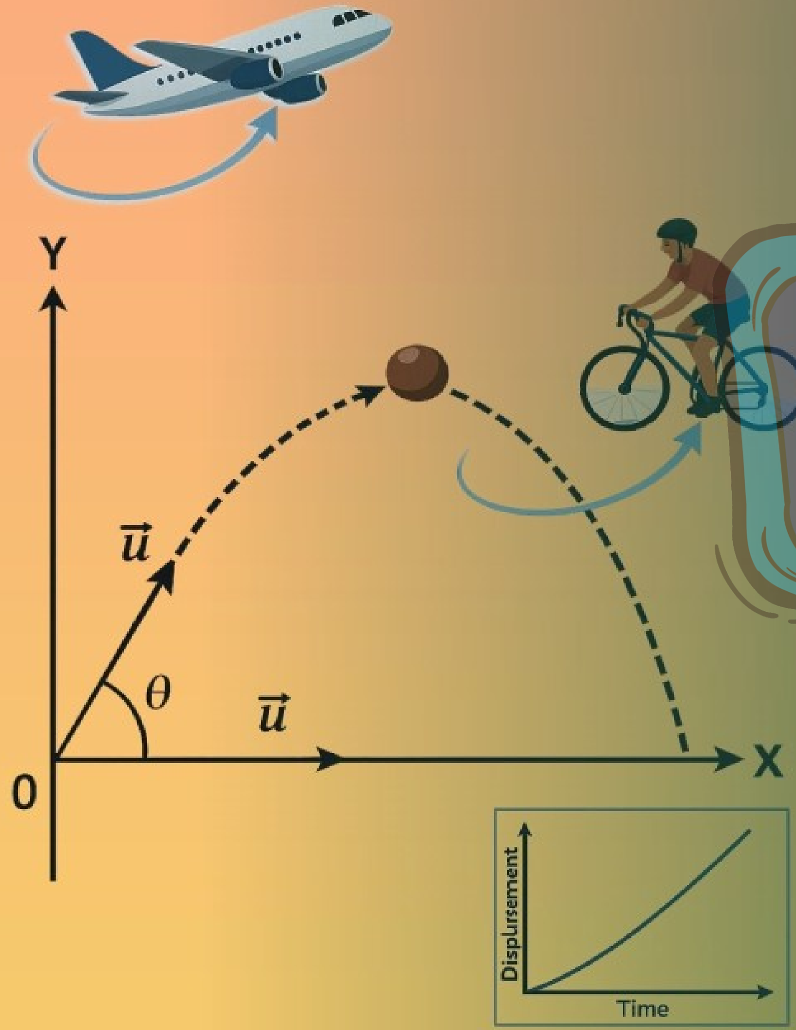


CLASS – 11

PHYSICS

Chapter – 3

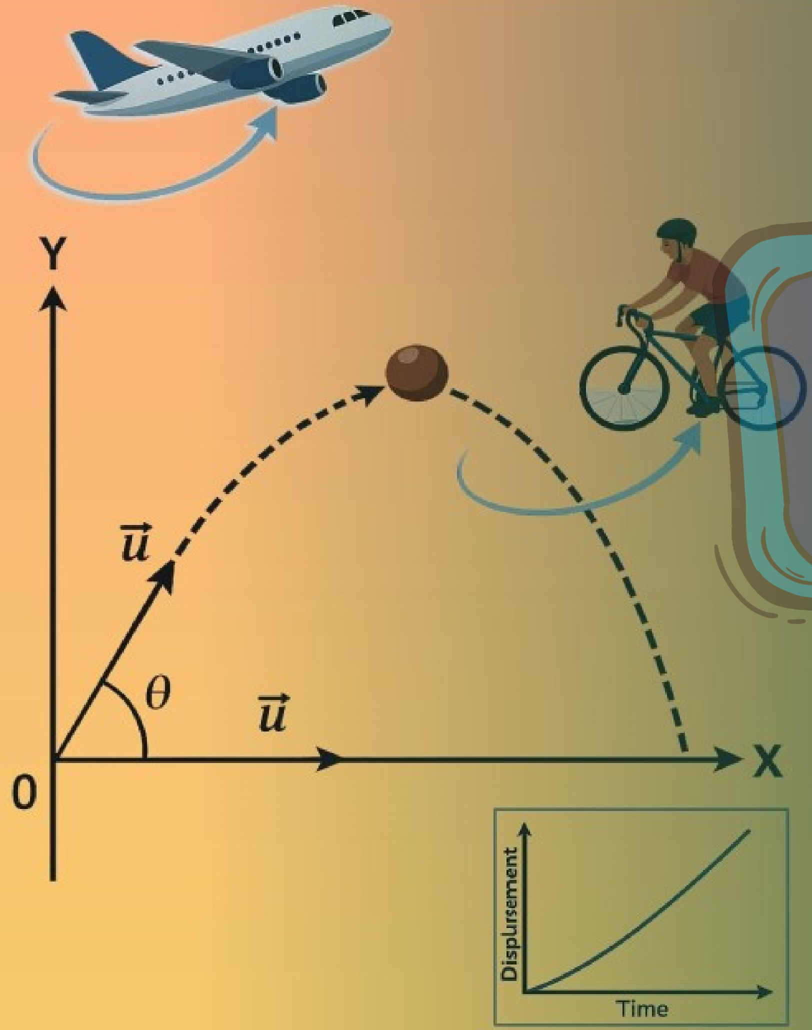
Motion in a Plane

Part – 5

Uniform circular Motion

Alok Gaur

OVERVIEW

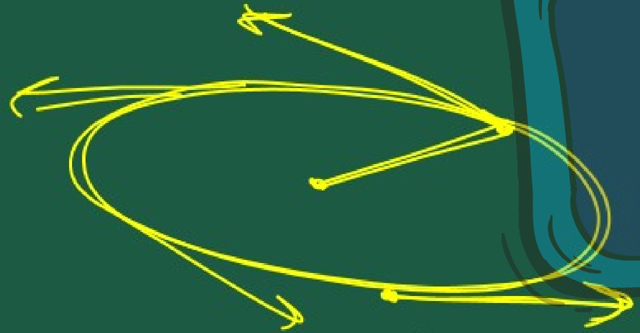


1. Scalars and Vectors
2. Addition of Vectors
