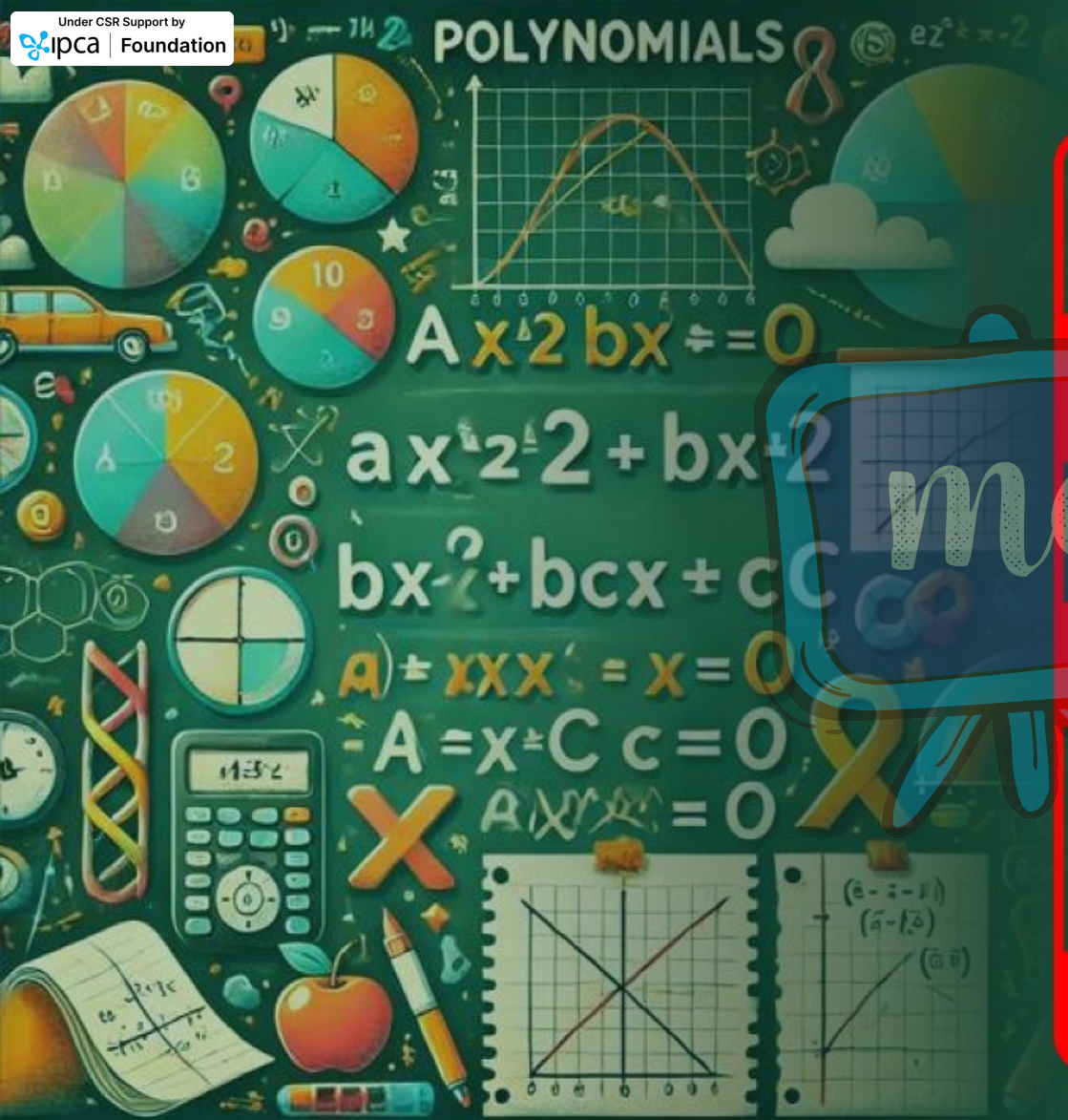

कक्षा – 9

गणित

अध्याय – 2

बहुपद

भाग – 5

उदाहरण 8 से 10

हर्ष गोयल

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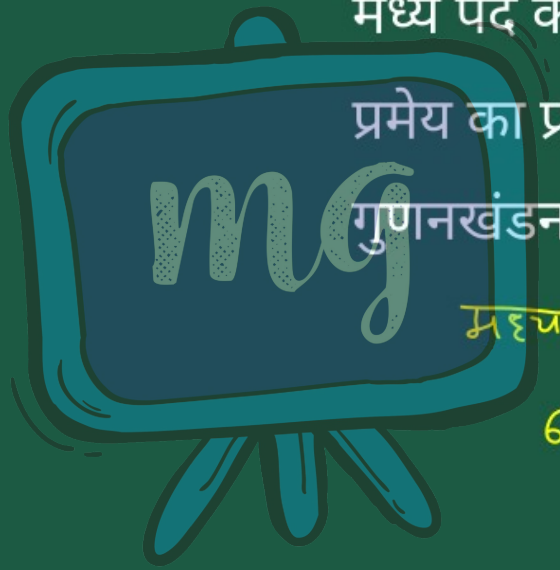
उदाहरण 8 :

