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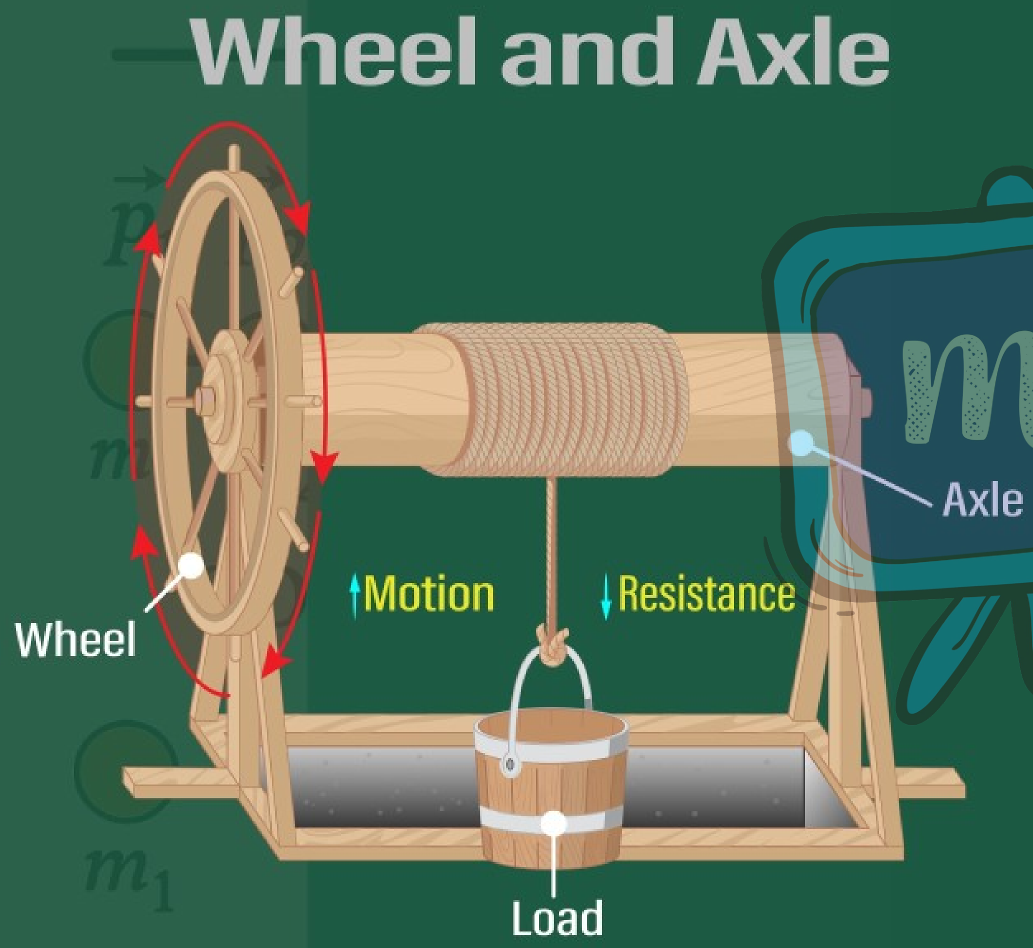

