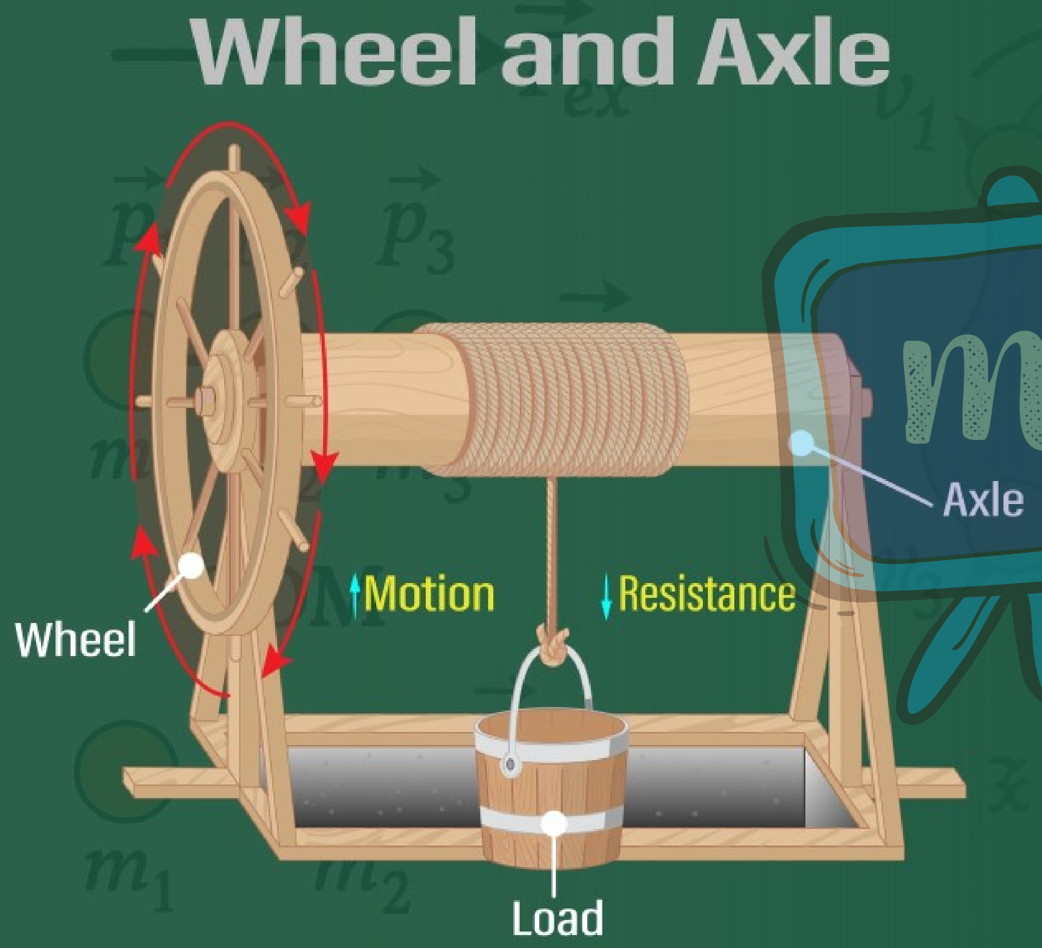


CLASS – 11

PHYSICS

Chapter – 6

Systems of Particles and Rotational Motion

Part – 7

Exercise (Questions 10 to 17)

Alok Gaur

EXERCISE

Q.10 Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time.

$\tau_{cylinder} = \tau_{sphere}$

$I_c \alpha_c = I_s \alpha_s$

$MR^2 \alpha_c = \frac{2}{5} MR^2 \alpha_s$

$\alpha_c = 0.4\, \alpha_s$

$\omega = \omega_0 + \alpha t$

$\omega = \alpha t$

$\therefore \alpha_s > \alpha_c$

$\omega_s > \omega_c$

EXERCISE

Q.11 A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

$m = 20$ kg

$\omega = 100$ rad/s

$R = 0.25$ m $= \frac{1}{4}$

K.E = ?

L = ?