मध्य पद को विभक्त करके तथा गुणनखंड प्रमेय का प्रयोग करके $6x^2 + 17x + 5$ का गुणनखंडन कीजिए ।



मध्य पद विभक्त करने

$6x^2 + 17x + 5$

मध्य पद $= 17x$

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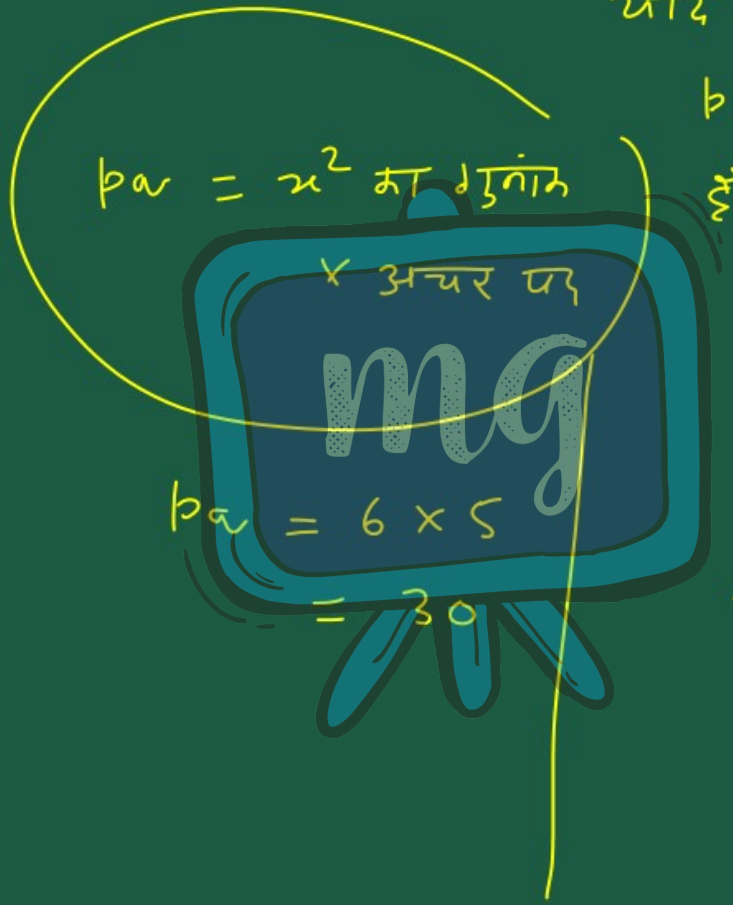

