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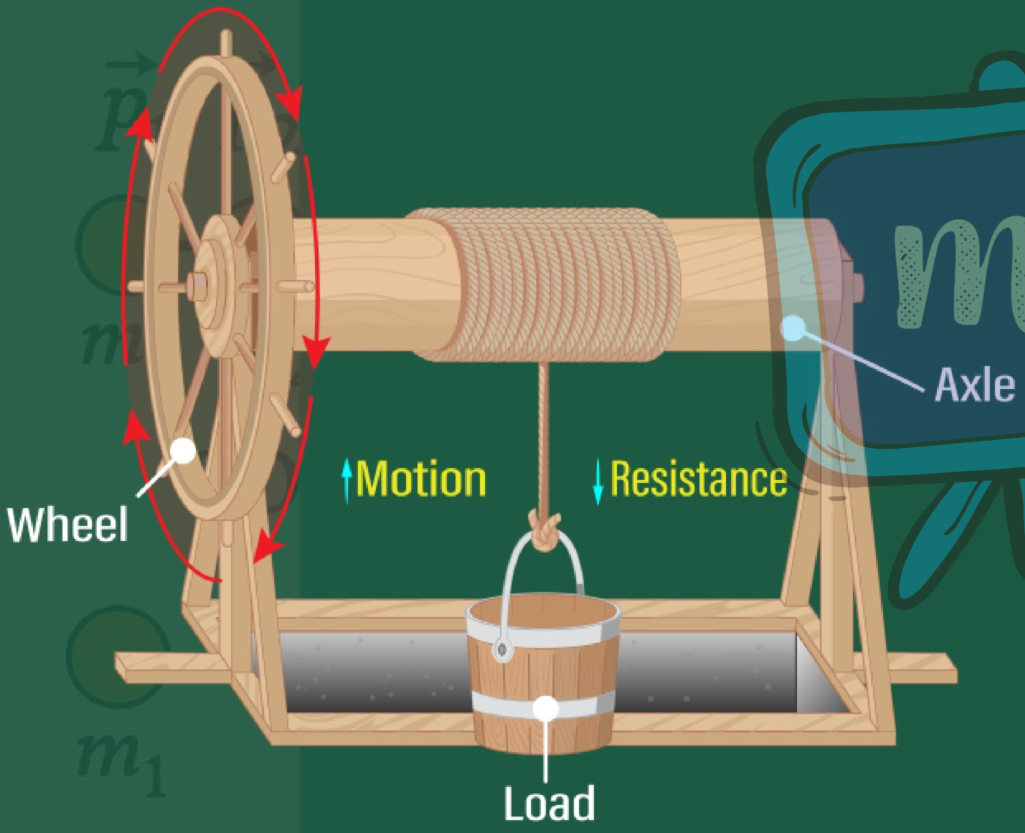
Wheel and Axle