$K.E = \frac{1}{2} I \omega^2$

$K.E = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2$

$K.E = \frac{1}{4} \times 20 \times \frac{1}{16} \times 100 \times 100$

$K.E = 5 \times 625$

$K.E = 3125$ J

$L = mvr = I\omega$

$L = \left(\frac{MR^2}{2}\right)\omega$

$L = \frac{20}{2} \times \frac{1}{16} \times 100$

$L = \frac{125}{2}$

$L = 62.5 \ kg\,m^2/s$

EXERCISE

Q.12 (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value ? Assume that the turntable rotates without friction.

$\omega_1 = \frac{40 \text{ rev}}{60 \text{ mnt}}$

$\omega_1 = 2\pi \times \frac{40}{60}$

L = Constant

$I\omega$

$I_1\omega_1 = I_2\omega_2$

$I_1 \times 2\pi \times \frac{40}{60} = \frac{2}{5} I_1 \omega_2$

$\frac{5}{2} \times 2\pi \times \frac{40}{60} = 2\pi \frac{\omega_2}{60}$

$\omega_2 = 100 \frac{\text{Rev}}{\text{mnt}}$

$\omega_2 = 2\pi \times \frac{100}{60} \text{ Rad/sec}$

EXERCISE

Q.12 (b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

$K.E_i = \frac{1}{2} I \omega^2 = \frac{1}{2} I (2\pi \times 40$

$K.E_f = \frac{1}{2} \left(\frac{2}{5} I\right) \omega^2$

$K.E_i = \frac{1}{2} I \omega^2$

$K.E_i = \frac{1}{2} I \left(2\pi \times \frac{40}{60}\right)^2$

$K_f E_f = \frac{1}{2}\left(\frac{2}{5} I\right)\left(2\pi \times \frac{100}{60}\right)^2$

$$\frac{K.E_i}{K.E_f} = \frac{4\pi^2 \times \left(\frac{40}{60}\right)^2}{\frac{2}{5} \times (2\pi)^2 \times \left(\frac{100}{60}\right)^2}$$

$$\frac{K.E_i}{K.E_f} = \frac{(40) \times (40) \times 5}{2 \times 100 \times 100}$$

$$\frac{K.E_i}{K.E_f} = 0.4$$

$K.E_i = 0.4\, K.E_f$

$K.E_f > K.E_i$

$\Delta K.E = K_f - K_i$

$\Delta K.E = 0.6\, K_f$

EXERCISE

Q.13 A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

$M = 3\ kg$

$r = 40\ cm = 40 \times 10^{-2}\ m$

$F = 30\ N$

$\alpha = ?$

$a = ?$

$\tau = I\alpha = F \times 2r$

$Mr^2 \times \alpha = F \times 2r$

$\alpha = \frac{2F}{Mr} = \frac{2 \times 30}{3 \times 0.4}$

$\alpha = \frac{20}{0.4} = 50\ Rad/s^2$

$a = r\alpha$

$a = 0.4 \times 50$

$= 20\ m/s^2$

EXERCISE

Q.14 To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 N m. What is the power required by the engine ? (Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

$\omega = 200$ Rad/s

$\tau = 180$ Nm

$P = ?$

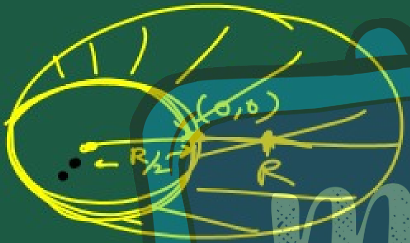
$P = \tau\omega$

$P = 180 \times 200$

$P = 36 \times 10^3$ Watt

EXERCISE

Q.15 From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.



$\pi R^2 \rightarrow M$

$1 \rightarrow \frac{M}{\pi R^2}$

$\pi\left(\frac{R}{2}\right)^2 \rightarrow \frac{M}{\pi R^2} \times \pi\left(\frac{R}{2}\right)^2$

