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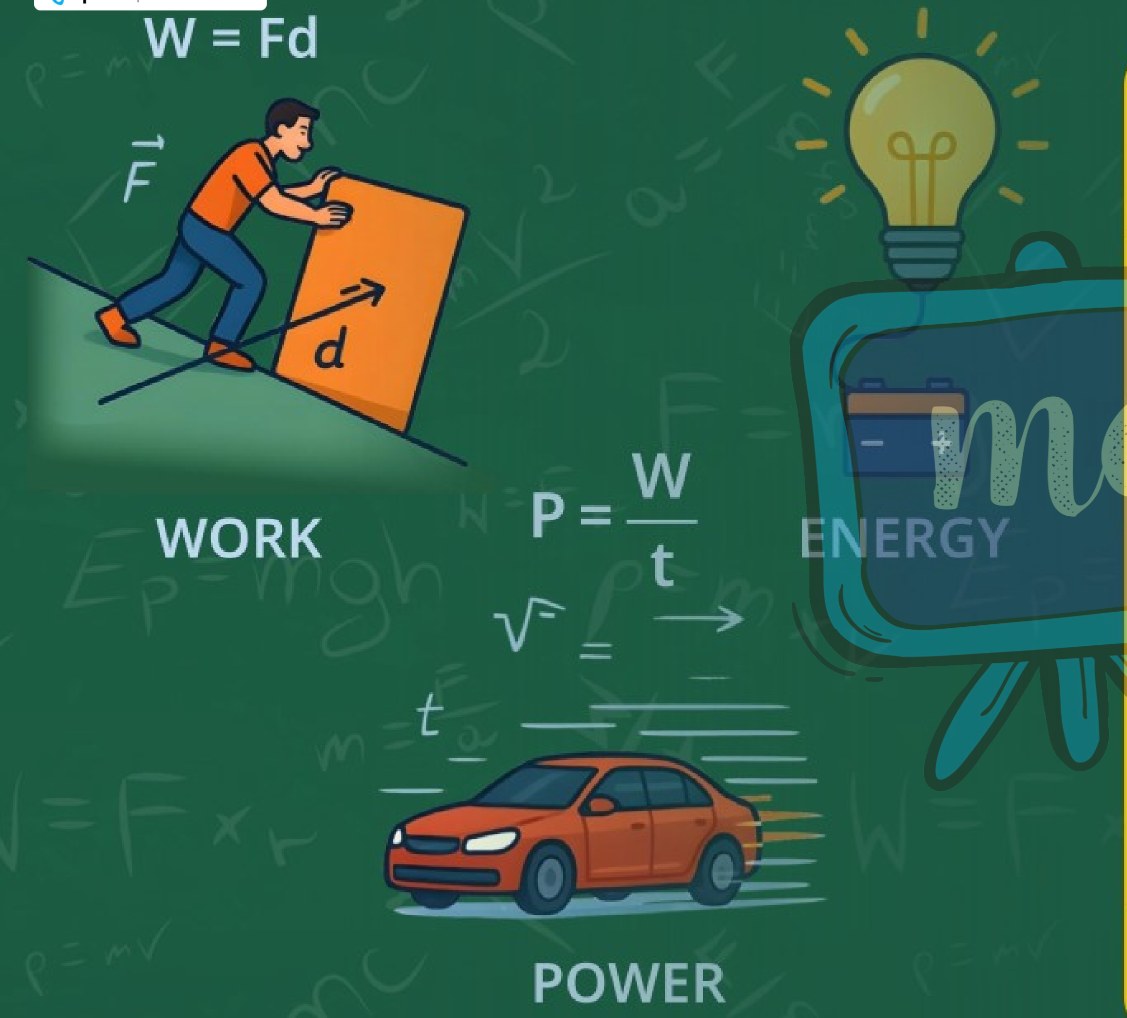