CLASS – 11

PHYSICS

Chapter – 6

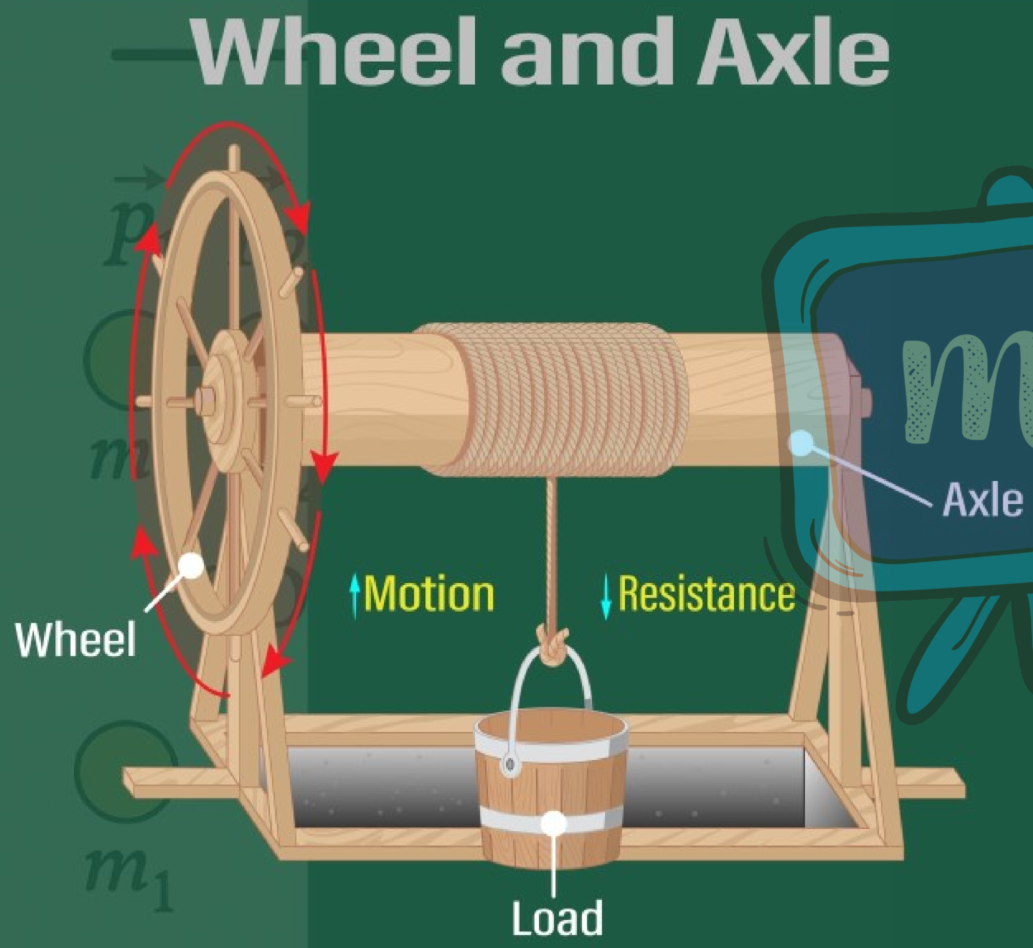
Systems of Particles and Rotational Motion

Part – 1

Centre of Mass

Alok Gaur

OVERVIEW



1. Centre of Mass
2. Motion of Centre of Mass and Vector Product
3. Torque and Angular momentum
4. Moment of Inertia
5. Law of conservation of angular momentum

RIGID BODY

- A rigid body is defined as a solid object in which the distance between any two particles remains constant, regardless of the external forces applied to it.

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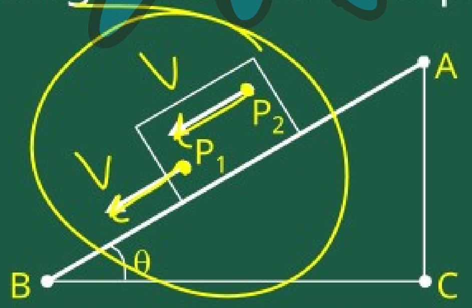
Motion of Rigid Body

Translatory Motion

In which each particle of the body moves with uniform velocity at a particular instant.

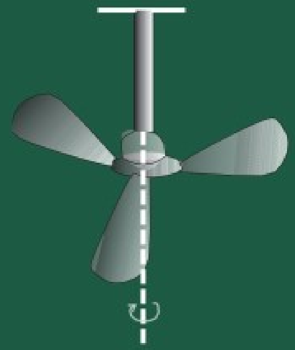
Ex.: Motion of a block sliding on an inclined plane.



Rotational Motion

In which a rigid body rotates around a fixed axis is called rotational motion.

Ex.: Motion of a ceiling fan.

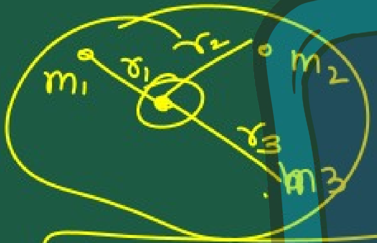


Rolling Motion =

Translatory motion + Rotational motion

CENTRE OF MASS

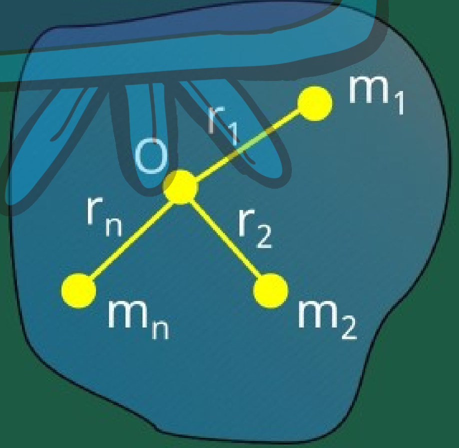
- That point in a body or system of particles at which whole mass of body or system is supposed to be concentrated is called Centre of Mass of the body or system.



