CLASS – 11

PHYSICS

Chapter – 4

Laws of Motion

Part – 5

Circular Motion

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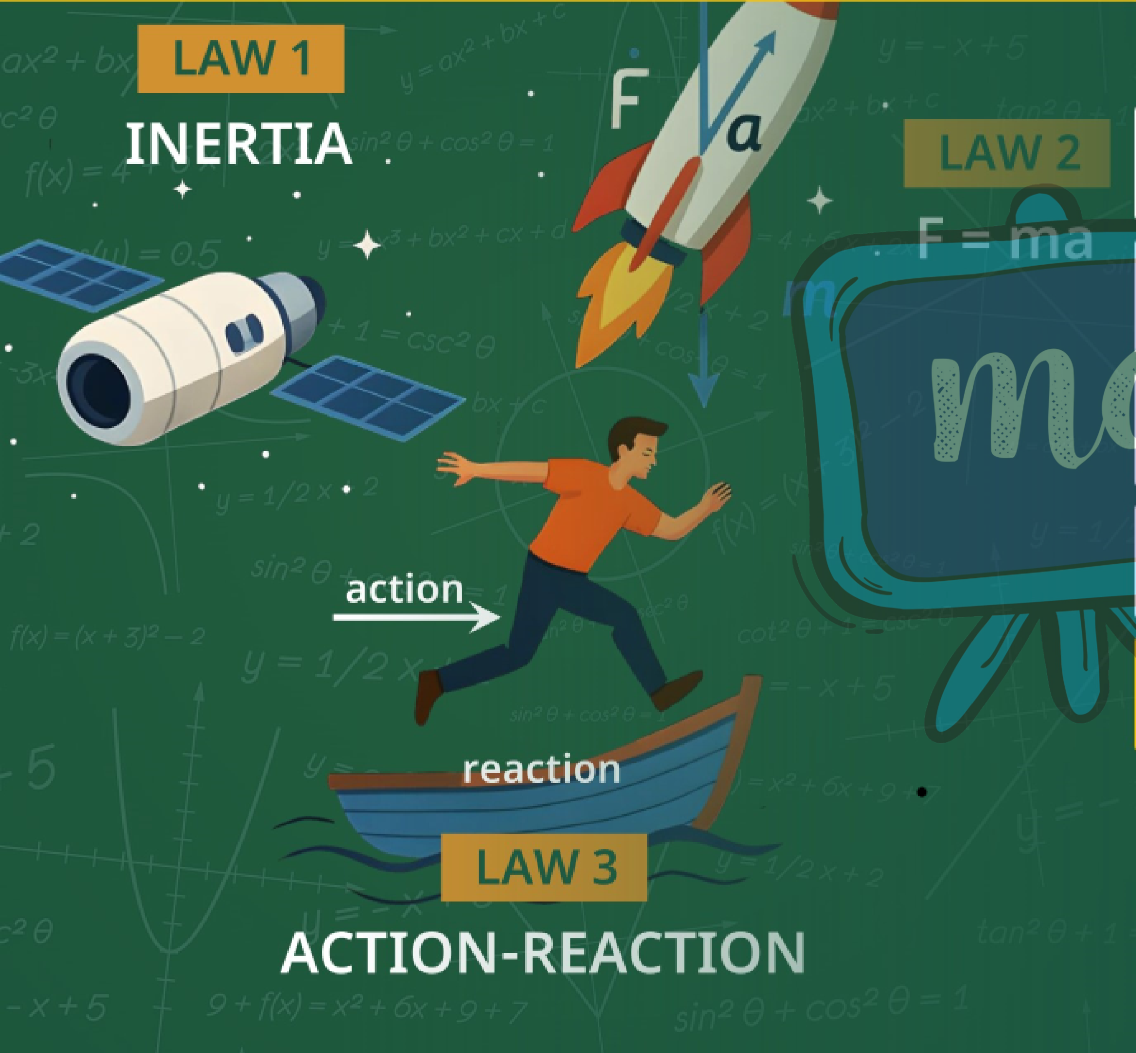
OVERVIEW



1. Newton's First Law of Motion
2. Newton's Second Law of Motion
3. Newton's Third Law of Motion
4. Common Forces in Mechanics
5. Circular Motion

