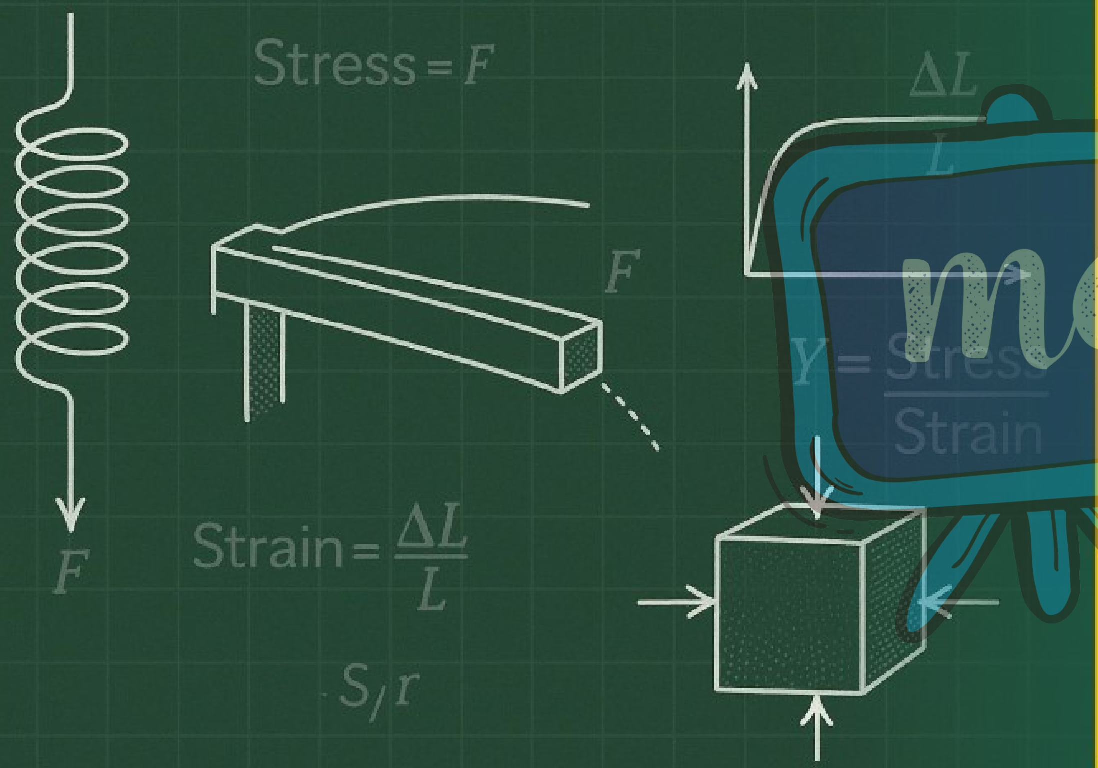

CLASS – 11

PHYSICS

Chapter – 8

Mechanical Properties of Solid

Part – 2

Elastic Moduli

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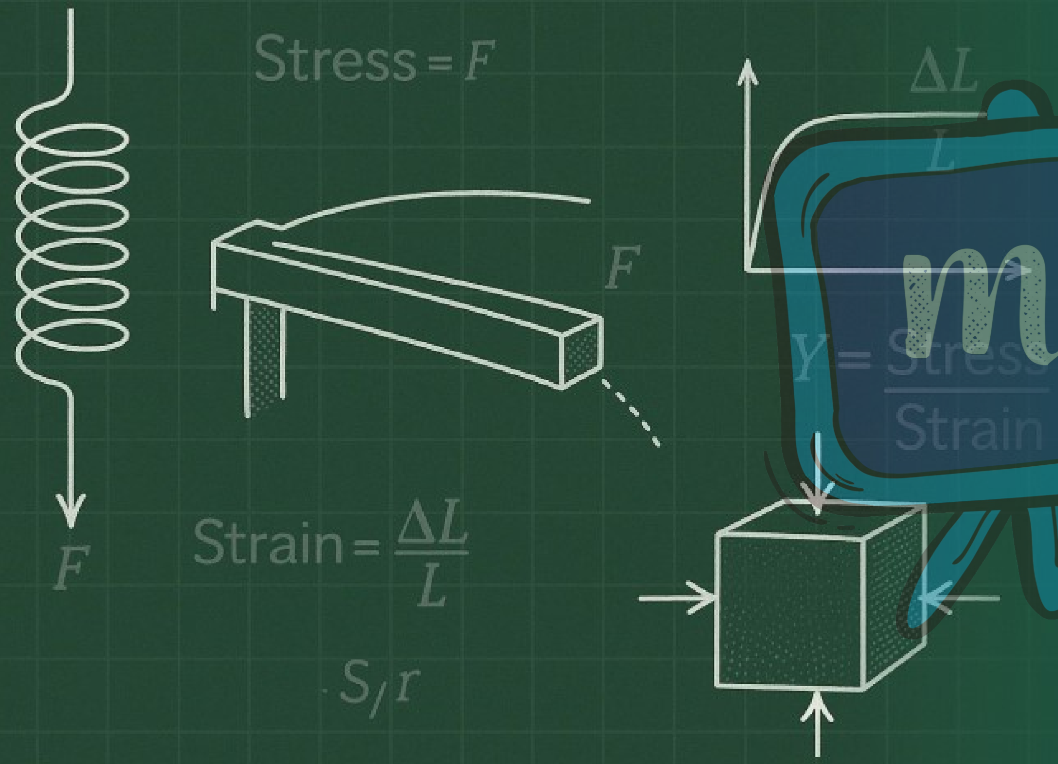
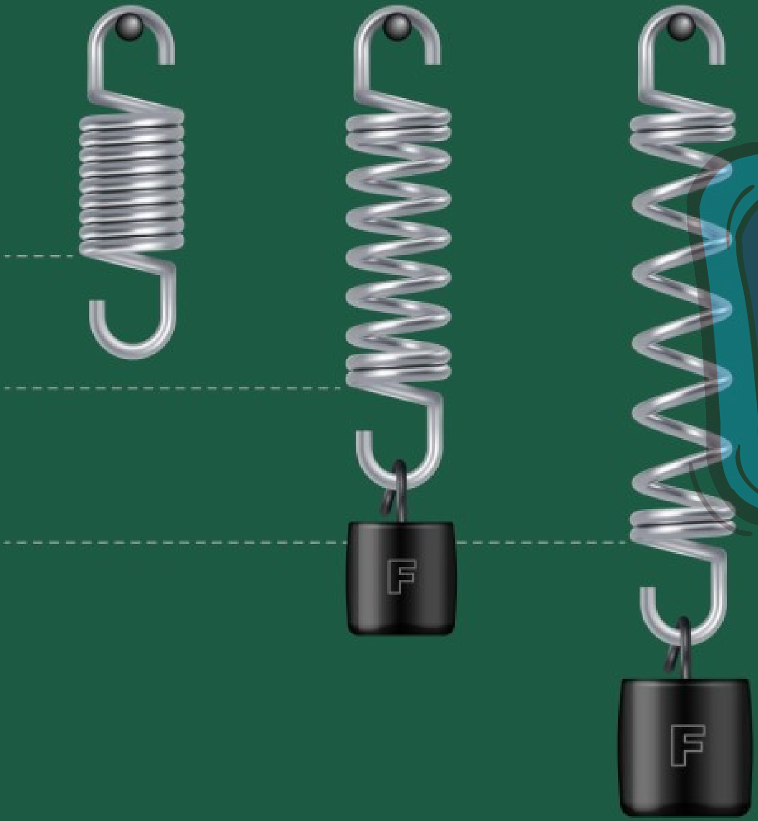
Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER

OVERVIEW



1. Stress and Strain
2. Elastic Moduli
3. Stress – Strain Curve



HOOKE'S LAW

- According to this law, "Stress in the limit of elasticity is directly proportional to the strain produced.

Stress $\propto$ strain

Under the limit of elasticity

Stress = constant (E) × strain

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$\sigma \propto e$

$\sigma = eE$

$E = \frac{\sigma}{e}$

E = Modulus of Elasticity

Where E is called co-efficient of elasticity.