$M' = \frac{M}{4}$

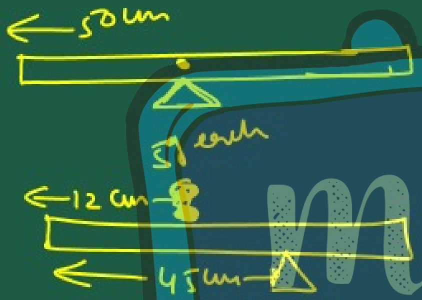
$M - \frac{M}{4} = \frac{3M}{4}$

$x_{cm} = \frac{M_1x_1 - M_2x_2}{M_1 - M_2}$

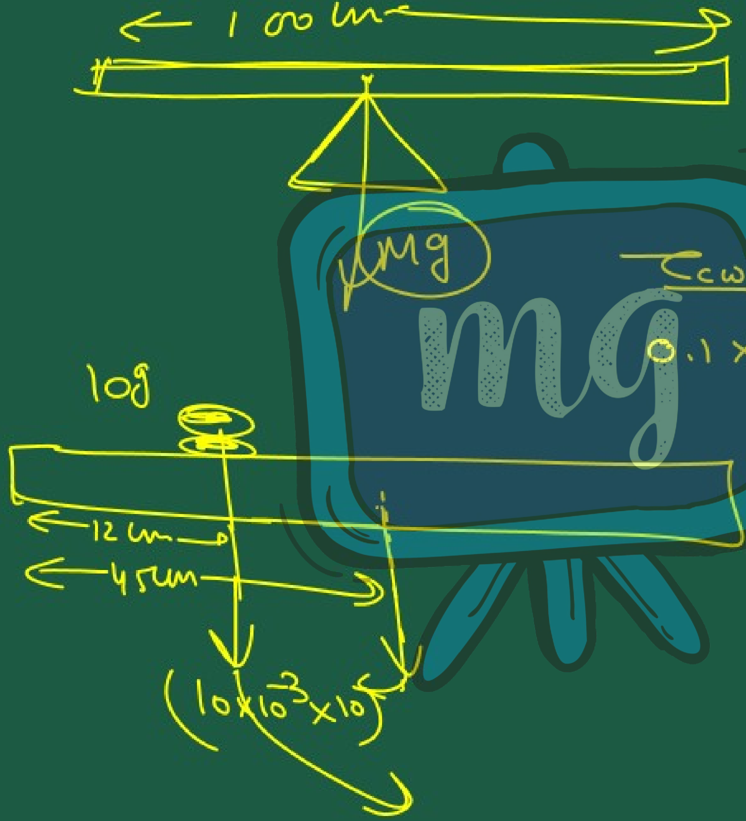
$x_{cm} = \frac{M(0) - \frac{M}{4}\left(\frac{R}{2}\right)}{M - \frac{M}{4}}$

$= \frac{0 + \frac{MR}{8}}{\frac{3M}{4}} = +\frac{R}{6}$

EXERCISE



Q.16 A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?



$$\tau_{cw} = \tau_{A.C.W}$$

$$0.1 \times 38 = M \times 10 \times 5$$

$$M = \frac{3.8}{50} \text{ kg}$$

EXERCISE

Q.17 The oxygen molecule has a mass of 5.30 × 10^{-26} kg and a moment of inertia of 1.94 × 10^{-46} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

O_2

$m = 5.3 \times 10^{-26}$ kg

$I = 1.94 \times 10^{-46}$ kg

$v = 500$ m/s

$K.E_r = \frac{2}{3} K.E_T$

$\omega = ?$

$I = mr^2$

$\frac{1}{2} I \omega^2 = \frac{2}{3} \times \frac{1}{2} m v^2$

$mr^2 \omega^2 = \frac{2}{3} m v^2$

$\omega^2 = \frac{2 v^2}{3 r^2}$

$$\omega^2 = \frac{1}{3}\frac{v^2}{r^2}$$

$$\omega = \frac{1}{\sqrt{3}}\frac{v}{r}$$

$$\omega = \frac{1}{\sqrt{3}}\frac{500}{\times 0.6 \times 10^{-10}}$$

$$\omega = \frac{500 \times 10^{10}}{0.6\sqrt{3}}$$

$$I = mr^2$$

$$r = \sqrt{\frac{I}{m}}$$

$$r = \sqrt{\frac{1.94 \times 10^{-46}}{5.3 \times 10^{-26}}}$$

$$r = \sqrt{0.36 \times 10^{-20}}$$

$$r = 0.6 \times 10^{-10}\ m$$