CIRCULAR MOTION

$\vec{V}$

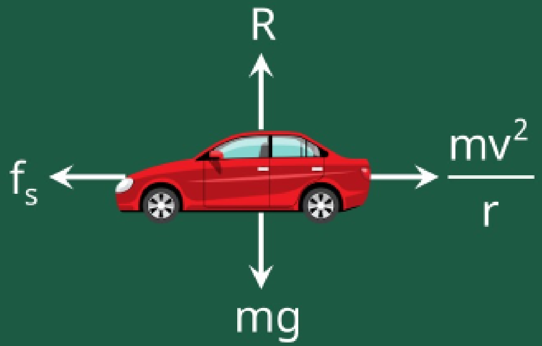
Centripetal Acceleration

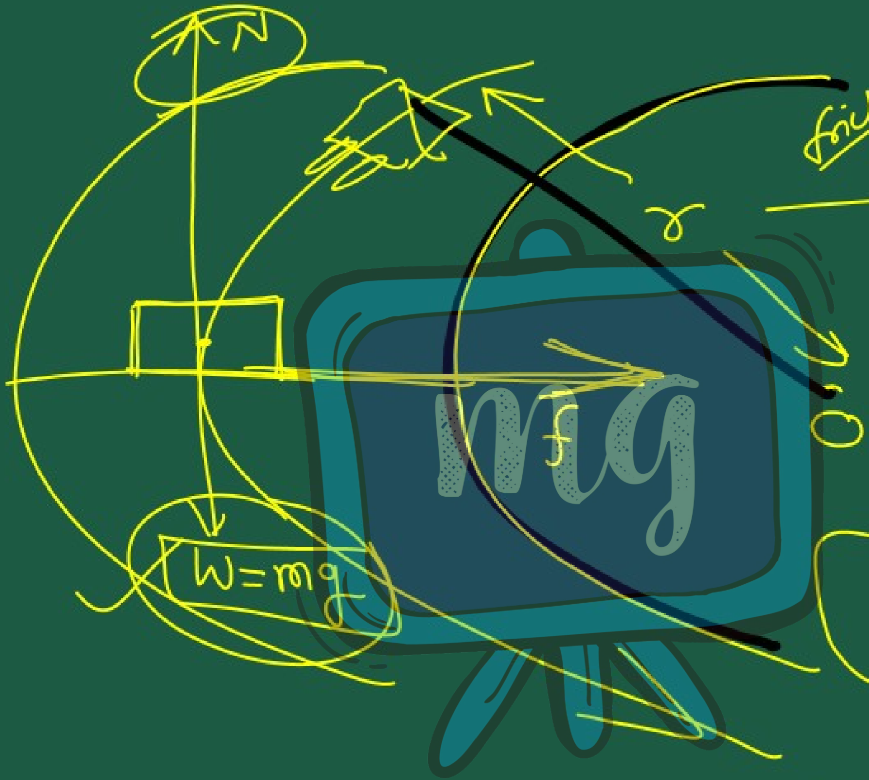
$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

MOTION OF A VEHICLE ON A PLANE CIRCULAR ROAD

- Let a car of mass m is moving with constant speed on a plane circular road of radius r.





friction $f = F_c = \frac{mv^2}{r}$ —— (1)

$N = mg$ —— (2)

$\frac{f}{N} = \frac{v^2}{rg}$

$\frac{v^2}{rg} = \mu_s$

$\mu = \tan\theta$

$V_{max} = \sqrt{rg\tan\theta}$

$v^2 = \mu r g$

$V_{max} = \sqrt{\mu r g}$

$V_{max} \propto \sqrt{r}$

The following forces act on car →

(i) Weight of car $w = mg$

(vertically downward)

(ii) Normal reaction force R

(vertically upward)

(iii) Centripetal force $f_e = \frac{mv^2}{r}$

(In moving direction)

(iv) Frictional force $f_s = \mu R$

(Opposite of moving directional)

In balancing conditions,

$$mg = R \quad \text{..........} \quad (i)$$

$$\frac{mv^2}{r} = \mu R \quad \text{..........} \quad (ii)$$

On dividing equation (ii) by equation (i)