CLASS – 11

PHYSICS

Chapter – 6

Systems of Particles and Rotational Motion

Part – 2

Motion of Centre of Mass and Vector Product

Alok Gaur

OVERVIEW

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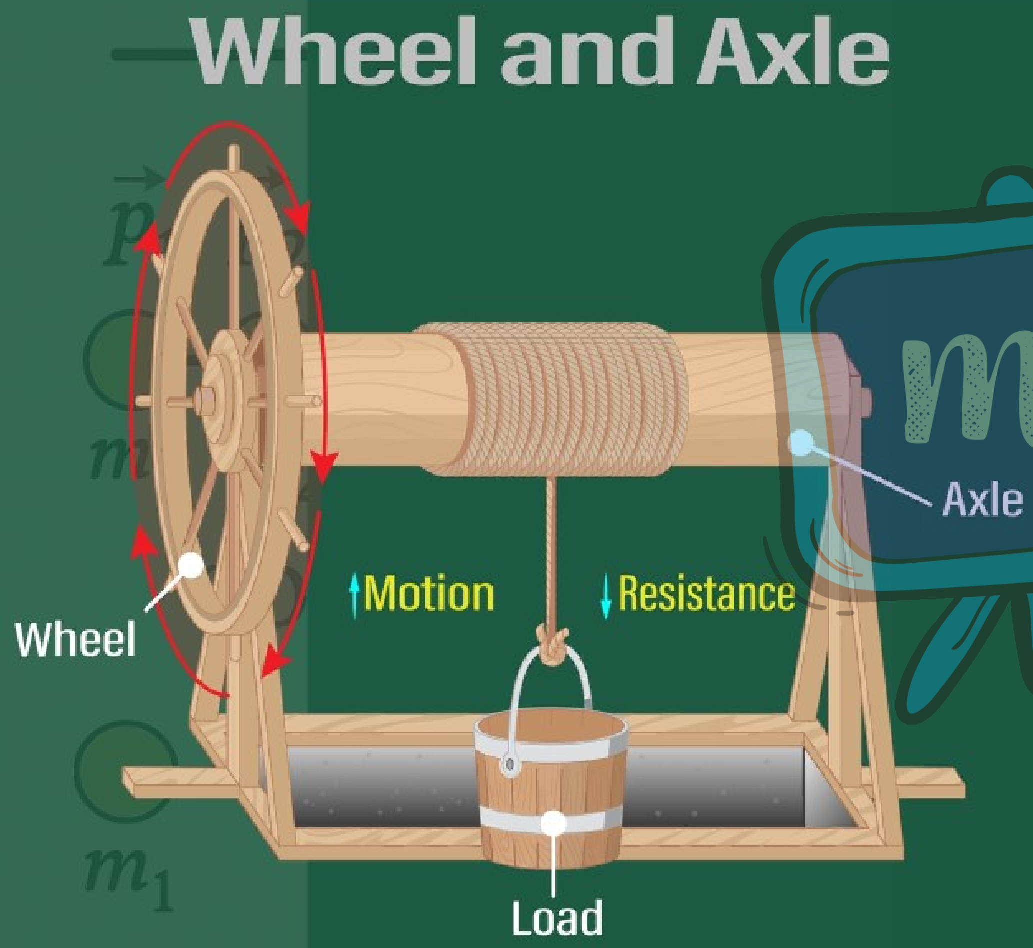


1. Centre of Mass
2. Motion of Centre of Mass and Vector Product
3. Torque and Angular momentum
4. Moment of Inertia
5. Law of conservation of angular momentum

MOTION OF CENTRE OF MASS

$$\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots.. + m_n\vec{r}_n}{m_1 + m_2 + \ldots.. + m_n}$$

$$m = m_1 + m_2 + \ldots\ldots. + m_n$$

$$\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots.. + m_n\vec{r}_n}{m}$$

$$m\vec{R}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots.. + m_n\vec{r}_n$$

Differentiating with respect to 't'

$$m\frac{d}{dt}\vec{R}_{cm} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \ldots\ldots + m_n\frac{d\vec{r}_n}{dt}$$

$$\because \vec{v} = \frac{d\vec{r}}{dt}$$

$$m\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n \qquad \ldots\ldots (i)$$

Here, $\vec{V}_{cm}$ is the velocity of centre of mass and $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots\ldots + \vec{v}_n$ are respectively velocities of particles.

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$\frac{dr_{cm}}{dt} = \frac{1}{m_1+m_2}\left[\frac{d}{dt}(m_1 r_1) + \frac{d}{dt}(m_2 r_2)\right]$$

$$\vec{V}_{cm} = \frac{1}{m_1+m_2}\left[m_1\vec{V}_1 + m_2\vec{V}_2\right]$$

$$\vec{V}_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1+m_2}$$

$$V_{cm} = \frac{\sum_{i=1}^{i=n} m_i V_i}{\sum_{i=1}^{i=n} m_i}$$

Differentiating equation (i) is with respect to 't'

$$m\frac{d\vec{v}_{cm}}{dt} = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + \ldots.. + m_n\frac{d\vec{v}_n}{dt}$$

$$m\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots.. + m_n\vec{a}_n \quad \ldots. \text{(ii)}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + - - - - + m_n\vec{a}_n}{m_1 + m_2 + - - - + m_n}$$

From Newton's second Law,

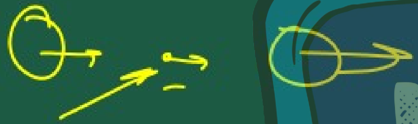
$\vec{F} = m\vec{a}$

$m\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots + \vec{F}_n$ .... (iii)

Where, $\vec{F}_1, \vec{F}_2, \ldots\ldots, \vec{F}_n$ are external forces acting on particles of masses m_1, m_2, ... m_n respectively.

