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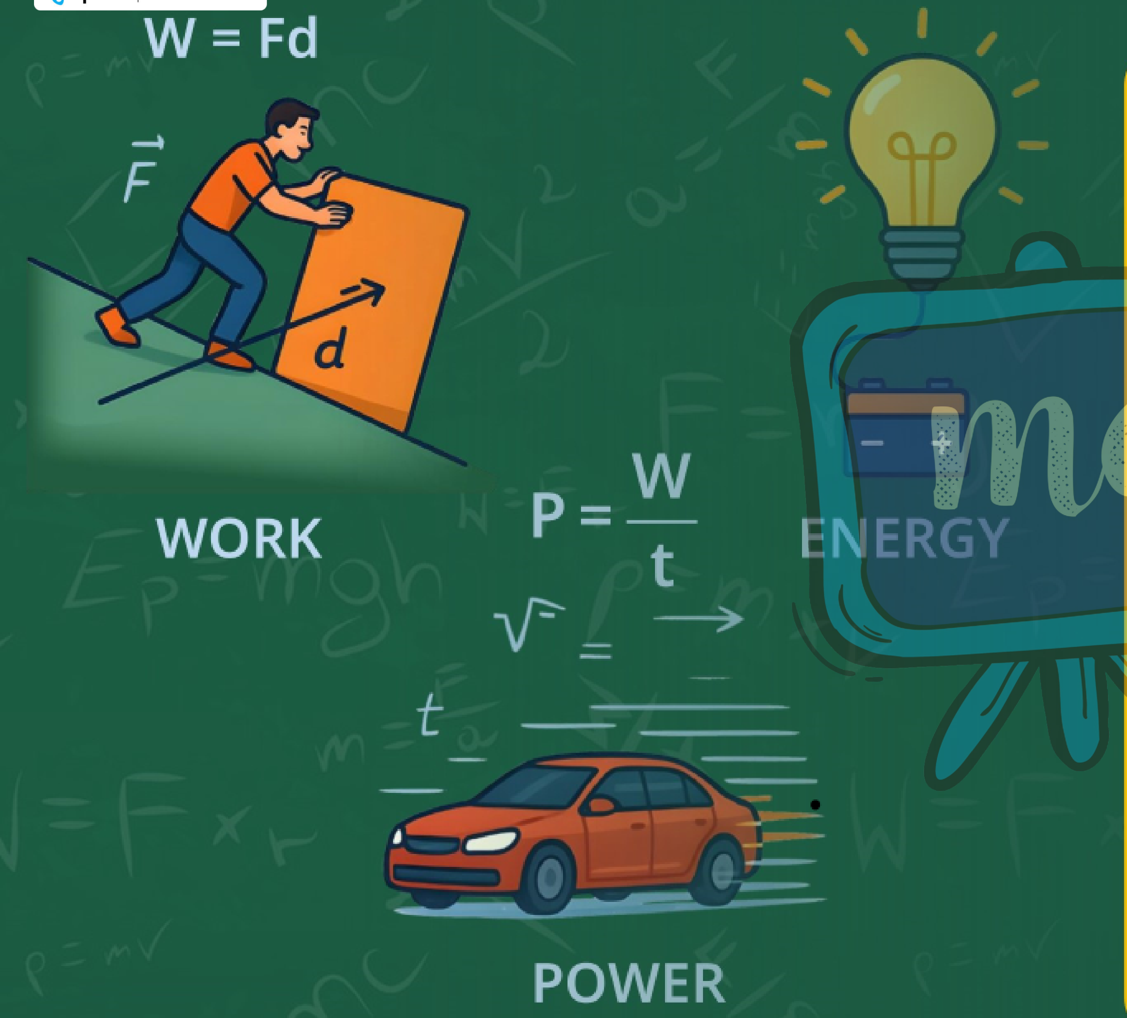