$$m_1 r_1 + m_2 r_2 + \dots + m_n r_n = 0$$

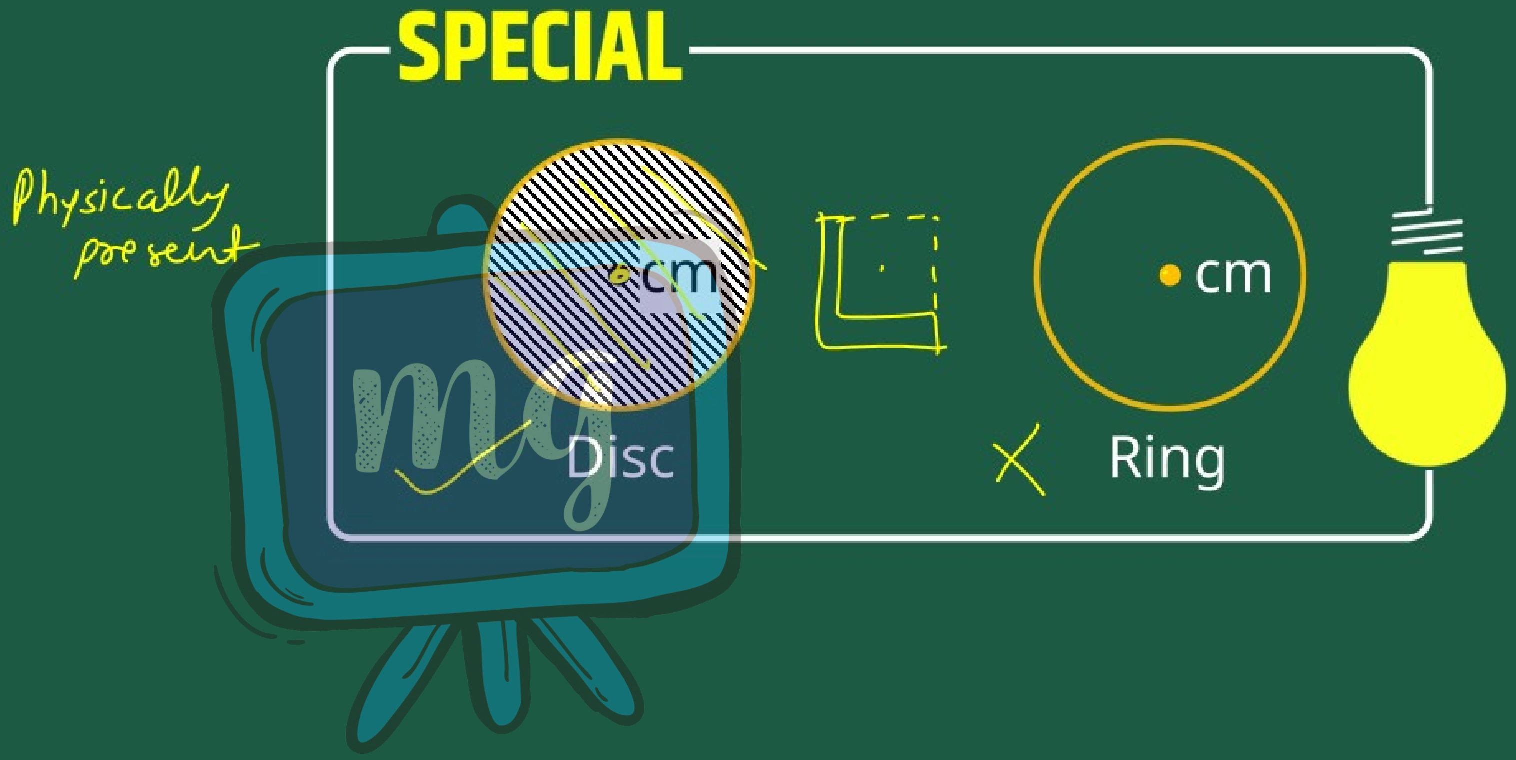
$$\sum_{i=1}^{i=n} m_i r_i = 0$$

- The Centre of is a mass of a body or system point, at which the vector sum of the moments of the masses of all particles of the body or system is always zero.