From equation (iii),

$m\vec{a}_{cm} = \vec{F}_{ext}$ .... (iv)



- Equation(iv) shows that centre of mass moves in such a way that the whole mass of the system is concentrated on centre of mass and the all external forces act on this point.

Linear Momentum of a System of Particles

- Momentum of the system

$$\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n$$

$$\vec{p} = m\vec{v}_{cm}$$

- Total linear momentum of the system is equal to the product of it's total mass and velocity of it's centre of mass.

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$\vec{V}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}_{cm}}{dt}$$

$$\frac{d\vec{p}}{dt} = m\vec{a}_{cm}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{ext}$$

$$\vec{F}_{ext} = m\vec{a}_{cm}$$

☛ If $\vec{F}_{ext} = 0$

$$\frac{d\vec{p}}{dt} = 0$$

$\vec{p}$ = constant

$\vec{p} = m\vec{v}_{cm}$

$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}_{cm}}{dt}$

$\vec{F}_{ext} = m\vec{a}_{cm}$

$m\vec{a}_{cm} = 0$

$\vec{a}_{cm} = 0$

$\frac{d\vec{p}}{dt} = 0$

$\vec{P}$ = Const

- So, when sum of external forces applied on a system is zero, then total total linear momentum of that system remains constant. It is called law of conservation of linear momentum of Centre of mass of the system.

$$\frac{d\vec{p}}{dt} = 0$$

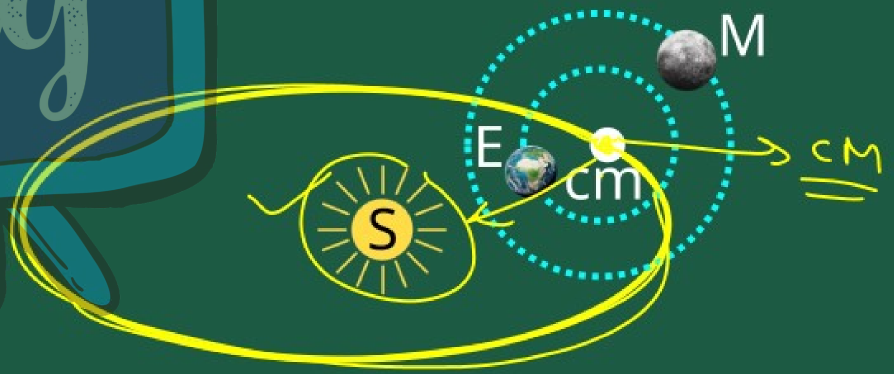
$$\frac{d}{dt}(m\vec{v}_{cm}) = 0$$

$$\vec{v}_{cm} = \text{constant}$$

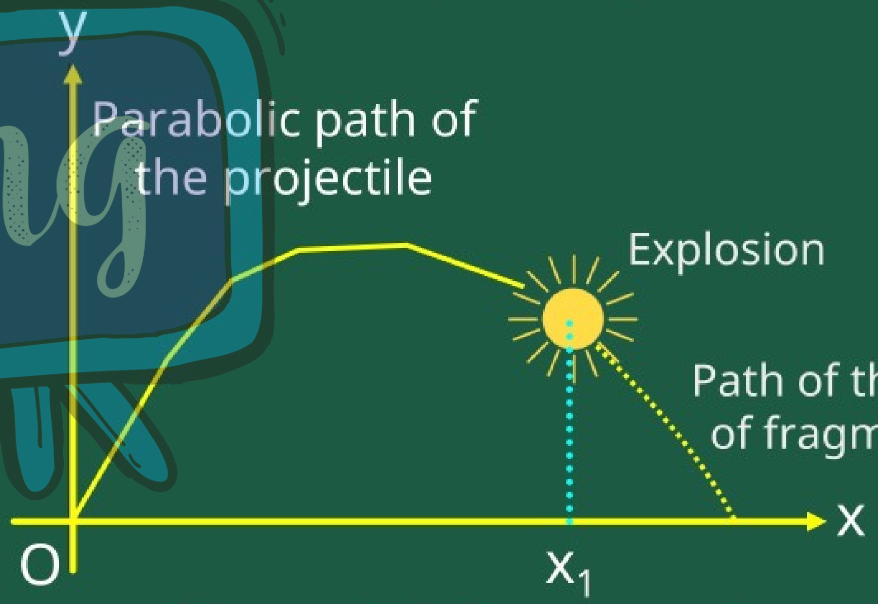
- When the sum of external forces applied on a system is zero, then velocity of that system remains constant.