CLASS – 11

PHYSICS

Chapter – 5

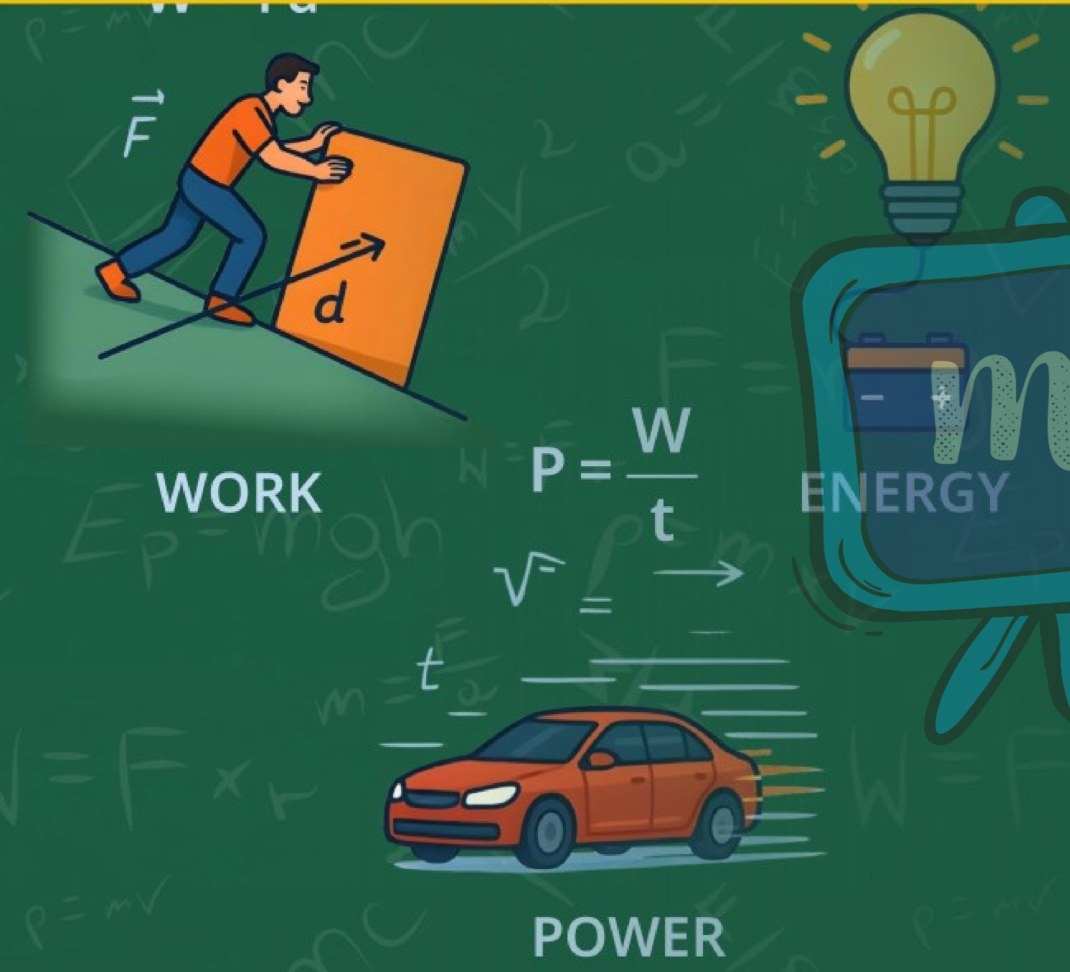
Work, Energy and Power

Part – 3

Law of Conservation of Mechanical Energy and Power

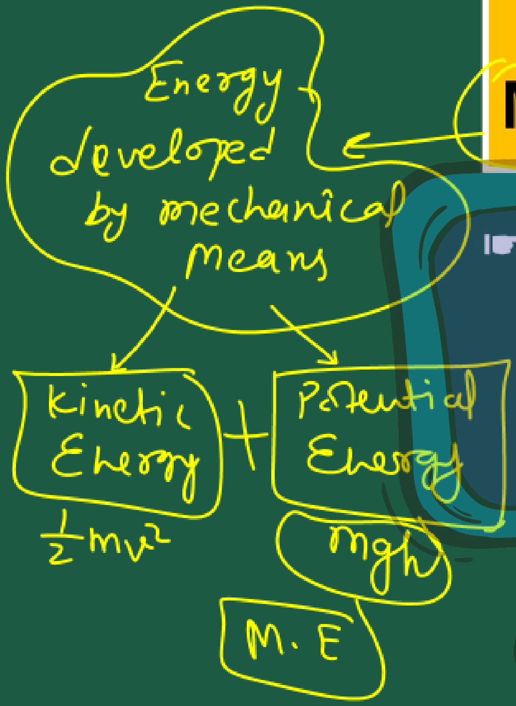
Alok Gaur

OVERVIEW



1. Work
2. Energy
3. Law of Conservation of Mechanical Energy and Power
4. Collision

LAW OF CONSERVATION OF MECHANICAL ENERGY AND POWER



☛ Energy function or sum of kinetic and potential energies of a particle remains constant for conservative forces. This principle is called law of conservation of mechanical energy.