$$\frac{V^2}{rg} = \mu$$

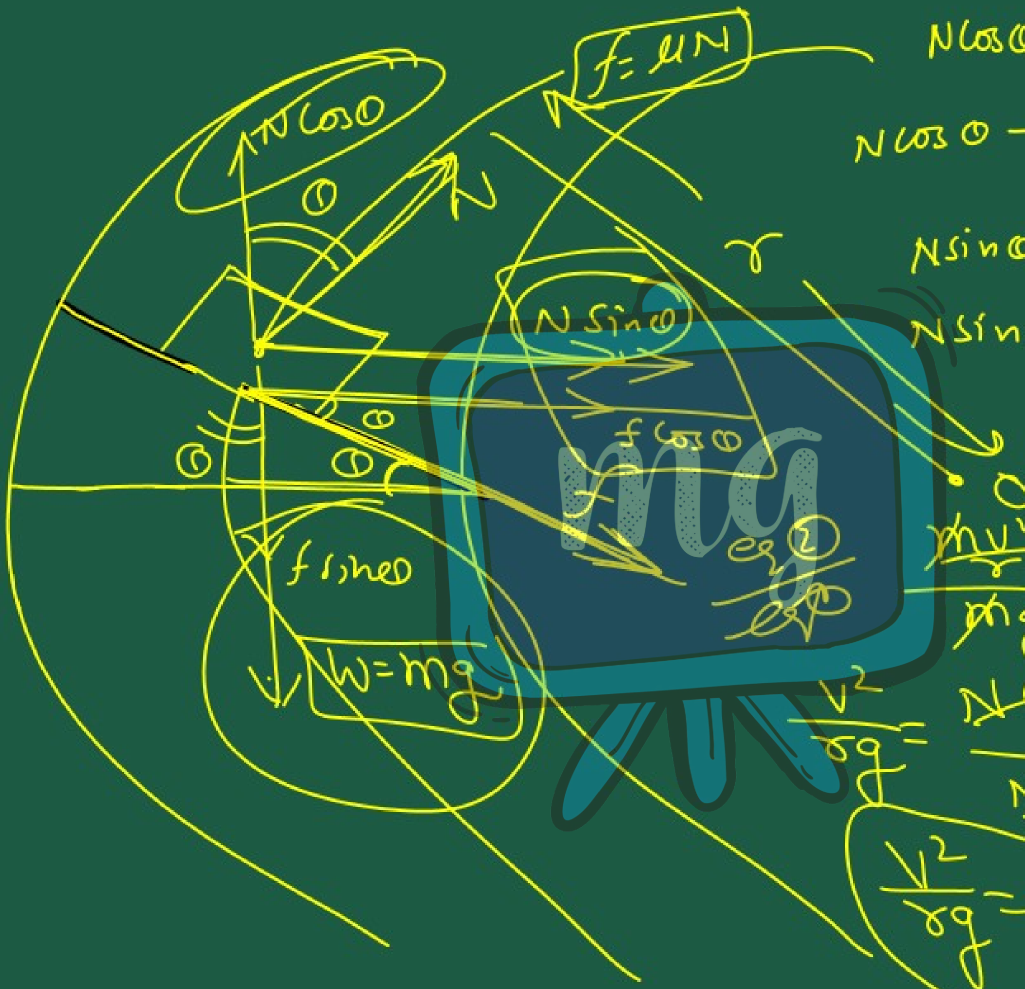
$$V^2 = \mu rg$$

$$V_{max} = \sqrt{\mu rg}$$

$$V < \sqrt{\mu rg}$$

MOTION OF A VEHICLE ON BANKED ROAD

- When a vehicle takes a circular turn on a road, so to provide essential centripetal force the outside part of the road is slightly upraised with respect to innerside. It is called banking of road.



$$N\cos\theta = mg + f\sin\theta$$

$$N\cos\theta - \mu N\sin\theta = mg \quad \text{—— (1)}$$

$$N\sin\theta + f\cos\theta = F_c$$

$$N\sin\theta + \mu N\cos\theta = \frac{mv^2}{r} \quad \text{—— (2)}$$

$$\frac{\frac{mv^2}{r}}{mg} = \frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta}$$

$$\frac{v^2}{rg} = \frac{N\cos\theta\left[\frac{\sin\theta}{\cos\theta} + \mu\right]}{N\cos\theta\left[1 - \mu\frac{\sin\theta}{\cos\theta}\right]}$$

$$\frac{v^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\frac{V^2}{rg} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$$

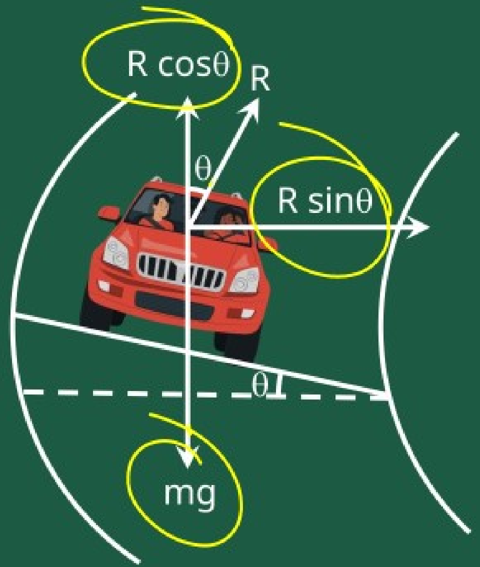
$$V^2 = rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$V_{max} = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

- The angle by which the outside part is raised, is called angle of banking.