Examples of the Motion of Centre of Mass

1. Earth-Moon System

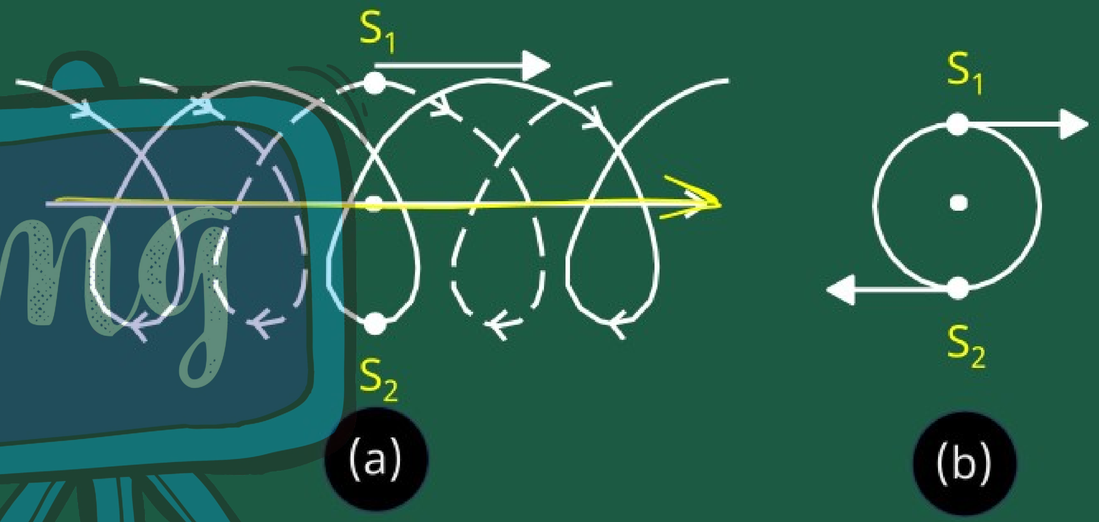


2. Motion of the centre of mass of firecrackers exploding in air



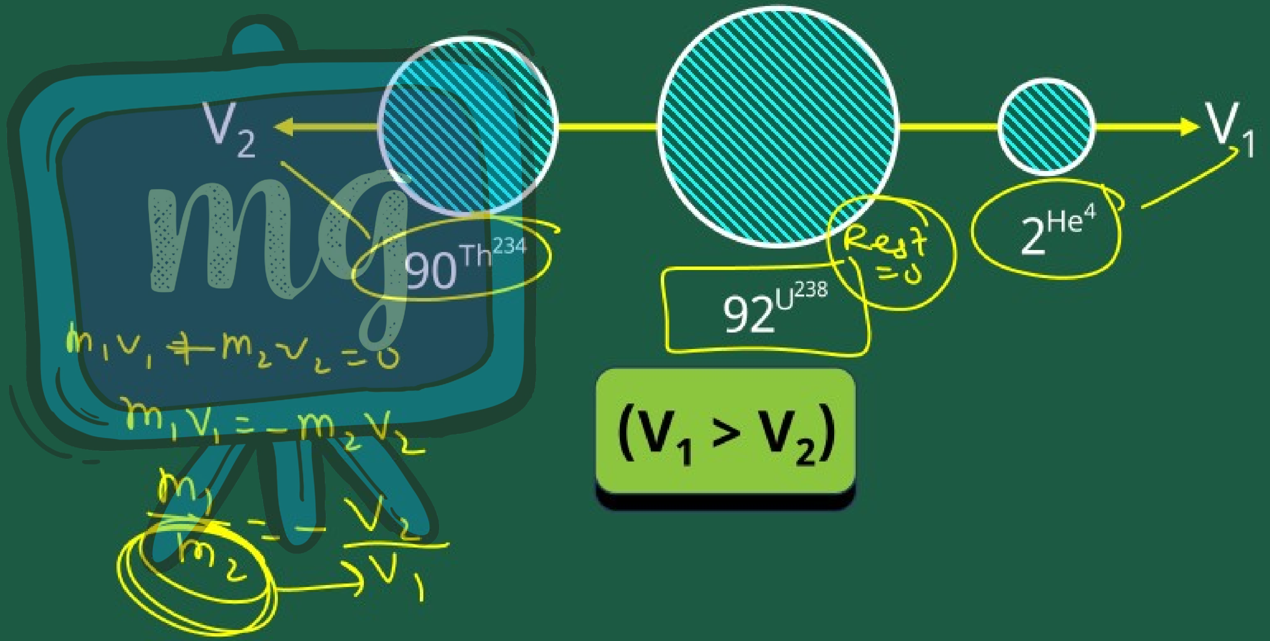
The centre of mass of the fragments of the projectile continues along the same parabolic path which it would have followed if there were no explosion.