$$M.E = K.E + U = \text{Constant}$$

$$\Delta M.E = 0$$

$$\Delta K.E + \Delta U = 0$$

$$\Delta K.E = -\Delta U$$

Change in K.E = −Change in P.E

$100J \rightarrow U$

Rest
$V = 0$
$K.E = 0$

$40J$ K.E ↑
+
$60J$ U ↓

$100J$

- According to work – energy theorem,

$$\Delta W = \Delta K \text{ .... (i)}$$

The work done on the body against the force is equal to increase in it's potential energy.

$$-\Delta W = \Delta U \text{ .... (ii)}$$

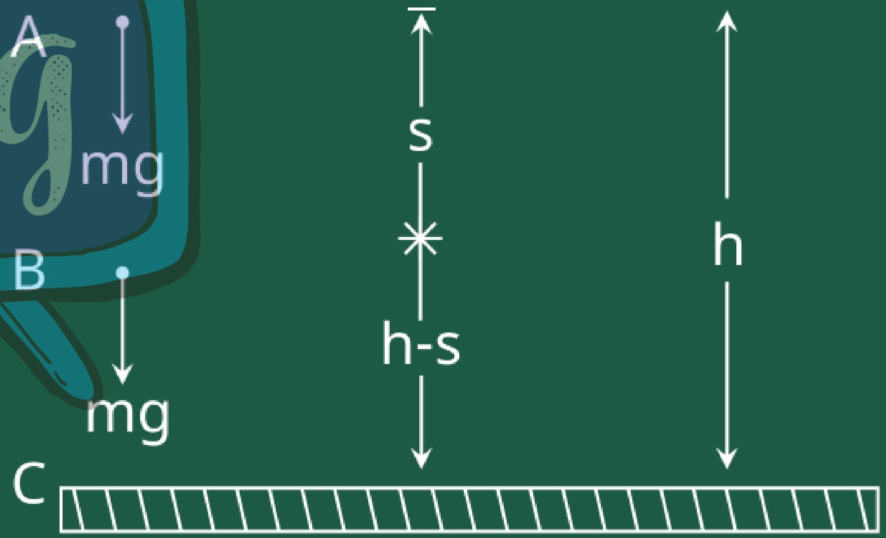
From eq^n (i) & (ii)

$$\Delta K + \Delta U = \Delta 0$$

$$K + U = \text{Constant}$$

Some examples :

☛ A body falling freely



Freely falling Body

at point A

$E_A = \frac{1}{2}m(0)^2 + mgh$

* $E_A = mgh$

at point B

$E_B = \frac{1}{2}mv_1^2 + mg(h-x)$

$v_1^2 = 0^2 + 2gx$

$E_B = \frac{1}{2}m(2gx) + mgh - mgx$

$E_B = mgx + mgh - mgx$

* $E_B = mgh$

at C

$v_2^2 = 0^2 + 2gh$

$v_2^2 = 2gh$

$E_C = \frac{1}{2}mv_2^2 + 0$

$E_C = \frac{1}{2}m(2gh)$

* $E_C = mgh$

$E_A = E_B = E_C$

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At point A :

$$U = mgh$$

$$K = \frac{1}{2} mv^2 = \frac{1}{2} m \times 0 = 0$$

$$M.E. = K + U = mgh \ldots\ldots\ldots\ldots\ldots (i)$$

At point B :

$$U = mg\,(h-s)$$

$$K = \frac{1}{2} mv^2$$

$$\therefore\ v^2 = u^2 + 2gs$$

$$v^2 = 0 + 2gs$$

$$v = \sqrt{2gs}$$

$$K = \frac{1}{2} m\,(2gs)$$

$$K = mgs$$

$$M.E. = K + U = mgs + mg\,(h-s) = mgh \quad \text{..... (ii)}$$

At point C :