$$m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots + m_n\vec{r}_n = 0$$

$$\sum_{i=1}^{n} m_i\vec{r}_i = 0$$


SPECIAL
Physically present
cm
Disc
cm
Ring

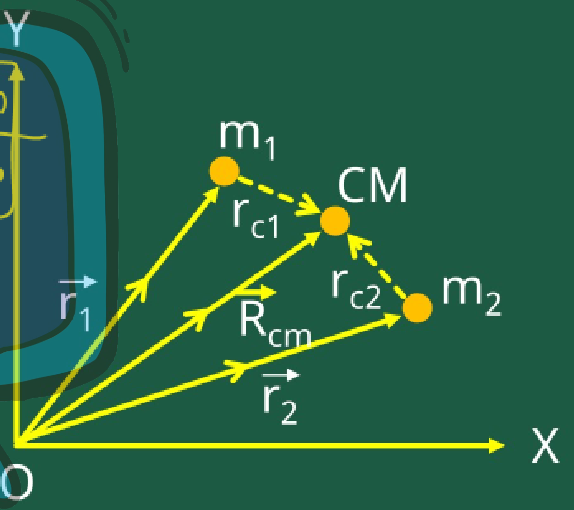
Centre of Mass of a Two Particle System

$m_1(x_1, y_1)$ ——— (x, y) ——— $m_2(x_2, y_2)$

$$x = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + \cdots + m_n}$$

$$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

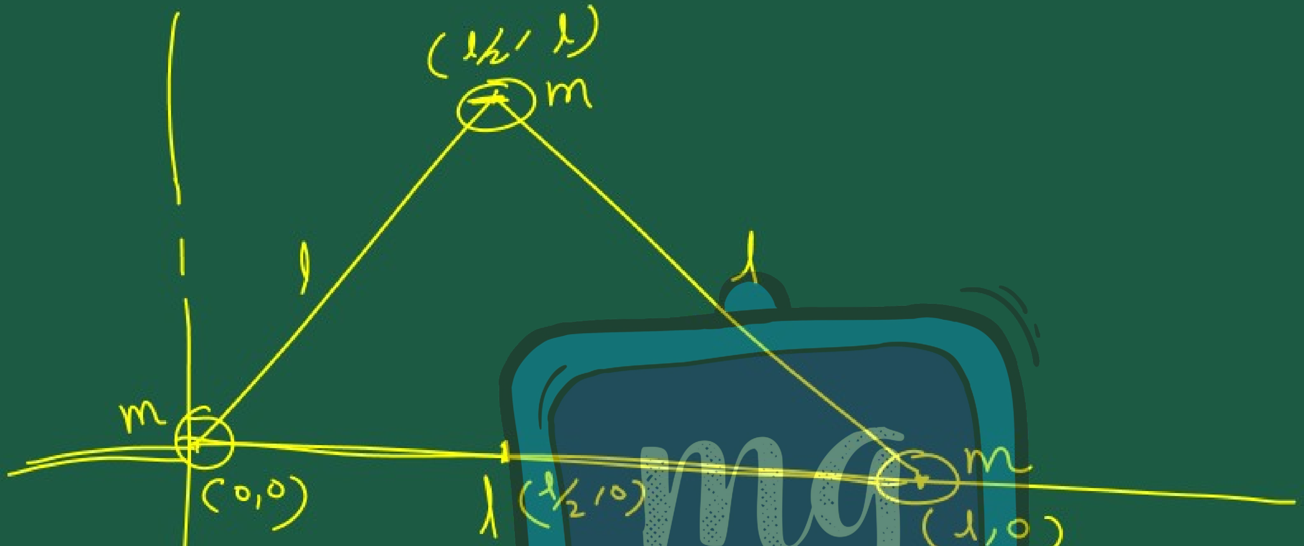
$$* \quad \vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1 + m_2 + \cdots + m_n}$$