3. Binary Stars



(a) Trajectories of two stars, S_1 (dotted line) and S_2 (solid line) forming a binary system with their centre of mass C in uniform motion.
(b) The same binary system, with the centre of mass C at rest.

4. Radioactive decay



$(V_1 > V_2)$

VECTOR PRODUCT

$\vec{A} \times \vec{B} = \vec{C}$

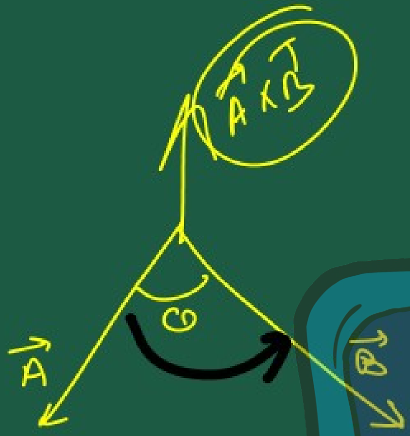
$\vec{A} \cdot \vec{B} = AB\cos\theta$

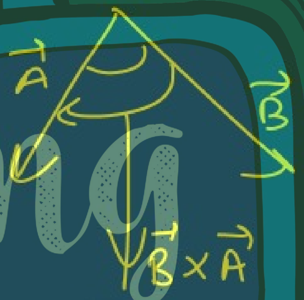
$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

Cross product

- The vector product of two vector quantities is equal to the product of magnitudes of those both vectors and sine of angle between them and the direction is perpendicular to the plane of both vectors.

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$





$$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$$

$$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

Right Hand Rule

Examples of Vector Product

(i) Torque = $\vec{\tau} = \vec{r} \times \vec{F}$

(ii) Lorentz force = $\vec{F} = Q(\vec{v} \times \vec{B})$

(iii) Angular Momentum = $\vec{J} = \vec{r} \times \vec{p}$

Determination of direction of vector product of two vectors

(i) Right Handed Screw Law :

- According to this law when a right handed screw, whose axis is perpendicular to the plane framed by $\vec{A} \times \vec{B}$ is rotated towards $\vec{A} \times \vec{B}$ then direction of its advancement gives the direction of $\vec{A} \times \vec{B}$.

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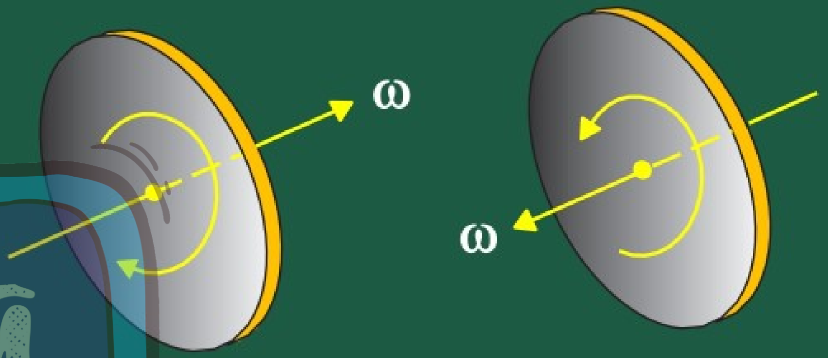


If the head of a right handed screw rotates with the body, the screw advances in the direction of the angular velocity ω.
If the sense (clockwise or anticlockwise) of rotation of the body changes, so does the direction of ω.

(ii) Right hand Law :

- Aceording to this law of the fingers of right hand are turned towards $\vec{A}$ to $\vec{B}$ then the stretched thumb. gives the direction of $\vec{A} \times \vec{B}$.