$$U = mg\,(o) = 0$$

$$K = \frac{1}{2} mv^2$$

$$v^2 = u^2 + 2gh$$

$$v^2 = 0 + 2gh$$

$$v = \sqrt{2gs}$$

$$K = \frac{1}{2} m\,(2gh) = mgh$$

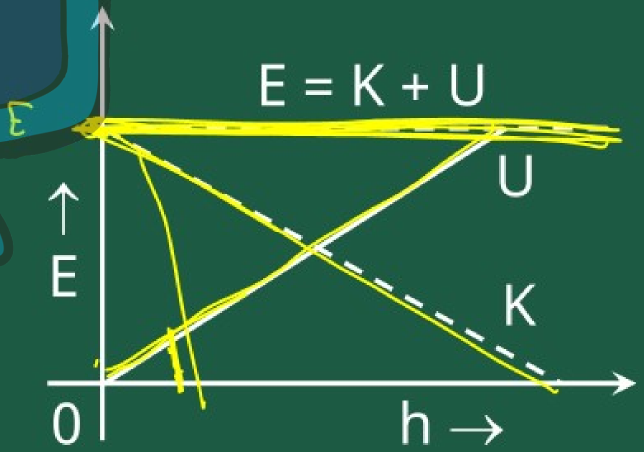
$$M.E. = K + U = mgh \;\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From eg^n (i), (ii) and (iii)

$$M.E. = mgh$$

$\therefore$ The total energy at earth's surface = mgh

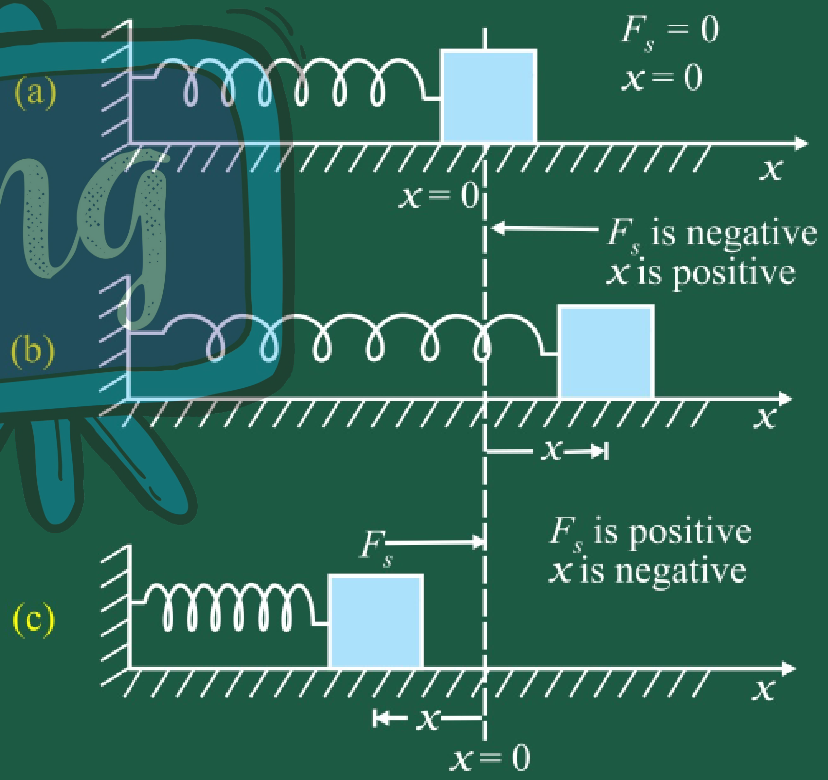
☛ It is clear that the sum of total energy at each point in the path of freely falling body in the three positions is constant.