CLASS – 11

PHYSICS

Chapter – 5

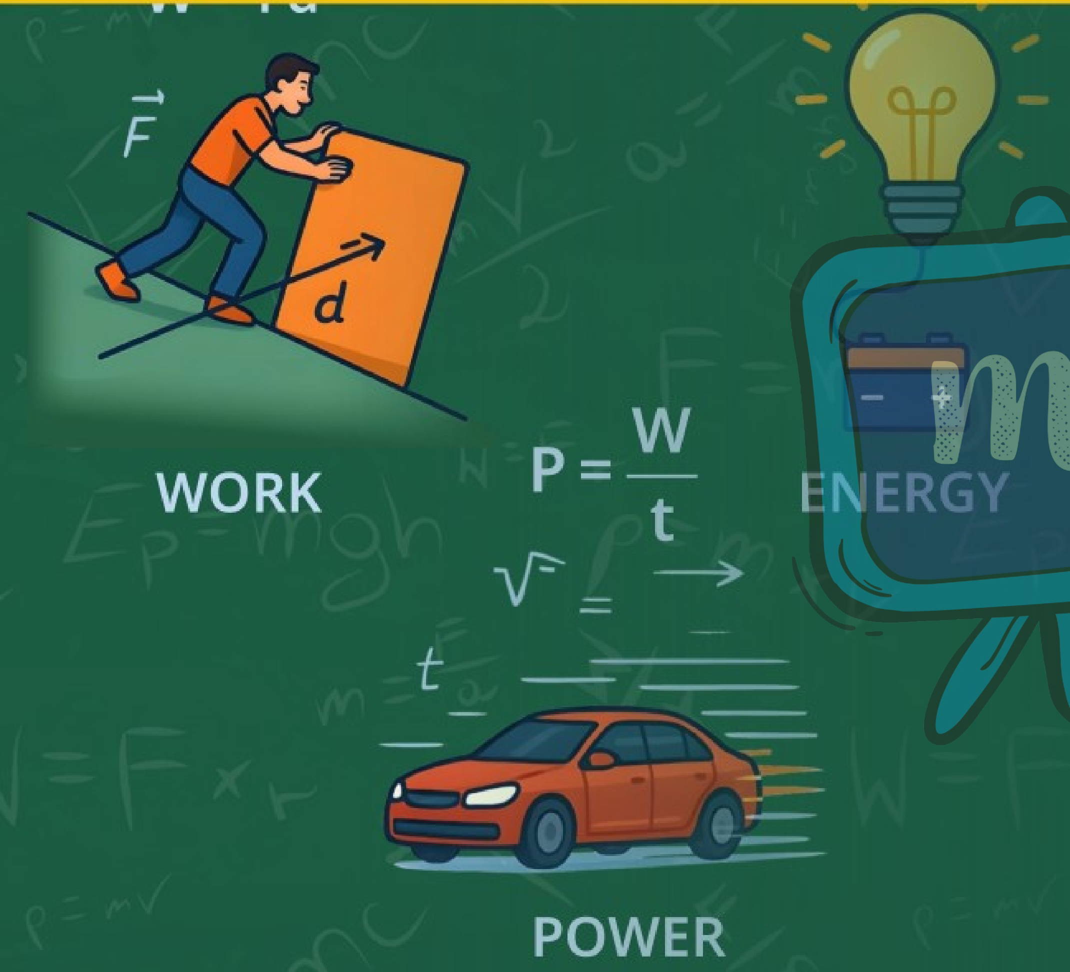
Work, Energy and Power

Part – 1

Work

Alok Gaur

OVERVIEW



1. Work
2. Energy
3. Law of Conservation of Mechanical Energy and Power
4. Collision

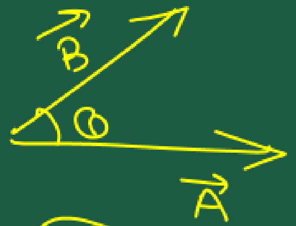
SCALAR PRODUCT

- Scalar product of two vectors is equal to the product of magnitudes of both vectors. It is called also dot product.