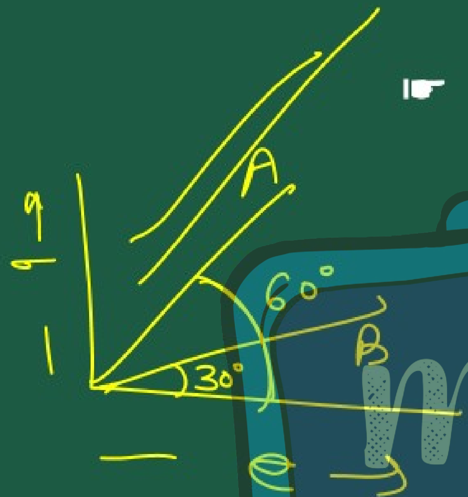
ELASTIC MODULI

- The ratio of stress and strain within the limit of elasticity is called the co-efficient of elasticity of the material or elastic moduli.

N/m^2 or pascal

$[M^1 L^{-1} T^{-2}]$

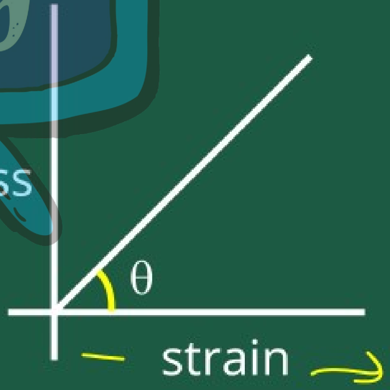
☛ In the limit of elasticity stress-strain graph is obtained as a straight line whose slope represents the elastic moduli.



$E_A = \text{slope}_A = \tan 60° = \sqrt{3}$

$E_B = \text{slope}_B = \tan 30° = \frac{1}{\sqrt{3}}$

$\frac{E_A}{E_B} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3/1$



$\text{Slope} = \tan\theta = \frac{\text{stress}}{\text{strain}} = E$

"$\tan \phi = E$"

- Unit : $\frac{\text{Newton}}{\text{Metre}^2}$
- Dimension : $[M^1L^{-1}T^{-2}]$
- Elastic moduli depends upon →
  (i) nature of the material of the body
  (ii) temperature of the body.
- Elastic moduli doesn't depend upon stress and strain.

TYPES OF MODULUS OF ELASTICITY

1. Young's modulus of elasticity (Y)

   $\frac{\text{Stress}}{\text{strain}}$

2. Bulk modulus of elasticity (K)
3. Modulus of rigidity or shear modulus (η)

i. Young's Modulus of Elasticity (Y)

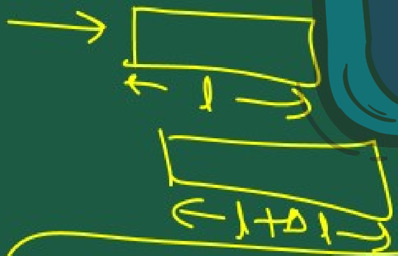
- Within elasticity limit the ratio of longitudinal stress and longitudinal strain is called. Young's modulus of elasticity.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$Y = \frac{F/A}{\Delta L/L}$$

$Y = \frac{Fl}{A\Delta l}$

$\Delta l = \frac{Fl}{AY}$



Steel $= 2 \times 10^{11}$ N/m²

$$Y = \frac{FL}{A\,\Delta L}$$

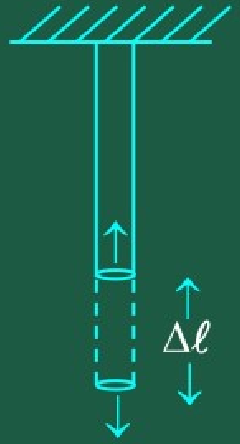
If the wire is cylindrical then

$A = \pi r^2$ $F = mg$

$$Y = \frac{MgL}{\pi r^2\,\Delta L}$$

If $\pi r^2 = 1m^2$ $\Delta L = L$

" $Y = Mg$ "



$F = mg$

- Y is equal to that force which when applied the length of the wire of unit area of cross = section will be doubled.
- The value of Y can only be obtained for solids.
- The value of Y for steel is more than that of the rubber.

