

CLASS – 11

PHYSICS

Chapter – 8

Mechanical Properties of Solid

Part – 4

Exercise (Q.1 - 16)

Alok Gaur

EXERCISE

Q.1 A steel wire of length 4.7 m and cross-sectional area $3.0 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper?

$$\begin{aligned} l_s &= 4.7 \text{ m} \\ A_s &= 3 \times 10^{-5} \text{ m}^2 \\ l_c &= 3.5 \text{ m} \\ A_c &= 4 \times 10^{-5} \text{ m}^2 \\ \Delta l_c &= \Delta l_s \end{aligned}$$

EXERCISE

Q.1

A steel wire of length 4.7 m and cross-sectional area $3.0 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper?

$$\Delta l = \frac{F l}{A Y}$$

$$\Delta l_s = \Delta l_c$$

$$\frac{F l_s}{A_s Y_s} = \frac{F l_c}{A_c Y_c}$$

$$\frac{Y_s}{Y_c} = \frac{A_c}{A_s} \times \frac{l_s}{l_c}$$

Answer :

$$\frac{Y_s}{Y_c} = \frac{4 \times 10^{-5} \times 4.7}{3 \times 10^{-5} \times 3.5}$$

$$\frac{Y_s}{Y_c} = 1.78 \approx 1.8$$

EXERCISE

Slope of $\sigma - \epsilon$ curve
→ γ

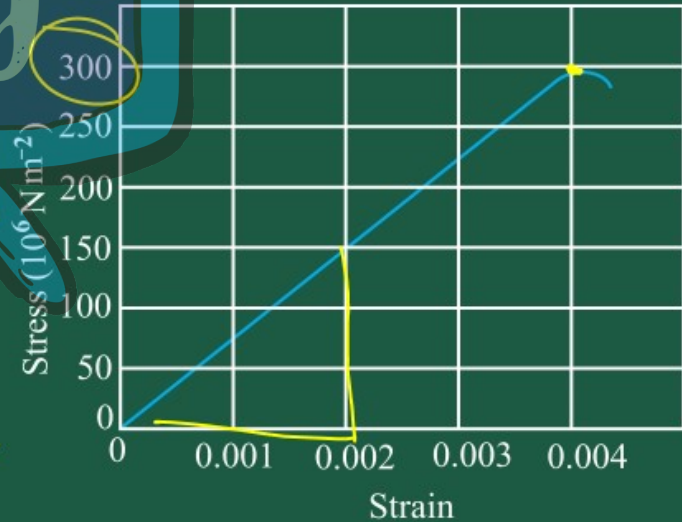
$$\gamma = \frac{150 \times 10^6}{2 \times 10^{-3}}$$

$$\gamma = 75 \times 10^9$$

$$\gamma = 7.5 \times 10^{10} \text{ N/m}^2$$

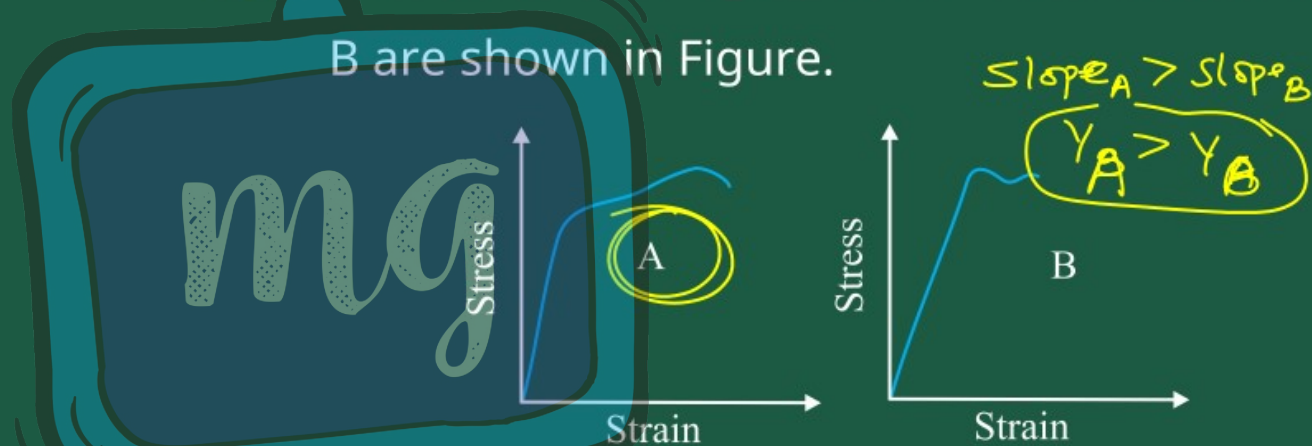
b) 300×10^6
 $= 3 \times 10^8 \text{ N/m}^2$

Q.2 Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?



