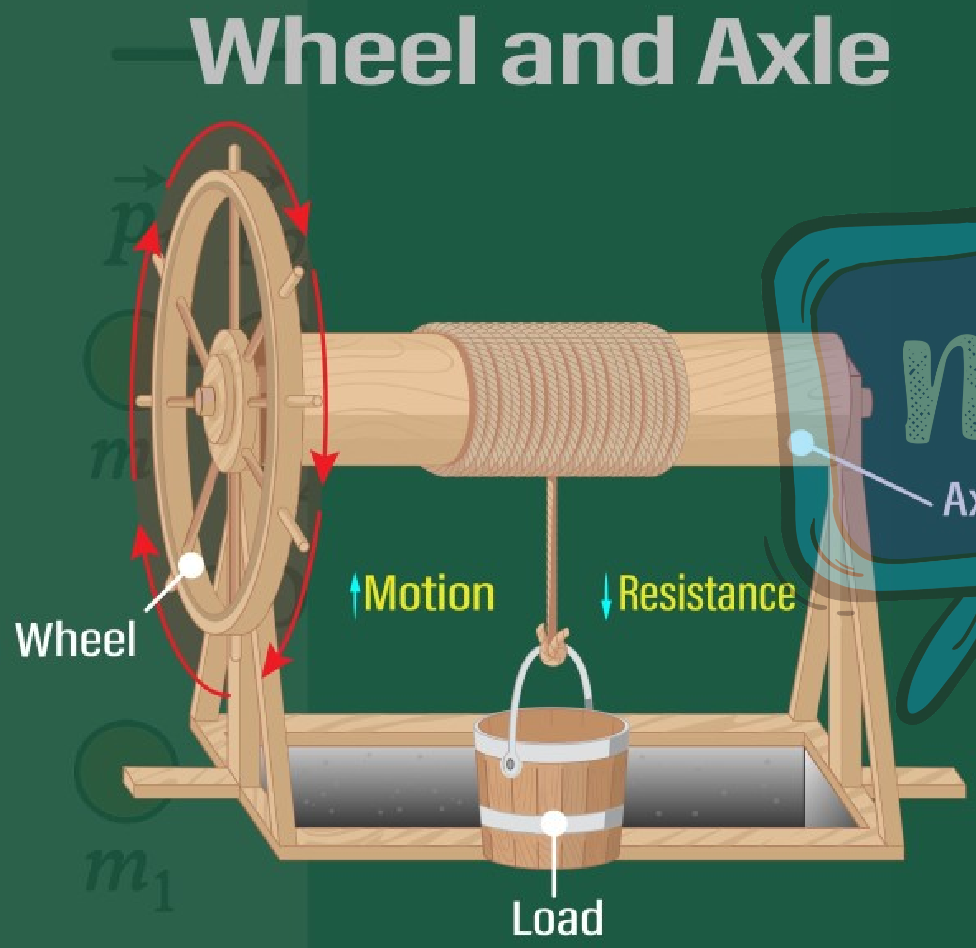
Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER



CLASS – 11

PHYSICS

Chapter – 6

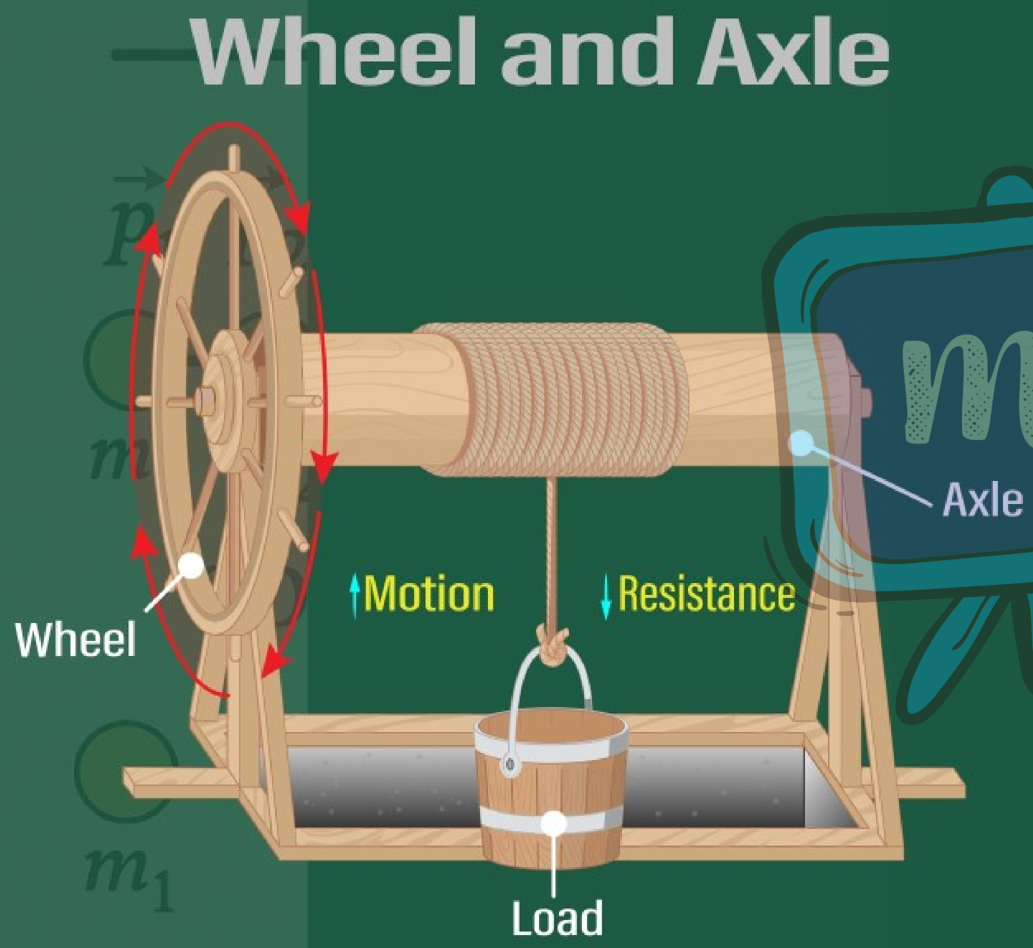
Systems of Particles and Rotational Motion