$$\vec{r}_1 + \vec{r}_{c1} = \vec{R}_{cm}$$

$$r_{c1} = R_{cm} - r_1$$

Similarly, $\vec{r}_{c2} = \vec{R}_{cm} + \vec{r}_2$



$$x_{cm} = \frac{m(0) + m(l) + m(l/2)}{3m} = \frac{3m\,l/2}{3m}$$

$$x_{cm} = l/2$$

$$y_{cm} = \frac{0 + m(0) + m(l)}{3m} = \frac{ml}{3m}$$

$$y_{cm} = l/3$$

$$\left(l/2,\ l/3\right)$$

From moments of masses

$$m_1(\vec{R}_{cm} - \vec{r}_1) + m_2(\vec{R}_{cm} - \vec{r}_2) = 0$$

$$R_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

X-axis, $$\vec{X}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1 + m_2}$$

Y-axis, $$\vec{Y}_{cm} = \frac{m_1\vec{Y}_1 + m_2\vec{Y}_2}{m_1 + m_2}$$

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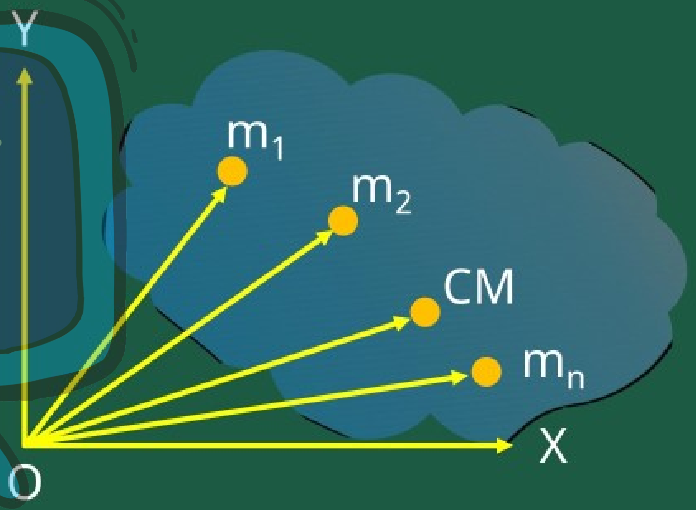
Centre of mass of a system of n particles



$$\vec{r}_1 + \vec{r}_{c1} = \vec{R}_{cm}$$

$$r_{c1} = R_{cm} - r_1$$

Similarly, $\vec{r}_{c2} = \vec{R}_{cm} + \vec{r}_2$

From Moments of Masses —

$$m_1(\vec{R}_{cm} - \vec{r}_1) + m_2(\vec{R}_{cm} - \vec{r}_2) + ... + m_n(\vec{R}_{cm} - \vec{r}_n) = 0$$

$$\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

X-axis, $$\vec{X}_{cm} = \frac{m_1\vec{X}_1 + m_2\vec{X}_2 + ... + m_n\vec{X}_n}{m_1 + m_2 + ... + m_n}$$

Y-axis, $$\vec{Y}_{cm} = \frac{m_1\vec{Y}_1 + m_2\vec{Y}_2 + ... + m_n\vec{Y}_n}{m_1 + m_2 + ... + m_n}$$

Z-axis, $$\vec{Z}_{cm} = \frac{m_1\vec{Z}_1 + m_2\vec{Z}_2 + ... + m_n\vec{Z}_n}{m_1 + m_2 + ... + m_n}$$

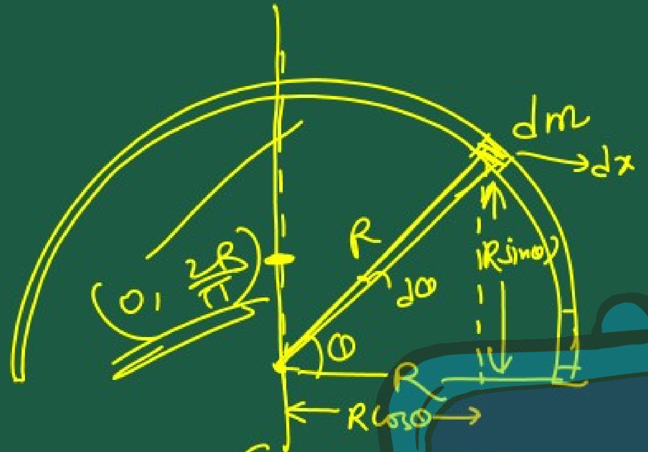
- If the mass of any such point mass Particle be dm and its position vector $\vec{r}$, then