EXERCISE

Q.3 The stress-strain graphs for materials A and B are shown in Figure.



The graphs are drawn to the same scale.

- (a) Which of the materials has the greater Young's modulus?
- (b) Which of the two is the stronger material?

EXERCISE

(a) Which of the materials has the greater Young's modulus?

Answer : In the two graphs it is that given that stress for A is more than that of B.

As, Young's modulus = $\frac{\text{Stress}}{\text{Strain}}$

Therefore, material A has greater Young's modulus.

EXERCISE



(b) Which of the two is the stronger material?



EXERCISE

(b) Which of the two is the stronger material?

Answer : The strength of a material is determined by the amount of stress required for fracturing a material, corresponding to its fracture point.

Fracture point is defined as the extreme point in a stress-strain curve.

From the graph it is clear that material A can withstand more strain than material B.

Therefore, material A is stronger than material B.

EXERCISE

Q.4 Read the following two statements below carefully and state, with reasons, if it is true or false.

(a) The Young's modulus of rubber is greater than that of steel; **F**

(b) The stretching of a coil is determined by its shear modulus. **T**



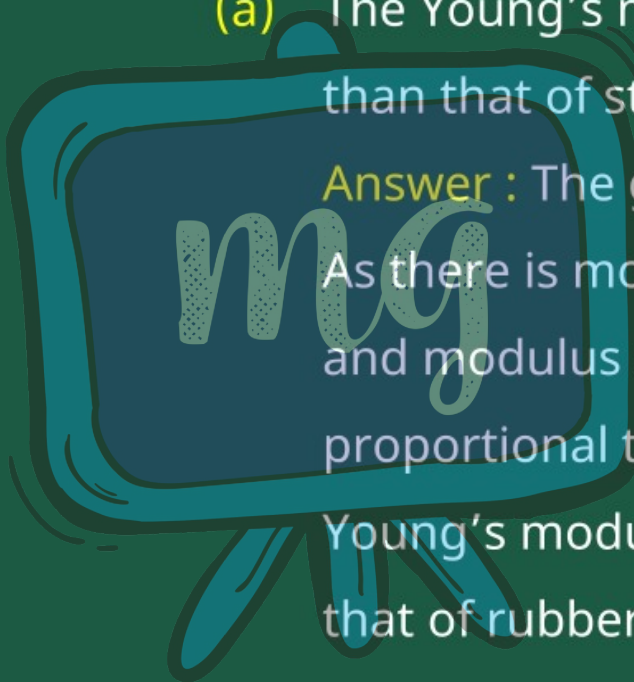
EXERCISE



(a) The Young's modulus of rubber is greater than that of steel;

Answer : The given statement is false.

As there is more strain in rubber than steel and modulus of elasticity is inversely proportional to strain. Therefore, the Young's modulus of steel is greater than that of rubber.



EXERCISE



(b) The stretching of a coil is determined by its shear modulus.



EXERCISE

(b) The stretching of a coil is determined by its shear modulus.

Answer : The given statement is true.

As the shear modulus of a coil relates with the change in shape of the coil and the stretching of coil changes its shape without any change in the length. Therefore, the shear modulus of elasticity is involved.

Hence the stretching of a coil is determined by its shear modulus.

EXERCISE

Q.5 Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Figure The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.

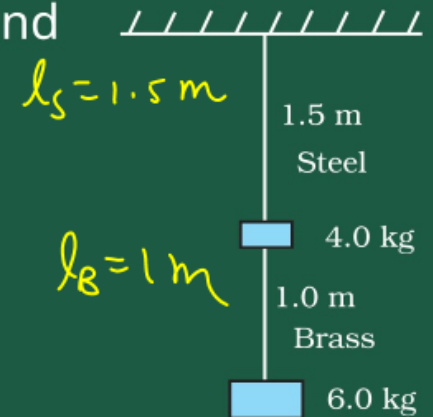


Diagram showing a vertical wire of length $l_s = 1.5 \text{ m}$ suspended from a ceiling. A mass of 4 kg is attached to the wire, and a mass of 6 kg is attached to the end of the wire. The total length of the wire is $1 \text{ m} = l_B$.