यदि हम दो ऐसी संख्याएं b तथा q ज्ञात कर सकते हैं कि

$b + q = 17$

$bq = 30$

तो हम गुणनखंड ज्ञात कर सकते हैं

30 के गुणनखंड

2, 3, 5

15, 2

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15, 2

तो मध्य पद $= 17x$

$= 15x + 2x$



दिया है, $6x^2 + 17x + 5$

$= 6x^2 + 15x + 2x + 5$

$= 3x(2x+5) + 1(2x+5)$

$= (2x+5)(3x+1)$

$6x^2 + 17x + 5 = (2x+5)(3x+1)$

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गुणनखंड प्रमेय के प्रयोग में

बहुपद $6x^2 + 17x + 5$

$$6\left[x^2 + \frac{17}{6}x + \frac{5}{6}\right]$$

$$= 6\,p(x)$$

जहाँ $p(x)$

$$= x^2 + \frac{17}{6}x + \frac{5}{6}$$

यदि a तथा b बहुपद $p(x)$ के शून्यक हैं तो

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$6x^2 + 17x + 5 = 6(x-a)(x-b)$

$6\left\{x^2 + \frac{17}{6}x + \frac{5}{6}\right\} = 6(x-a)(x-b)$

$p(a) = 0$

ऊपर पद $ab = \frac{5}{6}$

a तथा b के संभावित परिणाम

$+\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm 1$

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$$p(x) = x^2 + \frac{17}{6}x + \frac{5}{6}$$

$x = \frac{1}{2}$, $p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + \frac{17}{6}\left(\frac{1}{2}\right) + \frac{5}{6}$

$p\left(\frac{1}{2}\right) \neq 0$

$\left(x - \frac{1}{2}\right)$, बहुपद का गुणनखंड नहीं

$x = -\frac{1}{2}$, $p\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \frac{17}{6}\left(-\frac{1}{2}\right) + \frac{5}{6}$

$= \frac{1}{4} - \frac{17}{12} + \frac{5}{6}$

$= \frac{3 - 17 + 10}{12}$

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$p\left(\frac{1}{2}\right) \neq 0$

अत: $x + \frac{1}{2}$, गुणनखंड नहीं



$x = \frac{1}{3}$, $p(x) = x^2 + \frac{17}{6}x + \frac{5}{6}$

$p\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 + \frac{17}{6}\left(\frac{1}{3}\right) + \frac{5}{6}$

$p\left(\frac{1}{3}\right) \neq 0$

अत: $x - \frac{1}{3}$, बहुपद $p(x)$ का एक गुणनखंड नहीं है

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$$b(x) = x^2 + \frac{17}{6}x + \frac{5}{6}$$

$x = -\frac{1}{3}$, $b\left(-\frac{1}{3}\right) = \left(\frac{-1}{3}\right)^2 + \frac{17}{6}\left(\frac{-1}{3}\right) + \frac{5}{6}$

$$= \frac{1}{9} - \frac{17}{18} + \frac{5}{6}$$

$$= \frac{2 - 17 + 15}{18}$$

$b\left(-\frac{1}{3}\right) = 0$

अतः गुणनखंड प्रमेय से

$\left(x + \frac{1}{3}\right)$, बहुपद $b(x)$ का

एक गुणनखंड है।

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$x = -\frac{5}{2}, \frac{5}{3}$

$p\left(\frac{5}{2}\right) \neq 0$, $p\left(\frac{5}{3}\right) \neq 0$

$p(x) = x^2 + \frac{17}{6}x + \frac{5}{6}$

$x = -\frac{5}{3}$,

$p\left(-\frac{5}{3}\right) = \left(-\frac{5}{3}\right)^2 + \frac{17}{6}\left(-\frac{5}{3}\right) + \frac{5}{6}$

$= \frac{25}{9} - \frac{85}{18} + \frac{5}{6}$

$= \frac{50 - 85 + 15}{18}$

$p\left(-\frac{5}{3}\right) \neq 0$

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$$x = \frac{-5}{2}$$

$$p\left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right)^2 + \frac{17}{6}\left(\frac{-5}{2}\right) + \frac{5}{6}$$

$$= \frac{25}{4} - \frac{85}{12} + \frac{5}{6}$$

$$= \frac{75 - 85 + 10}{12}$$

$$= 0$$

$$p\left(\frac{-5}{2}\right) = 0$$



अत: $x + \frac{5}{2}$, बहुपद $p(x)$ का एक गुणनखंड है

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$\left(x+\frac{1}{3}\right)$, $\left(x+\frac{5}{2}\right)$