Part – 4

Moment of Inertia

Alok Gaur

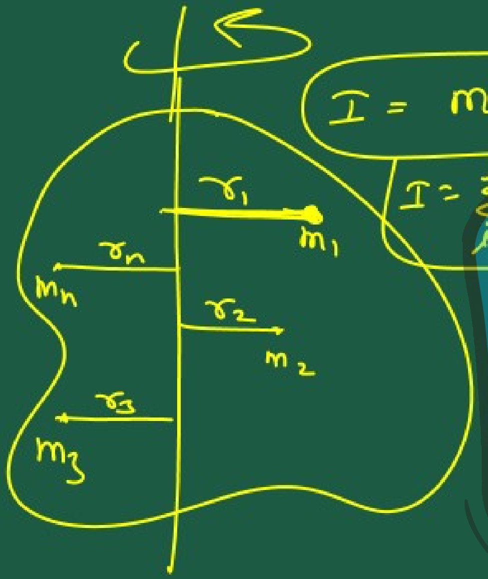
OVERVIEW



1. Centre of Mass
2. Motion of Centre of Mass and Vector Product
3. Torque and Angular momentum
4. Moment of Inertia
5. Law of conservation of angular momentum

Moment of Inertia

- The moment of Inertia (I) is a physical quantity that measures how difficult it is to change the rotational motion of an object.
- It is defined as the property of a body that opposes angular motion.



☛ Mathematically for a point mass

$$I = m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2$$

$$I = \sum_{i=1}^{i=n} m_i r_i^2$$

$$I = mr^2$$

kg-m²

Where,

m = the mass of the to point

r = the distance from ex the axis of rotation.

Tensor

Unit : kg . metre^2

Dimension : $[m^1L^2T^0]$

Dependence of moment of Inertia

(i) On the mass of the body.

(ii) On the position of rotational axis.

(iii) On the mass distribution about the rotational axis.

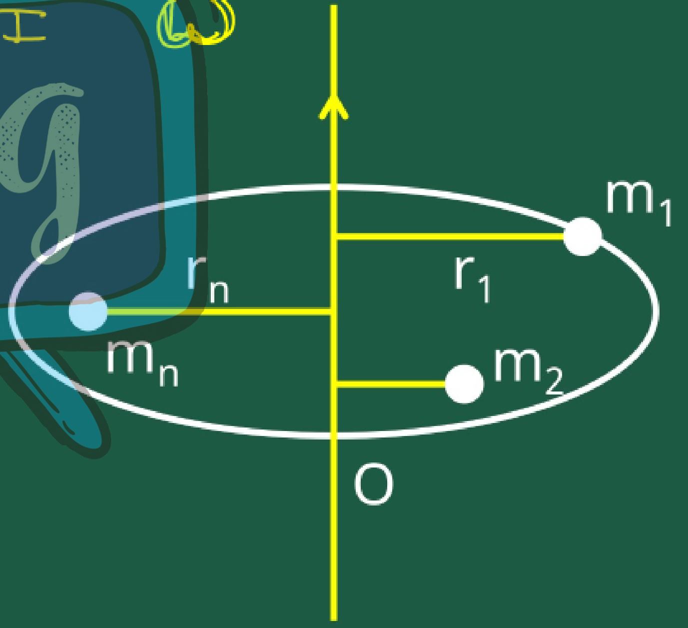
Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER

Physical Significance of M.O.I.