Formula for elongation: $\Delta l = \frac{F l}{A Y}$

Given: $d = 0.25 \text{ cm}$
 $r = \frac{0.25}{2} \times 10^{-2} \text{ m}$

Calculation for Δl_B :

$$\Delta l_B = \frac{60 \times 1}{\pi \times \frac{625}{4} \times 10^{-8} \times 0.91 \times 10^{11}}$$

$$\Delta l_B = \frac{60 \times 4}{0.9 \times \pi \times 625 \times 10^3}$$

$$= \frac{20 \times 4 \times 10^{-3}}{625}$$

$$= \frac{80}{625} \times 10^{-3} = \frac{32}{625} \times 10^{-4}$$

$$\Delta l_B = 1.28 \times 10^{-4} \text{ m}$$

Calculation for Δl_s :

$$\Delta l_s = \frac{10 \times 10 \times 1.5}{\pi \times \frac{25 \times 10^{-4}}{2} \times \frac{25 \times 10^{-4}}{2} \times 2 \times 10^{11}}$$

$$= \frac{10^2 \times 2}{\pi \times 625 \times 10^3}$$

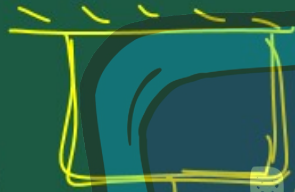
$$= \frac{4 \times 10^{-3}}{625}$$

$$= \frac{40 \times 10^{-4}}{255}$$

$$\Delta l_s = 1.6 \times 10^{-4} \text{ m}$$

EXERCISE

Q.6 The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?



100 kg

$$\eta = 25 \times 10^9 \text{ Pa}$$

$$\eta = \frac{F l}{A x}$$

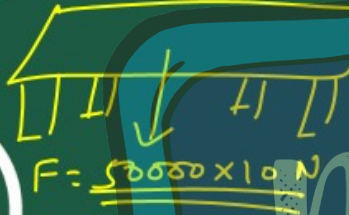
$$x = \frac{F l}{A \eta}$$

$$x = \frac{100 \times 10^{-4} \times 10 \times 10^{-2}}{25 \times 10^9}$$

$$x = 4 \times 10^{-7} \text{ m}$$

EXERCISE

Q.7 Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.



$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$e = \frac{\sigma}{Y}$$

$$e = \frac{F}{AY}$$

$$e = \frac{50000}{\pi [(60)^2 - (30)^2] \times 10^{-4} \times 2 \times 10^{11}}$$

$$e = \frac{5 \times 10^5}{\pi \times 900 \times 3 \times 2 \times 10^7}$$

$$e = \frac{1 \times 10^{-2}}{4 \times 900} = \frac{1 \times 10^{-4}}{36}$$

$$e = \frac{100 \times 10^{-6}}{36} = \underline{\underline{3 \times 10^{-6}}}$$

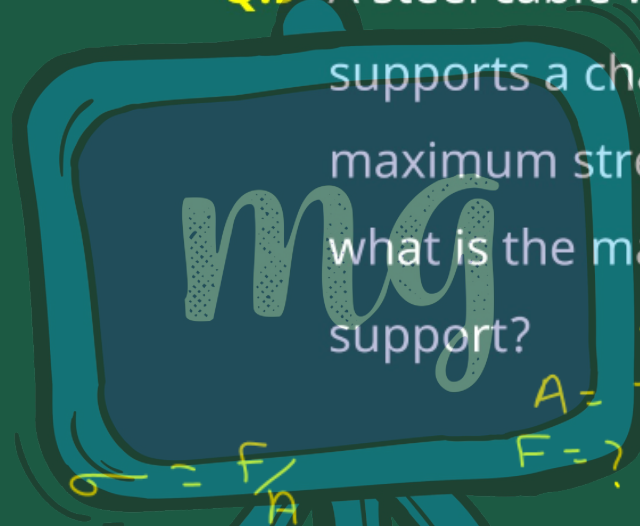
EXERCISE

Q.8 A piece of copper having a rectangular cross-section of $15.2 \text{ mm} \times 19.1 \text{ mm}$ is pulled in tension with $44,500 \text{ N}$ force, producing only elastic deformation. Calculate the resulting strain?

$$\begin{aligned} \epsilon &= \frac{\text{Stress}}{Y} = \frac{44500}{15.2 \times 19.1 \times 10^{-6} \times 1.1 \times 10^{11}} \\ &= \frac{44500 \times 10^{-5}}{20 \times 15} \\ &= \frac{445}{3} \times 10^{-5} = 148.33 \times 10^{-5} \end{aligned}$$

EXERCISE

Q.9 A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2} , what is the maximum load the cable can support?