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

DOT Product

$\vec{A}\cdot\vec{B} = C \rightarrow$ scalar



$\vec{A}\cdot\vec{B} = AB\cos\theta$

Magnitude of $\vec{A}$

Angle b/w $\vec{A}$ & $\vec{B}$

Magnitude of $\vec{B}$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\hat{i} \cdot \hat{i} = (1)(1)\cos 0^\circ$

$\boxed{\hat{i} \cdot \hat{i} = 1}$

$\boxed{\hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1}$

$\hat{i} \cdot \hat{j} = (1)(1)\cos 90^\circ = 0$

$\hat{i} \cdot \hat{j} = 0$

$\boxed{\hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0}$

If $\vec{A} \perp \vec{B}$

$\boxed{\vec{A} \cdot \vec{B} = 0}$

$\vec{A} = \boxed{A_x \hat{i}} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

Ex.

$\vec{A} = \boxed{2}\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{B} = \boxed{3}\hat{i} - \hat{j} + 4\hat{k}$

$\vec{A} \cdot \vec{B} = 6 - 2 + 12$

$= 16$

Example of scalar product :

i. $W = \vec{F} \cdot \vec{S}$

ii. $P = \frac{\vec{F} \cdot \vec{S}}{t} = \vec{F} \cdot \vec{v}$

iii. $\phi = \vec{E} \cdot \vec{A}$ Electric flux

iv. $\phi_m = \vec{B} \cdot \vec{A}$ magnetic flux

Characteristics of Scalar Product

1. Commutative Law

$$\vec{A}\cdot\vec{B} = AB\cos\theta = BA\cos\theta$$

$$\therefore \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

2. Distributive Law

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

3. Scalar product for

- parallel vectors $\theta = 0°$ $\vec{A} || \vec{B}$

$$\vec{A}\cdot\vec{B} = AB\cos 0° = AB$$

- equivalent vectors $\theta = 0°$ $\vec{A} = \vec{B}$

$$\vec{A}\cdot\vec{A} = AA\cos 0° = A^2$$

$$\vec{B}\cdot\vec{B} = BB\cos 0° = B^2$$

- perpendicular vectors $\theta = 90°$ $\vec{A} \perp \vec{B}$

$\vec{A} \perp \vec{B}$

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos 90° = 0}$$

4. scalar product form

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$\vec{A} = 2\hat{i} + \hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\theta = ?$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

$= \dfrac{2+2+12}{\sqrt{(2)^2+(1)^2+(4)^2}\,\sqrt{(1)^2+(2)^2+(3)^2}}$

$\cos\theta = \dfrac{16}{\sqrt{21}\sqrt{14}}$

$\theta = \cos^{-1}\dfrac{16}{\sqrt{21}\sqrt{14}} = \cos^{-1}\dfrac{16}{7\sqrt{6}}$

SPECIAL

$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta = AB\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

WORK

- Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of force.

Result

Effort

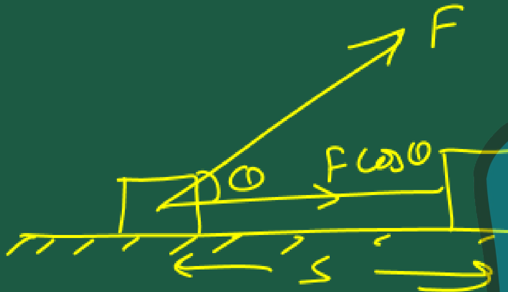
$W = 0$

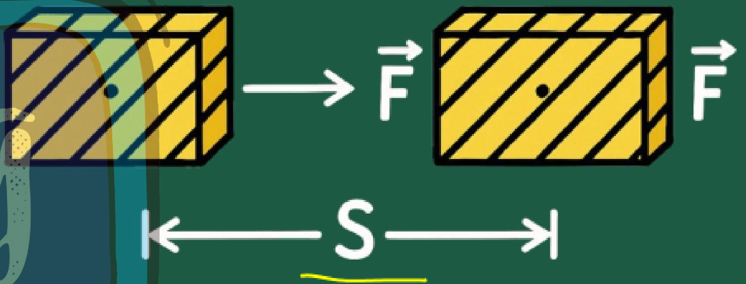
$x = 0$

$W = 0$

A horse pulls a cart.