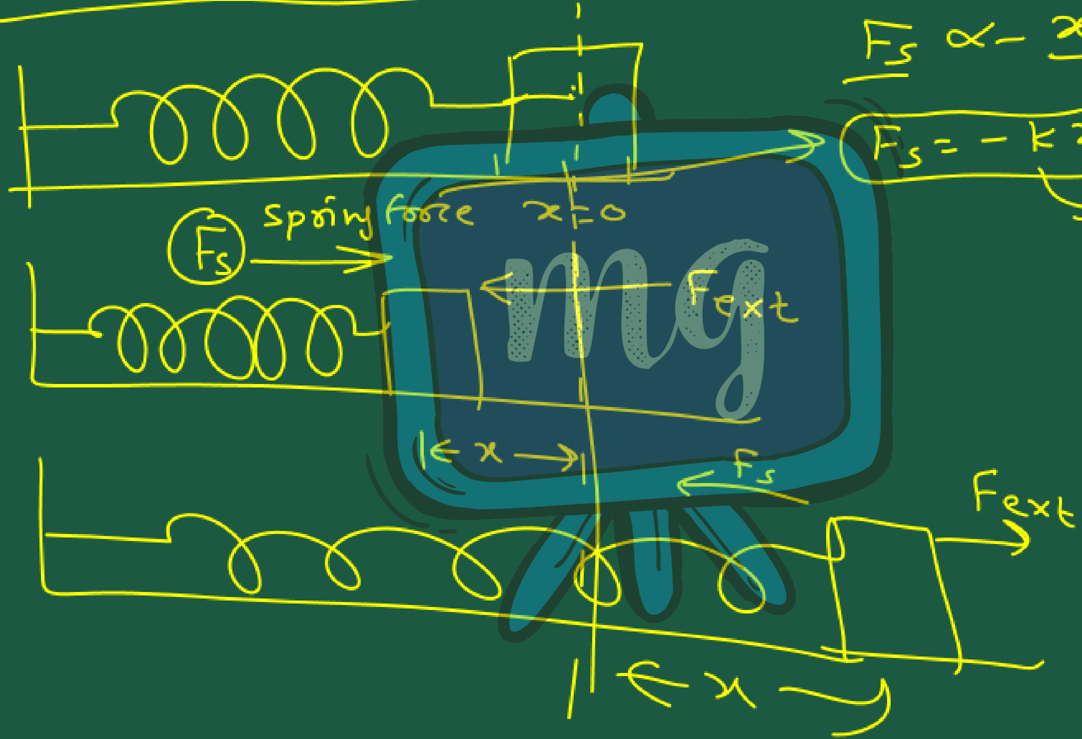
SPECIAL

☛ When the body stops suddenly after striking with earth's surface, then it's kinetic energy converts into heat, sound and light.

THE POTENTIAL ENERGY OF A SPRING



Potential Energy = Work against force



$F_s \propto -x$

$F_s = -kx$

k → spring const.

$$W = -U$$

$$W = \int_{x_1}^{x_2} F_s \, dx$$

$$W = \int_{x_1}^{x_2} -kx \, dx$$

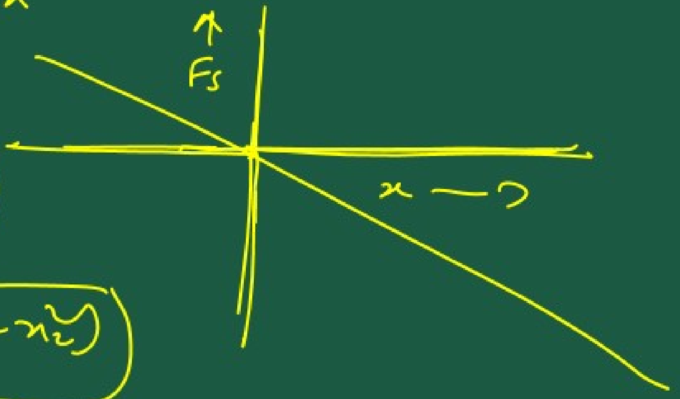
$$W = -k \left(\frac{x^2}{2} \right)_{x_1}^{x_2}$$