$$r = 1.5 \times 10^{-2} \text{ m}$$

$$A = \pi \times (1.5 \times 10^{-2})^2$$

$$\sigma = 10^8 \text{ N/m}^2$$

$$F = ?$$

$$\sigma = \frac{F}{A}$$

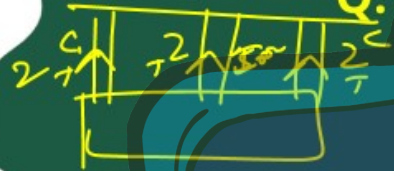
$$F = \sigma A$$

$$F = 10^8 \times \pi \times 2.25 \times 10^{-4}$$

$$F = 10^4 \times 7 \text{ N}$$

EXERCISE

Q.10 A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.



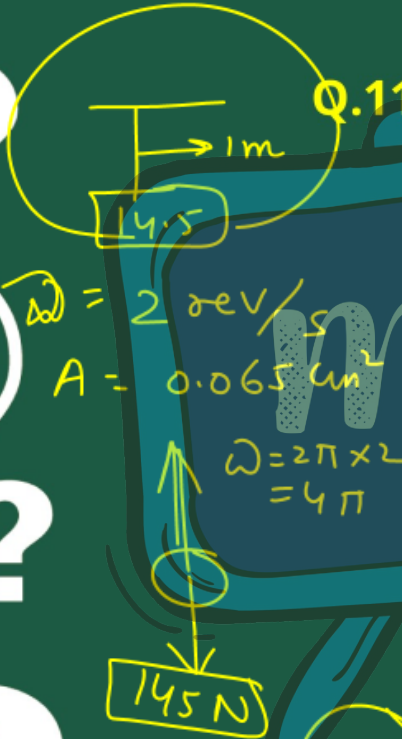
$$Y = \frac{F l}{A \Delta l}$$

$$A = \frac{F l}{Y \Delta l}$$

$$A \propto \frac{1}{Y}$$

EXERCISE

Q.11 A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm². Calculate the elongation of the wire when the mass is at the lowest point of its path.



$$\begin{aligned} m\omega^2 r &= 14.5 \times 1 \times 16 \times \pi^2 \\ &= 2320 \\ T &= 232 - 145 = 77 \text{ N} \end{aligned}$$

$$\begin{aligned} \Delta l &= \frac{F l}{A Y} \\ &= \frac{77 \times 1}{6.5 \times 10^7 \times 2 \times 10^{11}} \\ &= 1.1 \times 0.5 \times 10^{-4} \\ &= 0.55 \times 10^{-4} \text{ m} \end{aligned}$$

EXERCISE

Q.12 Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013×10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature).

Explain in simple terms why the ratio is so large.

$$V_i = 100 \text{ l}$$

$$m = 100 \text{ kg}$$

$$P = 100 \times 10^5 \text{ Pa}$$

$$V_f = 100.5 \text{ litre}$$

$$B = \frac{F \cdot V}{A \cdot \Delta V}$$

$$B = \frac{100 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$= 200 \times 10^7$$

$$B = 2 \times 10^9 \frac{\text{N}}{\text{m}^2}$$

$$\Delta V = 0.5 \text{ litre} = 0.5 \times 10^{-3} \text{ m}^3$$

EXERCISE

Q.13 What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is $1.03 \times 10^3 \text{ kg m}^{-3}$?



EXERCISE

Q.14 Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

$$\Delta V = \frac{FV}{AB}$$

$$\frac{\Delta V}{V} = \frac{P}{B}$$

$$\frac{\Delta V}{V} = \frac{10 \times 1.01 \times 10^5}{B_g}$$

EXERCISE

Q.15 Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

$$\Delta V = \frac{P V}{B}$$

$$\Delta V = \frac{7 \times 10^6 \times 10^3 \times 10^{-6}}{B_c}$$

EXERCISE

Q.16 How much should the pressure on a litre of water be changed to compress it by 0.10%? carry one quarter of the load.

$$\frac{\Delta V}{V} = \frac{0.10}{100} = \frac{1}{1000} = 10^{-3}$$

$$\frac{\Delta V}{V} = 10^{-3} \quad \left| \quad V = 10^{-3} \text{ m}^3 \right.$$

$$P = ?$$

$$B = \frac{F/V}{A \Delta V}$$

$$B = \frac{P}{\Delta V/V} \Rightarrow$$

$$P = B \times \left(\frac{\Delta V}{V} \right)$$

$$= 2 \times 10^9 \times 10^{-3}$$

$$= \underline{\underline{2 \times 10^6 \text{ Pa}}}$$