A man goes up a hill.

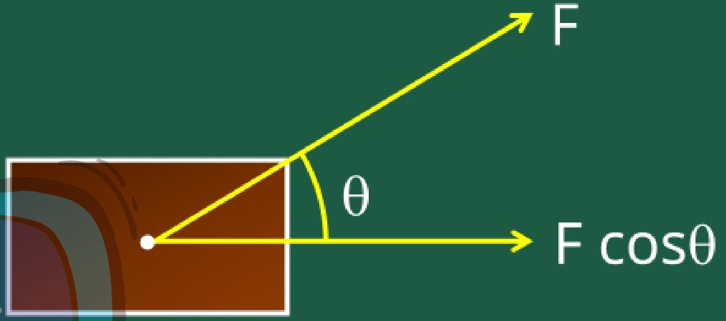




$$W = (F\cos\theta)\,s$$
$$= F\,s\cos\theta$$
$$* \boxed{W = \vec{F}\cdot\vec{s}}$$

"Work done = Force × Displacement"

$$\boxed{W = F\,S}$$



$$W = F\cos\theta \times S$$

$$W = FS\cos\theta$$

$$\boxed{W = \vec{F}\cdot\vec{S}}$$

- Work is a scalar quantity.

Nature of Work done in Different Situations

$\boxed{W = FS\cos\theta}$

$\cos\theta = +ve$

I III

$\theta < 90^\circ$

$W = +ve$

A. Positive work : If a force acting on a body has a component in the direction of the displacement then the work done by the force is positive.

$W = FS\cos\theta$

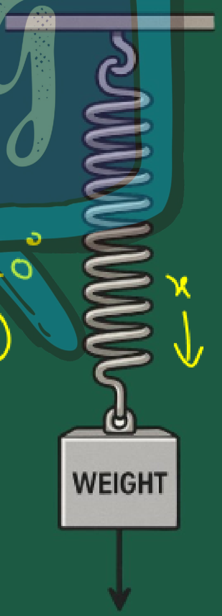
$\theta = 0^\circ$

$W = FS$

Example :

i. When a body falls freely under gravity.

ii. Stretching a spring



$W = mgx$

$\theta = 0^\circ$

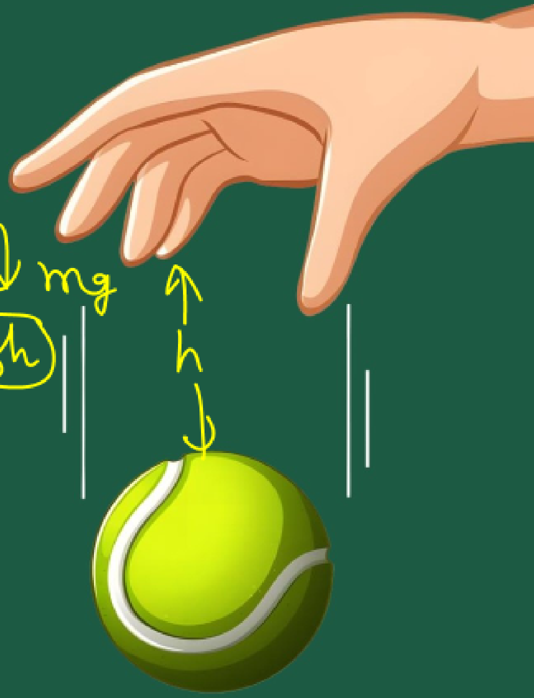
x

$\downarrow mg$

$\boxed{mgh}$

$\downarrow mg$

h

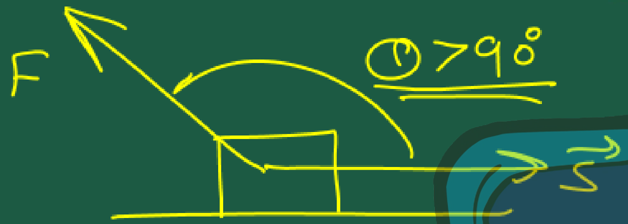


B. Negative work : If a force acting on the body has a component in the positive direction of displacement, the work done is negative.