Properties of Vector Product

(i) Commutative Law :

$$\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$$

(ii) Distributive Law :

$$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

(iii) Vector Product of Parallel Vectors :

$\theta = 0°$ $\vec{A} \times \vec{B} = AB \sin 0° = 0$

Vector product of equivalent vectors

$\theta = 0°$ $\vec{A} \times \vec{A} = 0$

$\vec{B} \times \vec{B} = 0$

Vector product of perpendicular vectors

$\theta = 90°$ $\vec{A} \times \vec{B} = AB \sin 90° = AB(\hat{n})$

$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

$\hat{i} \times \hat{i} = (1)(1)\sin 0^\circ = 0$

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

If $\vec{A} \parallel \vec{B}$ $\theta = 0^\circ$

$\vec{A} \times \vec{B} = 0$

$\hat{i} \times \hat{j}$

$\hat{j} \times \hat{i}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

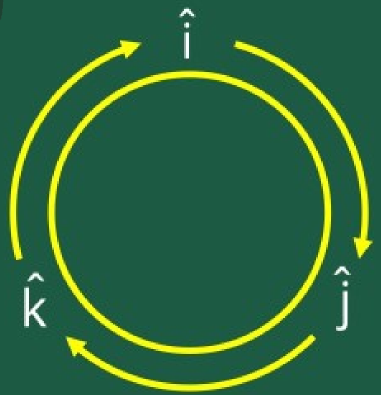
(iv) Vector Product of Unit Vectors

$$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$$

$$\hat{i}\times\hat{j}=\hat{k} \qquad \hat{j}\times\hat{i}=-\hat{k}$$

$$\hat{j}\times\hat{k}=\hat{i} \qquad \hat{k}\times\hat{j}=-\hat{i}$$

$$\hat{k}\times\hat{i}=\hat{j} \qquad \hat{i}\times\hat{k}=-\hat{j}$$



(v) Vector product in the form of components

$$\vec{A} = Ax\hat{i} + Ay\hat{j} + Az\hat{k}$$

$$\vec{B} = Bx\hat{i} + By\hat{j} + Bz\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}(AyBz - AzBy) - (AxBz - AzBx) + (AxBy - AyBx)$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}\begin{bmatrix} A_y & A_z \\ B_y & B_z \end{bmatrix} - \hat{j}\begin{bmatrix} A_x & A_z \\ B_x & B_z \end{bmatrix} + \hat{k}\begin{bmatrix} A_x & A_y \\ B_x & B_y \end{bmatrix}$$

$$\vec{A}\times\vec{B} = \hat{i}\left[A_yB_z - B_yA_z\right] - \hat{j}\left[A_xB_z - A_zB_x\right] + \hat{k}\left[A_xB_y - B_xA_y\right]$$

Example : Find the scalar and vector products of two vectors.

$\vec{A} = (3\hat{i} - 4\hat{j} + 5\hat{k})$ and $\vec{B} = (-2\hat{i} + \hat{j} - 3\hat{k})$

Answer : $\vec{A} \cdot \vec{B} = -6 - 4 - 15 = -25$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix} = 7\hat{i} - \hat{j} - 5\hat{k}$$

$$\hat{i}[12-5] - \hat{j}[-9+10] + \hat{k}[3-8] = 7\hat{i} - \hat{j} - 5\hat{k}$$

LEARNING OUTCOME

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1. To study for motion of the centre of mass
2. To study for vector product

ASSESSMENT

1 An asteroid enters the atmosphere of the earth and breaks into two pieces. one of the pieces is larger than the other. which of the following is true considering the centre of mass the pieces together? of both

A The centre of mass shifts horizontally towards the larger piece.

B The centre of mass shifts horizontally towards the smaller piece.

C The centre of mass remains in the same trajectory as before breaking apart of the asteroid.

D Depends on the velocity of the asteroid.

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ASSESSMENT

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2 Consider a system on which these are external forces acting. If the vector sum of all these external forces is zero, then the centre of mass ....

A must not have

B may move

C may accelerate

D may accelerate

ASSESSMENT

3 Two bodies of masses 5kg and 3 kg are moving towards each other at velocities of 3m/s and 5m/s respectively. what is the velocity of their centre of mass ?

A 0.25m/s towards 3 kg

B Upredictable

C The centre of mass is stationary

D 0.5 m/s towards skg

+3 → ← −5 m/s

$$\frac{5\times3 + 3\times-5}{5+3} = 0$$