(A) $\frac{\text{Safe speed}}{\text{Optimum speed}}$ (The friction force is ineffective)

- Suppose banking angle of banked road is θ and the radius of circular banked road is r.



In balancing condition

$$mg = R\cos\theta \text{ ....(i)}$$

$$\frac{mv^2}{r} = R\sin\theta \text{.....................(ii)}$$

On dividing equation (ii) by equation (i)

$$\frac{mv^2}{mrg} = \frac{R\sin\theta}{R\cos\theta}$$

$$\frac{v^2}{rg} = \tan\theta$$

$$v^e = rg\tan\theta$$

$$V = \sqrt{rg\tan\theta}$$

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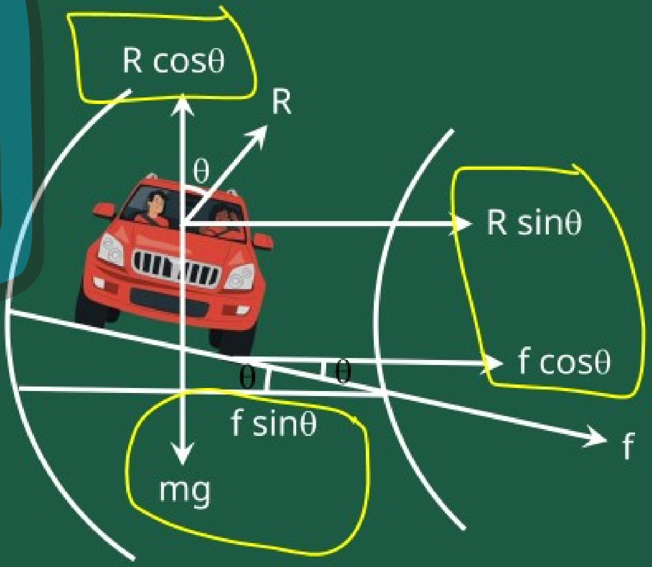
(B) Maximum Safe Speed (When friction force is effective)



In balancing condition

$$mg + f\sin\theta = R\cos\theta$$

$$mg = R\cos\theta - f\sin\theta \quad \text{..........(i)}$$

$$\frac{mv^2}{r} = R\sin\theta + f\cos\theta \quad \text{..............(ii)}$$

On dividing eq^n (ii) by eq^n (i)

$$\frac{mv^2}{rmg} = \frac{R\sin\theta + \mu R\cos\theta}{R\cos\theta - f\sin\theta}$$

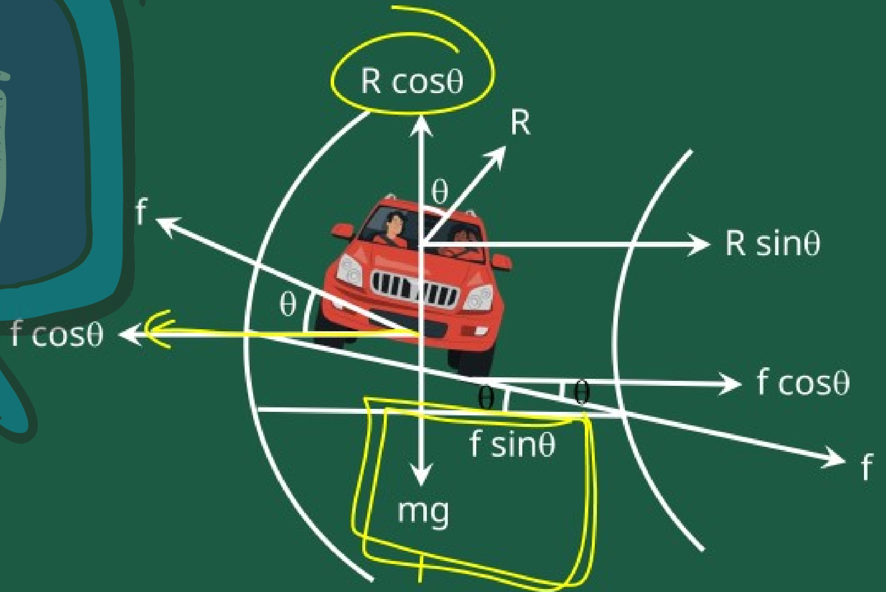
$$\frac{V^2}{rg} = \frac{R\sin\theta + \mu R\cos\theta}{R\cos\theta - \mu\sin\theta}$$