$W = FS\cos\theta$

$\theta = \pi$

$$W = -FS$$



$W = -ve$

$\cos\theta = -ve$

$W = F s \cos\theta$

$\boxed{W = -ve}$

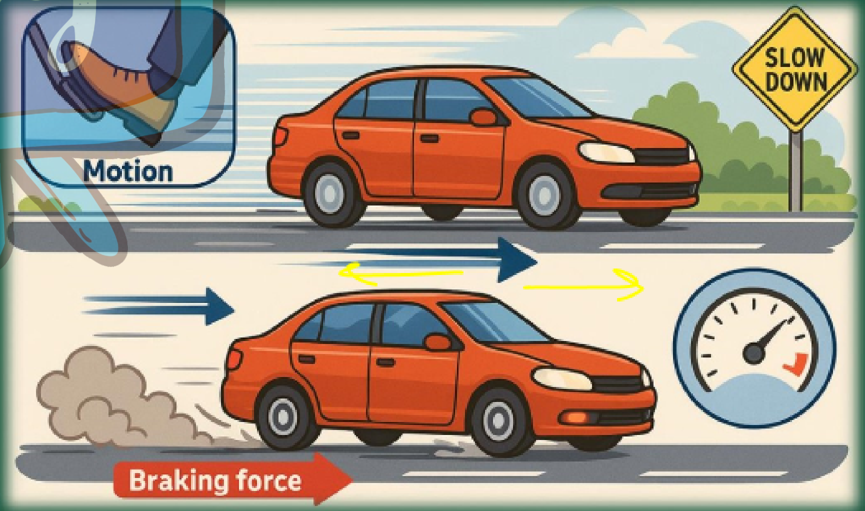
$\theta = 180^\circ$ $\cos 180^\circ = -1$

$\boxed{W = -FS}$

Example :

i. Brakes applying in a moving vehicle.

ii. A body slides against a rough horizontal surface.



C. Zero work : Work done by force is zero if the body gets displaced along a direction perpendicular to the direction of the applied force.

- The work done is zero if $\vec{F} = 0$ or $\vec{S} = 0$ or both $\vec{F}$ and $\vec{S}$ are 0.

$$W = FS\cos\theta$$

$$\theta = \frac{\pi}{2}$$

$$W = 0$$

$\theta = 90^\circ$

$\vec{A} \perp \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = 0$

$\vec{F} \perp \vec{S} \Rightarrow W = 0$

90°

mg

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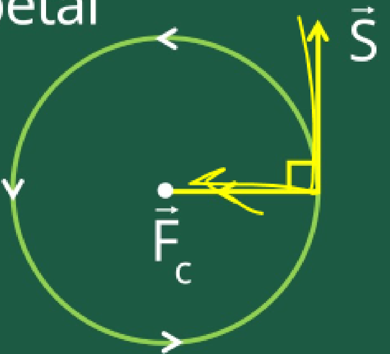
Example :

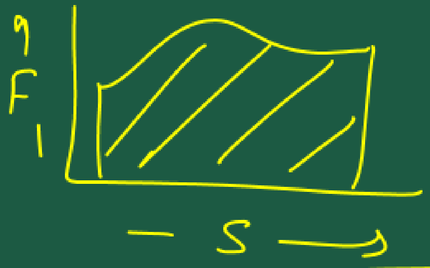
i. For a moving body in a circular path, the centripetal force and displacement are perpendicular to each other.

- Work done by centripetal force is always zero.