$$W = -\frac{k}{2} \left(x_2^2 - x_1^2 \right)$$

$x_1 = 0 \quad x_2 = x$

$$W = -\frac{1}{2} k x^2$$

$F_s \propto -x$



$U = -W =$

$$U = +\frac{1}{2} k x^2$$

$$U = \frac{1}{2} k (x_1^2 - x_2^2)$$

$$U \propto x^2$$

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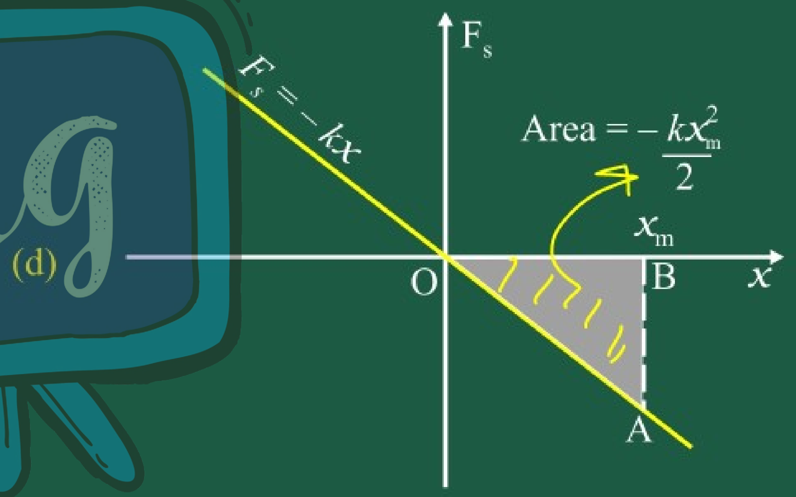


- Suppose that we pull the block outwards as in diagram. If the extension is X_m- the work down by the spring force is

$$W_s = \int_0^{x_m} F_s d_x$$

- Spring force is proportional to the displacement of the block from the equilibrium position.

$$F_s \times x$$

$$F_s = -kx$$

- Spring force is proportional to the displacement of the block from the equilibrium position.

$$F_s \times x$$

$$F_s = -kx$$

- The constant k is called the spring constant :

Unit : $N.m^{-1}$

Dimension : $[M^1L^0T^{-2}]$

$\frac{M^1L^1T^{-2}}{L^1}$

$k = F/x$

- The spring is said to be stiff if k is large and soft if k is small.

$$W_s = \int_0^{x_m} F_s d_x$$

$$W_s = -\int_0^{x_m} k_x d_x$$

$$W_s = \frac{1}{2} k x_m^2$$

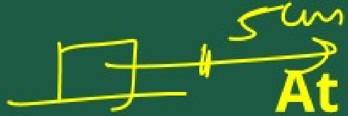
At the extreme position :

$x = \pm x_m$ and velocity $v = 0$

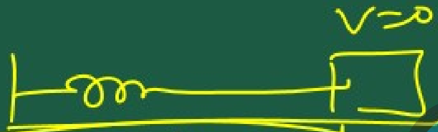
$$K.E. = \frac{1}{2} mv^2 = 0$$

$$U = \frac{1}{2} Kx_m^2 = \text{maximum}$$

x → max
K.E → 0
U → maximum



At an intermediate position :



$E = U = \frac{1}{2} k x_m^2$

$E = K + U$

$$\frac{1}{2} K x_m^2 = \frac{1}{2} m v^2 + \frac{1}{2} K x^2$$

$\frac{1}{2} m v^2 = \frac{1}{2} k (x_m^2 - x^2)$

$$K.E. = \frac{1}{2} K (x_m^2 - x^2)$$

$v^2 = \frac{k}{m} (x_m^2 - x^2)$

$$\text{Velocity } v = \sqrt{\frac{K}{m}(x_m^2 - x^2)}$$

$V = \sqrt{\frac{k}{m}(x_m^2 - x^2)}$

At equilibrium position here x = 0

$$U = \frac{1}{2} K (0)^2 = 0$$

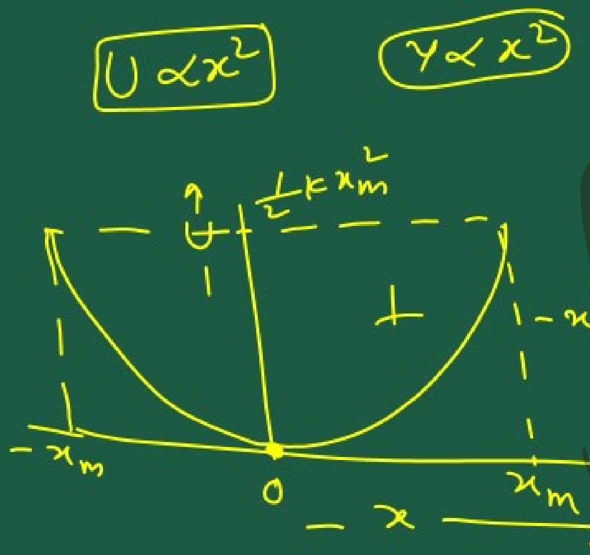
$$K.E. = \frac{1}{2} m{v_m}^2 = \frac{1}{2} K{x_m}^2$$