$$\frac{V^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$$

(C) Minimum Safe Speed (When friction force is effective)

$$R\sin\theta - f\cos\theta = \frac{mv^2}{r}$$



In balancing condition,

$$mg = R\cos\theta + f\sin\theta \quad \text{...........(i)}$$

$$\frac{mv^2}{r} + f\cos\theta = R\sin\theta$$

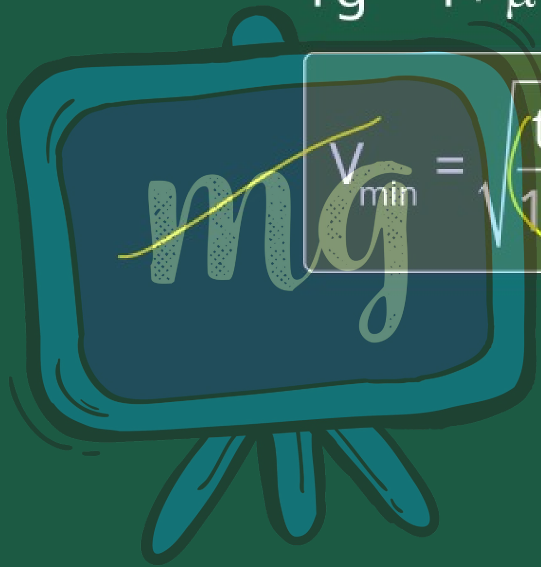
$$\frac{mv^2}{r} = R\sin\theta - f\cos\theta \quad \text{...........(ii)}$$

On dividing eq^n(ii) by eq^n (i)

$$\frac{mv^2}{rmg} = \frac{R\sin\theta - f\cos\theta}{R\cos\theta + f\sin\theta}$$

$$\frac{v^2}{rg} = \frac{R\sin\theta - \mu R\cos\theta}{R\cos\theta + \mu R\sin\theta}$$

$$\frac{V^2}{rg} = \frac{\tan\theta - \mu}{1 + \mu\tan\theta}$$



$$V_{min} = \sqrt{\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right) rg}$$

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EXAMPLE

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10. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

$r = 3\,m$, $\mu = 0.1$

$V_{max} = \sqrt{\mu r g}$

$= \sqrt{0.1 \times 3 \times 10}$

$= \sqrt{3}$

$V_{max} = 1.732\ m/s$

$18\ km/hr \rightarrow 5\ m/s$

EXAMPLE

Answer :

$$V = 18km/h = 18 \times \frac{5}{18} = 5m/s$$

$$r = 3m$$

$$\mu = 0.1$$

$$v = \sqrt{\mu rg}$$

$$v = \sqrt{0.1 \times 3 \times 9.8}$$

$$v = \sqrt{2.94}$$

$$v \approx 1.71 m/s$$

$V > V_{max}$

It can not take turn with this speed.

EXAMPLE

$V_{max} < v$ according to the question, so condition is not fulfilled the cyclist will slip while taking the circular turn.

EXAMPLE

11. A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the racecar to avoid wear and tear on its tyres, and (b) maximum permissible speed to avoid slipping?

EXAMPLE

Answer :

$r = 300m$

$\theta = 15°$

$m = 0.2$

(a) The optimum speed of the race-car

$$v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{300 \times 9.8 \times \tan 15°}$$

$$v \approx 28.1 m/s$$

EXAMPLE

(b) The maximum permissible speed of

$$V_{max} = \sqrt{\frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$$

$$= \sqrt{\frac{300 \times 9.8(0.2 + \tan 15^\circ)}{(1 - 0.2\tan 15^\circ)}}$$

* $\tan 15^\circ = 0.2679$

$$V_{max} = \sqrt{\frac{300 \times 9.8 \times 0.4679}{0.946}}$$

$$V_{max} = \sqrt{1454.15}$$

$$V_{max} \approx 38.1 m/sec$$

LEARNING OUTCOME

To study for motion in circular path

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ASSESSMENT

1 What should be the radius at least of a plane turn for safe cycling of a cyclist with uniform speed of 7m/s without learning? $\left(\mu = \frac{1}{4}\right)$

- A 9.8m
- B 19.6m
- C 4.9m
- D 0m

$v = 7\ m/s,\ \mu = 1/4$

$r = ?$

$v = \sqrt{\mu r g}$

$v^2 = \mu r g$

$r = \frac{v^2}{\mu g} = \frac{49 \times 4}{1 \times 9.8}$

$r = 20m$

ASSESSMENT

2 | Define safe speed or optimum speed?