- Increase in the length of the wire

$$\Delta L = \frac{FL}{\pi r^2 Y}$$

$$\Delta L \propto \frac{L}{r^2}$$

ii. Bulk Modulus of Elasticity (K)

- The ratio between volume stress and volume strain within the elasticity limit is called the bulk modulus of elasticity.

$$K = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$K = \frac{\frac{F}{A}}{\frac{-\Delta V}{V}} = \frac{-FV}{A\ \Delta V}$$

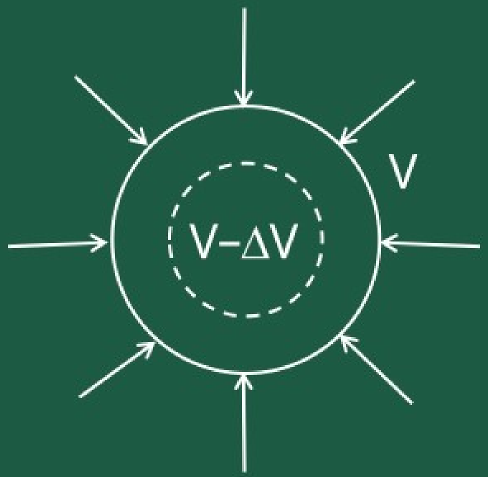
$$k = \frac{F/A}{\Delta V/V}$$

$$k = \frac{FV}{A\Delta V}$$

$$k = \frac{FV}{A\Delta V}$$

- (–) sign shows the decrease in volume.
- Compressibility (β) : The reciprocal of K is called compressibility.

$$\beta = \frac{1}{K}$$



- Gases only have bulk elasticity because change in pressure changes their volumes.

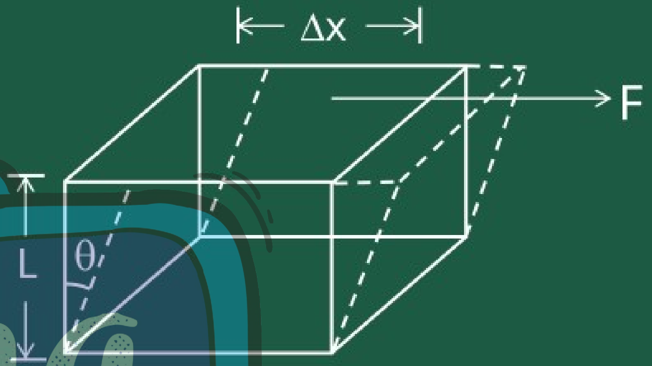
iii. Modulus of Rigidity or Shear Modulus (η)

- The ratio between shearing stress and shearing strain within the elasticity limit is called the modulus of rigidity.

$$\eta = \frac{\text{Shearing shtress}}{\text{Shearing strain}}$$

$$\eta = \frac{F/A}{x/l}$$

$$\eta = \frac{Fl}{Ax}$$



$$\eta = \frac{\frac{F}{A}}{\phi}$$

$$\eta = \frac{F}{A\phi}$$

$$\phi = \tan\phi = \frac{\Delta x}{L}$$

$$\eta = \frac{FL}{A\,\Delta x}$$

- Modulus of rigidity is lesser than Young's modulus of elasticity.
- For most of liquids $\eta = \frac{Y}{3}$