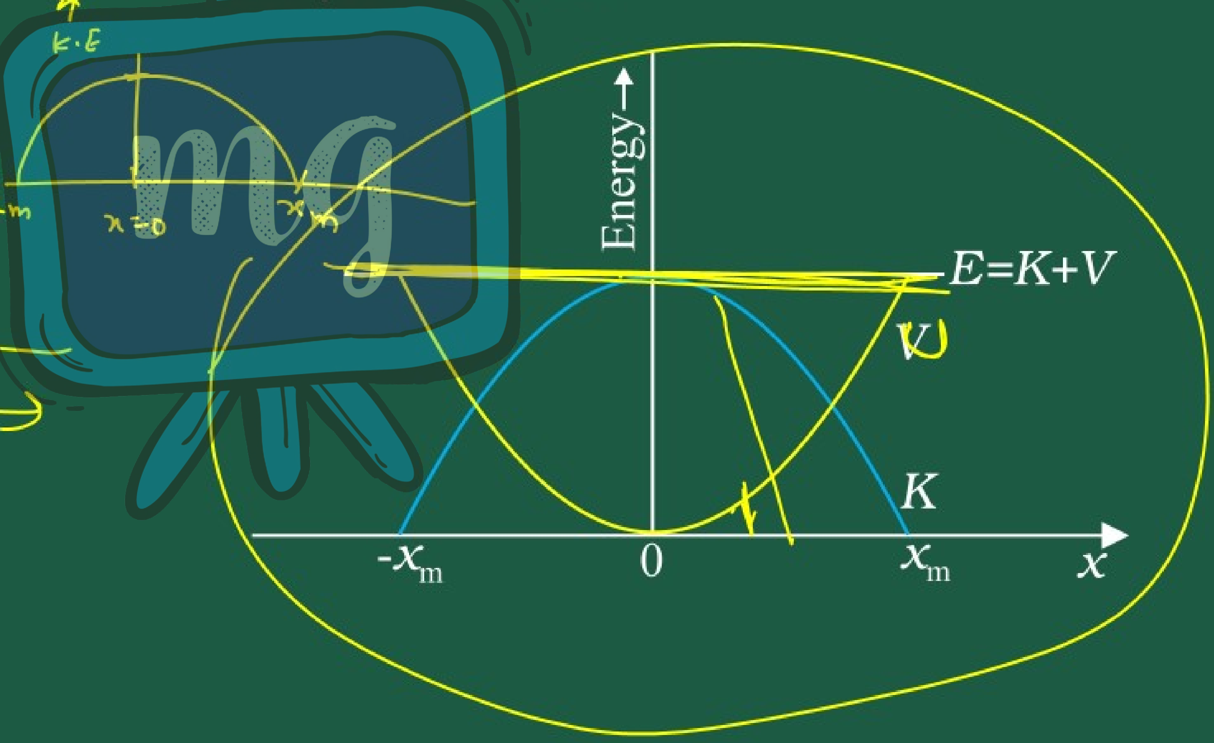
Maximum velocity $v_m = x_m \sqrt{\frac{K}{m}}$

When $x = 0 \Rightarrow$
K.E $\Rightarrow$ maximum
$U \Rightarrow$ min

When $x = max = \pm x_m$
$\Rightarrow V = 0 \Rightarrow K.E_{min} = 0$
$\Rightarrow U_{max} = \frac{1}{2} k x_m^2$

☛ Graph of K.E., P.E. and total energy with the displacement:





POWER

- Power is defined as the time rate at which work is done or energy is transferred.

$$P = \frac{W}{t}$$

- Power is a scalar quantity.
- Unit : Watt = Joule – second^{-1}
- Dimension : $[M^1L^2T^{-3}]$

P = Rate of doing work

$P = W/t$ → scalar

SI unit → $\frac{J}{sec}$ or watt

1 Horse power = 746 watt

$\frac{M^1L^2T^{-2}}{T^1} = [M^1L^2T^{-3}]$

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Average power :