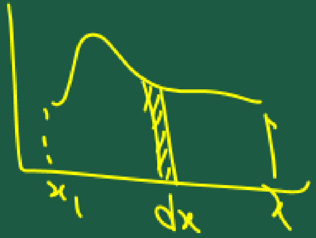


WORK DONE BY A VARIABLE FORCE

W = F S



Area of F-s graph = Work

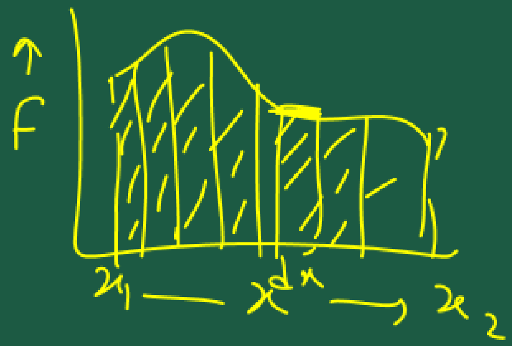


$$W = \sum_{x_i}^{x_f} F\Delta x$$

= sum of areas of all rectangles erected over all the small displacement

$$W = \lim \Delta x \to 0 \sum_{x_i}^{x_f} F\Delta x = \int_{x_i}^{x_f} F dx$$

= Area under the force – displacement curie.



$$dW = F\,dx$$

$$\int dW = \int_{x_1}^{x_2} F\,dx$$

$$W = \int_{x_1}^{x_2} F\,dx$$

EXAMPLE

1. Find the angle between force $\vec{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})$ unit and displacement $\vec{d} = (5\hat{i} + 4\hat{j} + 3\hat{k})$ unit. Also find the projection of F on d.

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{\vec{F}\cdot\vec{d}}{Fd}$$

$$= \frac{\cancel{15} + 16 - \cancel{15}}{\sqrt{9+16+25}\sqrt{25+16+9}}$$

$$\cos\theta = \frac{16}{\sqrt{50}\sqrt{50}} \quad \frac{16}{50}$$

$$\cos\theta = \frac{8}{25}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{8}{25}\right)}$$

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EXAMPLE

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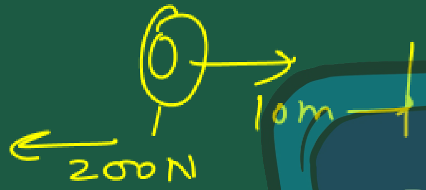
1. Find the angle between force $F = (3\hat{i} + 4\hat{j} - 5\hat{k})$ unit and displacement $d = (5\hat{i} + 4\hat{j} + 3\hat{k})$ unit. Also find the projection of F on d.

EXAMPLE

3. A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion.

a. How much work does the road do on the cycle?

b. How much work does the cycle do on the road?



$W = -200 \times 10$
$= -2000\ J$

EXAMPLE

Answer : Work done on the cycle by the road is the work done by the stopping (frictional) force on the cycle due to the road.

a. The stopping force and the displacement make an angle of 180° (π rad) with each other. Thus, work done by the road,

EXAMPLE

$W_r = Fd\cos\theta$

$= 200 \times 10 \times \cos\pi$

$= -2000\ J$

It is this negative work that brings the cycle to a halt in accordance with WE theorem.

b. From Newton's Third Law an equal and opposite force acts on the road due to the cycle.

EXAMPLE

Its magnitude is 200 N. However, the road undergoes no displacement. Thus, work done by cycle on the road is zero.

The lesson of Example 3 is that though the force on a body A exerted by the body B is always equal and opposite to that on B by A (Newton's Third Law);

EXAMPLE

the work done on A by B is not necessarily equal and opposite to the work done on B by A.

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LEARNING OUTCOMES

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1. To calculate scalar product
2. To define work

ASSESSMENT

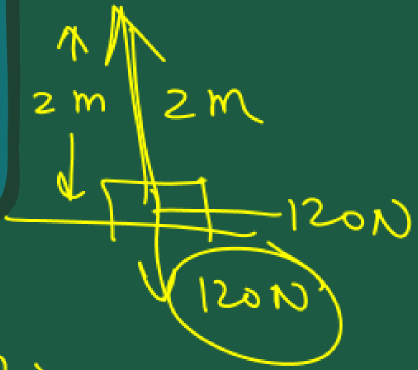
1 Work done can be–

- A Zero
- B Positive
- C Negative
- D All of these

ASSESSMENT

2 A boy raises a box with a weight of 120N from a height of 2m. The work done by him is–

- A 60 J
- B 120 J
- C 240 J
- D 130 J



W = 120 × 2

= 240 J