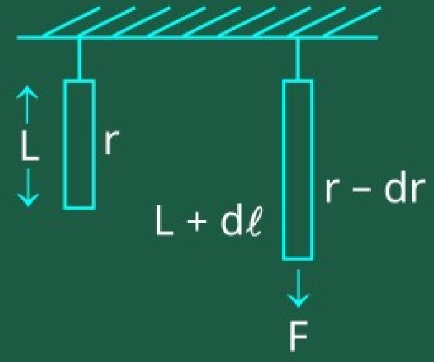
$Y = 3\eta$

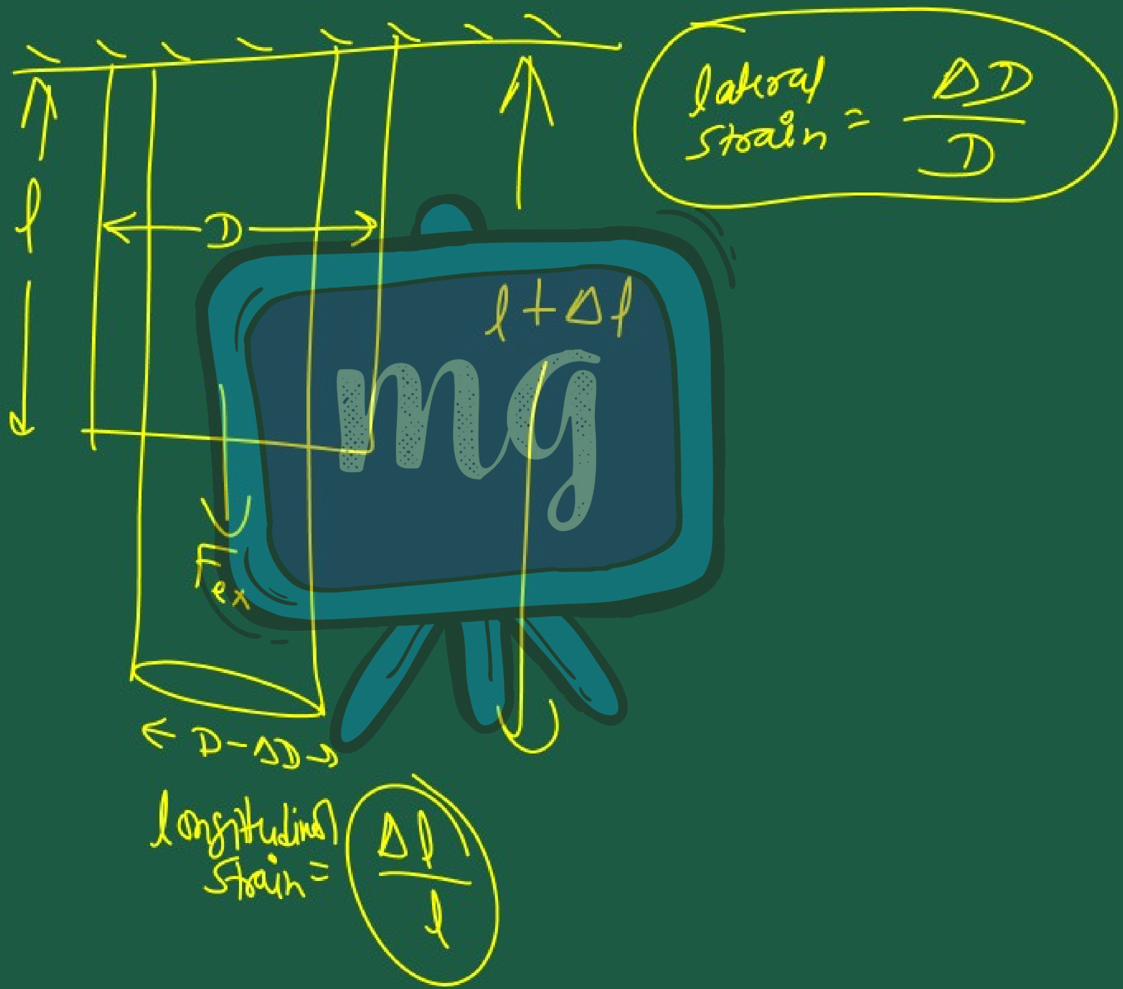
POISSON RATIO

- The ratio of lateral strain and longitudinal strain is called Poisson ratio.

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\sigma = \frac{-\frac{dr}{r}}{\frac{dL}{L}}$$




lateral strain = ΔD/D
l
D
l+Δl
F_{ex}
D−ΔD
longitudinal strain = Δl/l

- (–) sign shows that on stretching the wire it's radius decreases.
- Poisson ratio is unitless and dimensionless.

Relation between Y, K, η and σ

$$Y = 3K(1 - 2\sigma)$$

$$Y = 2\eta(1 + \sigma)$$

$$Y = \frac{9K\eta}{3K + \eta}$$

$$\sigma = \frac{3K - 2\eta}{6K + 2\eta}$$

- The value of Y is always more than η.
- In practice the value of σ for any material is less than 0.5.
- For all materials all elasticity constants Y, K, η and σ are positive.

LEARNING OUTCOMES

To study for “Elastic Moduli”

ASSESSMENT

1 The dimension of modulus of elasticity is of :

A Force

B Work

C Pressure

D Power

ASSESSMENT

2 | The Young's modulus of elasticity is equal to that stress numerically which :

A Can increase the length of wire by 25%

B Can increase the length of wire by 100%

C Can increase the length of wire by 50%

D Can increase the length of wire by 75%