- The importance that mass has in linear motion is the same importance that moment of Inertia has in rotational motion.
- MOI is very important in our daily life, to increase the moment of inertia in the wheels of motor car, scooter, rickshaw-cycle and toys etc.

Relation between torque, MOI and angular momentum



Torque (τ), MOI (I), Angular acceleration (α)

$\tau = F \times \perp$ dist.

$\tau = F r$

$\tau = m a r$

$\tau = m(r\alpha) r$

$\tau = mr^2 \alpha$

$$\tau = I\alpha$$

$F = mg$

- Linear acceleration of the particle of mass m_I.
- $a_1 = r_1 \alpha$
- $F_1 = m_1 \, a_1$
- $F_1 = m_1, r_1 \, \alpha$

Torque :

- $\tau_1 = F_1 \times r_1$
- $\tau_1 = (m_1 r_1 \alpha) \times r_1$
- $\tau_1 = (m_1 r_1^2 \alpha)$

Similarly, for other particles

$$\tau_2 = (m_1 r_2^2 \alpha)$$

$$\tau_3 = (m_1 r_3^2 \alpha)$$

$$\tau_n = (m_n r_n^2 \alpha)$$

Reasultant Torque :

- $\tau = \tau_1 + \tau_2 + \tau_3 + \ldots\ldots\ldots + \tau_n$
- $\tau = m_1 r_1^2 \alpha + m_1 r_2^2 \alpha + \ldots\ldots\ldots + m_n r_n^2 \alpha$
- $\tau = \alpha(m_1 r_1^2 + m_1 r_2^2 \ldots\ldots + m_n r_n^2)$
- $\tau = \alpha I$
- $\tau = I\alpha$

Angular Momentum (L), MoI (I), A. velocity (ω)

$L = rp = mvr$

$L = m(r\omega)r$

$L = \boxed{mr^2}\omega$

$\boxed{L = I\omega}$

$P = mv$

K.E. & M.O.I

$K.E = \frac{1}{2}mv^2$

$K.E = \frac{1}{2}m(r\omega)^2$

$= \frac{1}{2}\boxed{mr^2}\omega^2$

$\boxed{K.E = \frac{1}{2}I\omega^2}$

Relation between Angular Momentum, MOI and Angular Velocity

- Linear velocity of the particle of mass m_1 is

$$v_1 = r_1, \omega$$

- Linear momentum = $p_1 = m_1, v_1 = m, r, \omega$
- Angular momentum = $L_1 = r_1 p_1 = r_1 m_1 r_1 \omega$

- $L_1 = m_1 r_1^2 \omega$
- Similarly for other particles

$L_2 = m_2 r_2^2 \omega$

………………

………………

………………

$L_n = m_n r_n^2 \omega$

Resultant angular momentum

$$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + \ldots\ldots + m_n r_n^2 \omega$$

$$L = \omega(m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2)$$

$$L = \omega I$$

$$L = I \omega$$

Rotational Kinetic Energy

- Kinetic energy of the particle of mass m_1

$$E_1 = \frac{1}{2} m_1 r_1^2$$

$$= \frac{1}{2} m_1 (r_1 \omega)^2$$

$$= \frac{1}{2} m_1 r_1^2 \omega$$

Under CSR Support by ipca Foundation

missiongyan LET'S LEARN TOGETHER

- Similarly, for other particles

$$E_2 = \frac{1}{2} m_2 r_2^2 \omega$$

....

....

$$E_n = \frac{1}{2} m_n r_n^2 \omega$$

- Total Kinetic Energy —

$$E = E_1 + E_2 + \dots\dots\dots\dots E_n$$

$$E = \frac{1}{2}m_1r_1^2\omega + \frac{1}{2}m_1r_2^2\omega + \dots + \frac{1}{2}m_nr_n^2\omega$$

$$E = \left(\frac{1}{2}(m_1r_1^2 + \frac{1}{2}m_1r_2^2 + \dots\dots + \frac{1}{2}m_nr_n^2\right)\omega$$