$6x^2+17x+5 = 6\left[\left(x+\frac{1}{3}\right)\left(x+\frac{5}{2}\right)\right.$

$= 6\left[\left(\frac{3x+1}{3}\right)\left(\frac{2x+5}{2}\right)\right.$

$= \frac{6}{6}\left[(3x+1)(2x+5)\right.$

$6x^2+17x+5 = (3x+1)(2x+5)$

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उदाहरण 9 :

गुणनखंड प्रमेय की सहायता से $y^2 - 5y + 6$ का गुणनखंडन कीजिए।



$p(y) = y^2 - 5y + 6$

यदि a तथा b बहुपद के शून्यक हैं तो

$y^2 - 5y + 6 = (y - a)(y - b)$

अचर पद $= ab = 6$

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6 के गुणनखंड

= 1, 2, 3, 6



$p(y) = y^2 - 5y + 6$

$p(1) = (1)^2 - 5(1) + 6$

$= 1 - 5 + 6$

$p(1) \neq 0$

अतः $x-1$, बहुपद $p(y)$ का एक गुणनखंड नहीं है

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$p(y) = y^2 - 5y + 6$

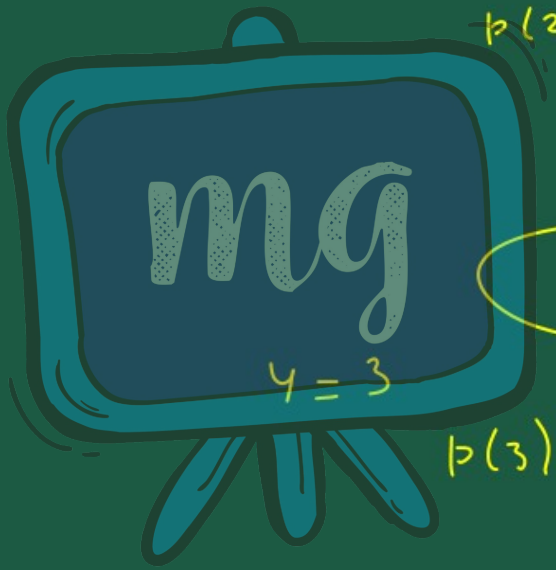
$y = 2$

$p(2) = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6$

$= 10 - 10 = 0$

$p(2) = 0$



$p(3) = (3)^2 - 5(3) + 6$

$= 9 - 15 + 6$

$p(3) = 0$

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गुणनखंड प्रमेय



$y-2$ तथा $y-3$, बहुपद $P(y)$ के गुणनखंड होंगे

$$\boxed{y^2-5y+6=(y-2)(y-3)}$$

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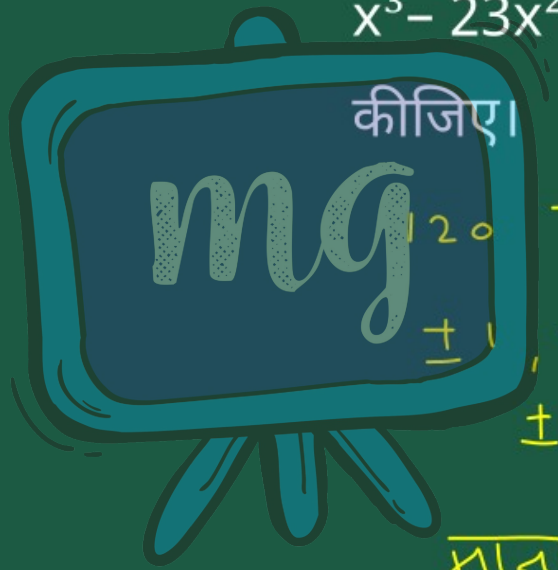
उदाहरण 10 :