↓ Time Interval

$$P = \frac{\Delta W}{\Delta t}$$

$$P = \frac{dW}{dt}$$

$$= \frac{d}{dt}(F s \cos\theta)$$

$$= F\cos\theta \frac{ds}{dt}$$

$$= F\cos\theta\, v$$

$$P = F v \cos\theta$$

$$P = \vec{F}\cdot\vec{v}$$

Instantaneous power :

↓ Particular Instant

$$Pin = \lim_{\Delta t \to r} \frac{\Delta W}{\Delta t}$$

$$P = \frac{dw}{dt}$$

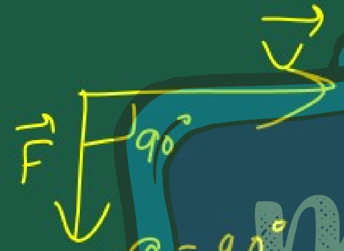
$$dw = \vec{F}.d\vec{s}$$

$$P = \frac{\vec{F}.d\vec{s}}{dt}$$

$$P = \vec{F}.\vec{V}$$

$$P = FV\cos\theta$$

Power = force . velocity



$\theta = 90°$

$P = 0$

for maximum power

$\theta = 0°$

Other unit of power :

☛ The bigger units of power are kilowatt (kw) and horse power (hp)

1 kw = 1000 watt or 1kw = 10^3w

1 horse power 746 watt or 1 hp = 746w

Relation between kwh and joule :

1 kWh = 1000 W - 3600 sec

= 3.6×10^6 W-sec

1 kWh = 3.6×10^6 J

→ B.T.U

Board of Trade Unit

"1 kwh = 3.6×10^6 joule"

Unit → Electrical Energy

$P = \frac{W}{t}$

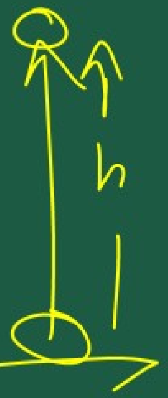
$W = Pt$

J = Watt - sec

SPECIAL

$$P = \frac{W}{t}$$

$$P = \frac{mgh}{t}$$

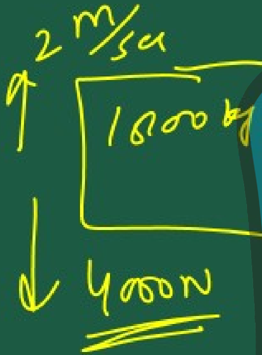


Mass – Energy Relation :

$$E = mc^2$$

Where C = 3×10^8 m/s is the velocity of light in vacuum.

EXAMPLE



Q. An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of $2ms^{-1}$.The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.

EXAMPLE

Anwer : The downward force on the elevator is $F = mg + F_f = (1800 \times 10) + 4000 = 22000$ N

The motor must supply enough power to balance this force. Hence,

$P = F.v = 22000 \times 2 = 44000$ W $= 59$ hp

LEARNING OUTCOMES

Prove the principle of conservation of mechanical energy with examples.

ASSESSMENT

1 On applying a force F on a body, it gains a velocity v, then power will be–

A F/v

B Fv^2

C Fv

D F/v^2

ASSESSMENT

2 A body is kept in motion along a straight line by a machine of constant power. Distance covered by the body in time t is proportional to

- A $t^{1/2}$
- B $t^{3/4}$
- C $t^{3/2}$
- D t^{2}

$x \propto t^{n}$ ($n = ?$)

$P = \frac{W}{t}$

$P = \frac{Fx}{t}$

$P = \frac{max}{t}$

$P = \frac{mvx}{t^2}$

$P = \frac{mx^2}{t^3}$

$x^2 = \frac{P}{m} t^3$

$x \propto t^{3/2}$

ASSESSMENT

3 An engine produces 10 kw energy. What time will it take in lifting a mass of 200 kg to a height 40 m? ($g = 10ms^{-2}$)

$P = 10 \text{ kW} = 10000 W$

$m = 200 \text{ kg}, \ h = 40 m$

$P = \frac{W}{t} = \frac{mgh}{t}$

$t = \frac{mgh}{P} = \frac{40 \times 10 \times 200}{10 \times 1000}$

$t = 8 \text{ sec}$