$$E = \frac{1}{2}I\,\omega^2$$

- If ω = 1 rad/sec

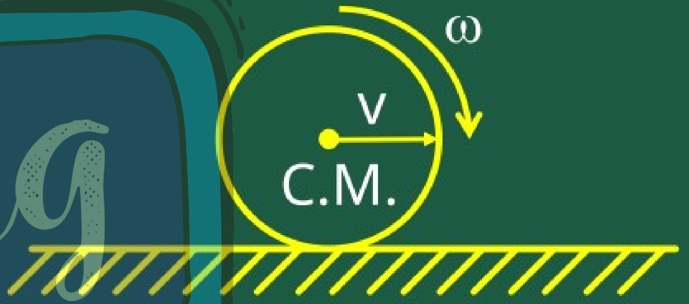
$$I = 2E$$

Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER

SPECIAL

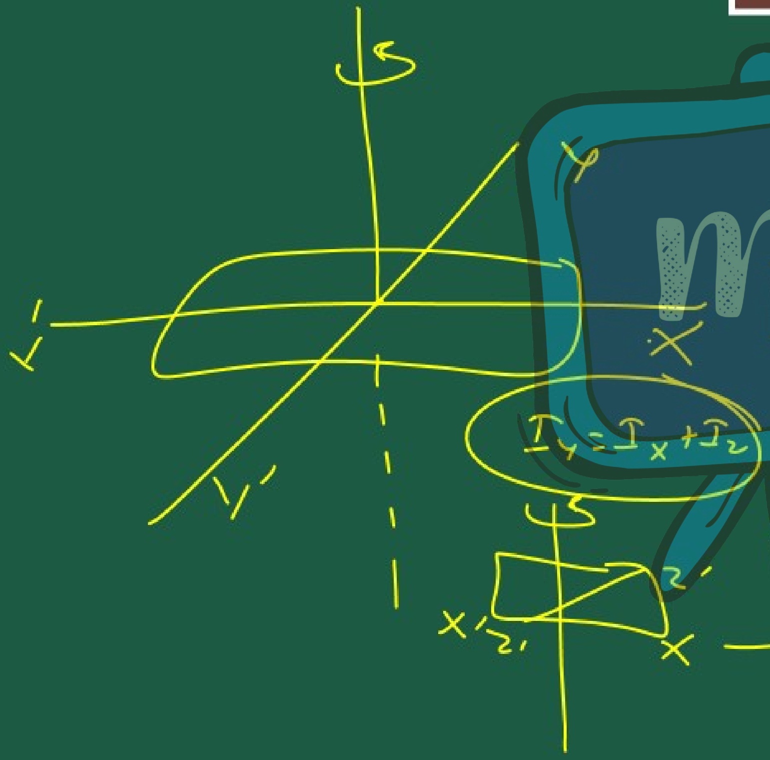
- Rolling motion of the body.



$$K.E. = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Theorems of MOI

Theorems of MOI



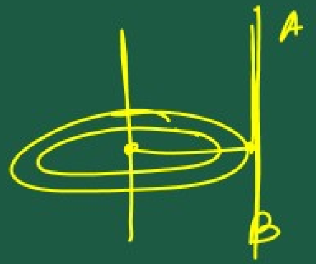
Theorem of perpendicular axes

$$I_z = I_x + I_y$$

Theorem of Parallel axes

$$I = I_G + md^2$$

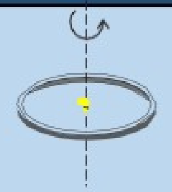
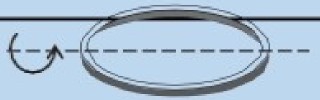
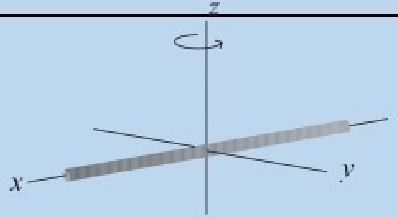
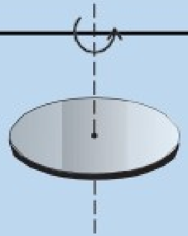
Moments of inertia of some regular Shaped bodies about specific axes

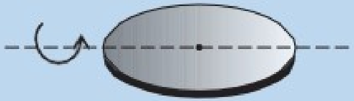
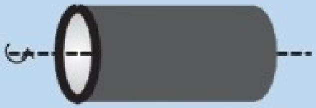
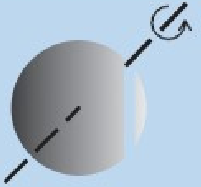


$I_{AB} = I_{cm} + Md^2$

$= MR^2 + MR^2$

$= 2MR^2$

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre		MR^2
(2)	Thin circular ring, radius R	Diameter		$MR^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$ML^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre		$MR^2/2$

Z	Body	Axis	Figure	I
(5)	Circular disc, radius R	Diameter		$M R^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder		$M R^2$
(7)	Solid cylinder, radius R	Axis of cylinder		$M R^2/2$
(8)	Solid sphere, radius R	Diameter		$2 M R^2/5$

LEARNING OUTCOME

1. To study for moment of Inertia
2. To derive for mulda — $L = I\omega$; $t = I\alpha$; $ER = \frac{1}{2} I \omega^2$

ASSESSMENT

1 Linear velocities of all particles in rotational motion —

- A are zero
- B are different
- C are equal
- D nothing can be said

ASSESSMENT

2 wrong formula is —

A $I = \frac{\tau}{\alpha}$

B $I = \frac{L}{\omega}$

C $I = \frac{E}{2\omega^2}$

D $I = \sum mr^2$