$x^3 - 23x^2 + 142x - 120$ का गुणनखंडन कीजिए।



120 के गुणनखंड

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10$ ----

माना $P(x) = x^3 - 23x^2 + 142x - 120$

$x = 1$ पर $P(1) = (1)^3 - 23(1)^2 + 142(1) - 120$

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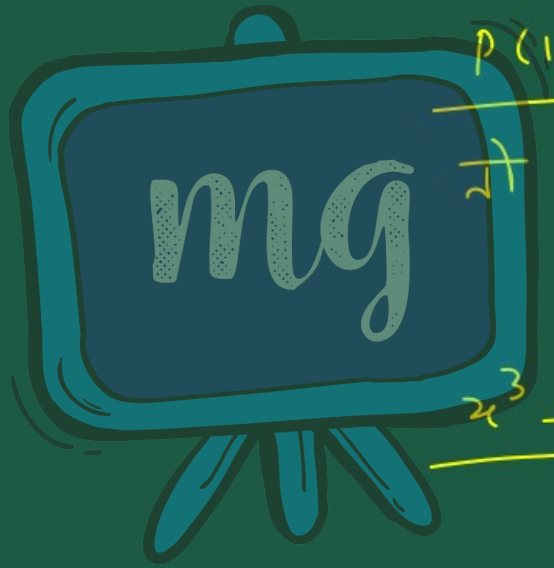
$P(1) = 1 - 23 + 192 - 120$

$= 143 - 143$

$P(1) = 0$

$\therefore$ $x - 1$, बहुपद $P(x)$ का एक गुणनखंड है।

$x^3 - 23x^2 + 192x - 120$



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$$x^2 - 22x + 120$$

$$x - 1 \;\overline{)\; x^3 - 23x^2 + 142x - 120}$$

$$-x^3 + x^2$$

$$x^2(x-1)$$

$$-22x^2 + 142x - 120$$

$$-22x^2 + 22x$$

$$(x-1)(-22x)$$

$$120x - 120$$

$$-120x - 120$$

$$+$$

$$0$$



$$x^3 - 23x^2 + 142x - 120$$

$$= (x-1)(x^2 - 22x + 120)$$

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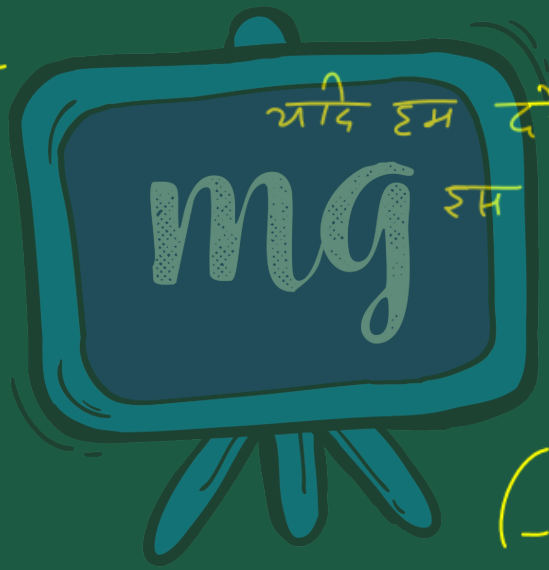
$x^2 - 22x + 120$

मध्य पद विभक्त करने

मध्य पद $= -22x$

2	120
2	60
2	30
3	15
5	5
	1



यदि हम दो ऐसी संख्याएँ p तथा q इस प्रकार ज्ञात कर सकते हैं कि

$p + q = -22$

$pq = 120$

$(-12, -10)$

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$$x^2 - 22x + 120$$

$$= x^2 - 12x - 10x + 120$$

$$= x(x-12) - 10(x-12)$$

$$= (x-12)(x-10)$$

$x^2 - 22x + 120 =$

$$x^3 - 23x^2 + 142x - 120$$

$$= (x-1)(x^2 - 22x + 120)$$

$$= (x-1)(x-12)(x-10)$$