$$R_{cm} = \frac{1}{m}\int \vec{r}\,dm$$

$$X_{cm} = \frac{1}{m}\int X\,dm$$

$$Y_{cm} = \frac{1}{m}\int Y\,dm$$

$$Z_{cm} = \frac{1}{m}\int Z\,dm$$



$\pi R \rightarrow M$

$1 \rightarrow \dfrac{M}{\pi R}$

$dx \rightarrow \dfrac{M}{\pi R}\, dx$

$dm = \dfrac{M}{\pi R}\, dx$

$$x_{cm} = \frac{\int x\, dm}{M}$$

$dx = R\, d\theta$

$$x_{cm} = \frac{\int R\cos\theta \, \frac{M}{\pi R}\, dx}{M}$$

$$x_{cm} = \int_0^{\pi} \frac{\cos\theta \, R\, d\theta}{\pi}$$

$$= \frac{R}{\pi}\left[\sin\theta\right]_0^{\pi}$$

$$x_{cm} = \frac{R}{\pi}\left[\sin\pi - \sin 0\right]$$

$$x_{cm} = 0$$

$$y_{cm} = \frac{\int y\, dm}{M}$$

$$= \frac{\int_0^{\pi} R\sin\theta \, \frac{M}{\pi R}\, R\, d\theta}{M}$$

$$= \frac{R}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

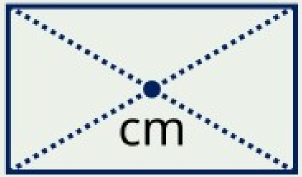
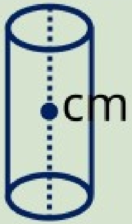
$$= -\frac{R}{\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{R}{\pi}\left(-1 - 1\right)$$

$$y_{cm} = \frac{2R}{\pi}$$

$$\left(0, \frac{2R}{\pi}\right)$$

S.No.	Symmetrical Object	Figure	Position of Centre of Mass
1.	Uniform thin Rod		Mid point of the Rod
2.	Circular Ring		Centre of the Ring
3.	Circular Disc		Centre of the Disc

S.No.	Symmetrical Object	Figure	Position of Centre of Mass
4.	Sphere		Centre of the Sphere
5.	Rectangular Plane		Interesection Point of Diagonals
6.	Cylinder		Mid point of the axis of the cylinder

Centre of Mass of a Thin Rod

- Let the mass of a thin uniform rod is m and it's length is L, the rod is placed along X-axis in such a way that its left end is on the origin point O.
- Now we consider a small segment of the rod of thickness dx at a distance x from O.

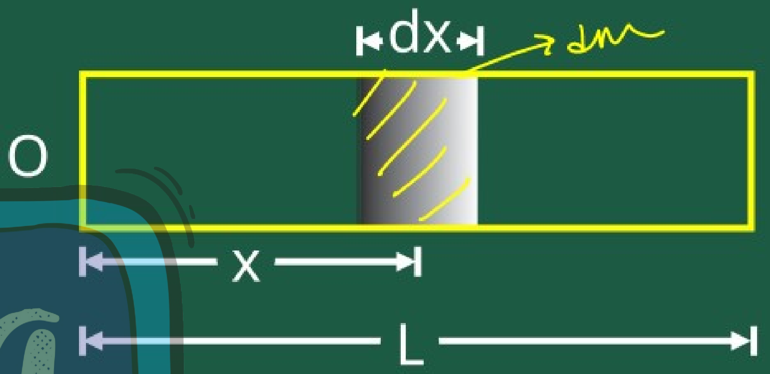
$L \rightarrow M$

$1 \rightarrow \frac{M}{L}$

$dx \rightarrow \frac{M}{L}dx$

$dm = \frac{M}{L}dx$

$$x_{cm} = \frac{\int_0^L x \frac{M}{L} dx}{M} = \frac{1}{L}\left(\frac{x^2}{2}\right)_0^L = \frac{L^2}{2L} = \frac{L}{2}$$



- Mass of unit Length of the rod = $\frac{m}{L}$
- mass of the segment dm = $\frac{m}{L}dx$

Position of Centre of Mass —

$$X_{cm} = \frac{1}{m}\int_{0}^{L} x\,dm$$

$$X_{cm} = \frac{1}{m}\int_{0}^{L} x\frac{m}{L}dx$$

$$X_{cm} = \frac{1}{L}\int_{0}^{L} x\,dx$$

$$X_{cm} = \frac{1}{L}\left[\frac{x^2}{2}\right]_{0}^{L}$$

$$X_{cm} = \frac{1}{L}\left[\frac{L^2}{2} - 0\right]$$

$$X_{cm} = \frac{L}{2}$$

EXAMPLE

1. Find the Centre of mass of a triangular lamina.

LEARNING OUTCOMES

To study for "Centre of Mass"

ASSESSMENT

1 The Centre of mass of a body —

A Lies inside the body

B Lies outside the body always

C Lies on the surface of the body always

D None of the option

ASSESSMENT

2 The combination of rotational Motion and the translation motion of a rigid body is Known as —

A Frictional Motion

B Axis Motion

C Angular Motion

D Rolling Motion