3. Resolution of Vectors
4. Projectile Motion
5. Uniform Circular Motion

UNIFORM CIRCULAR MOTION

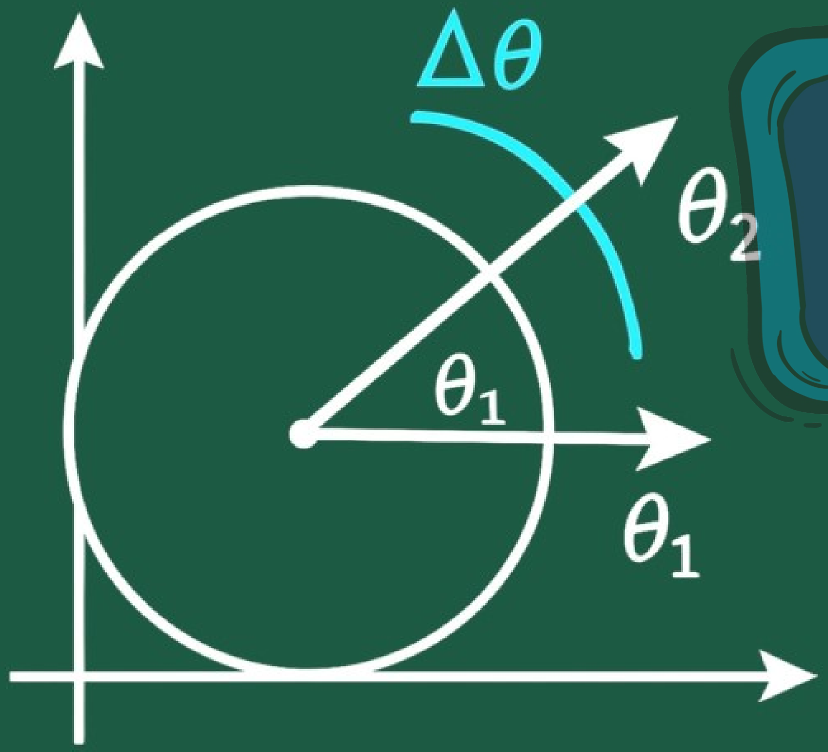
When a particle moves with a uniform linear speed on a circular path the motion of the particle is called uniform circular motion.

$\boxed{\text{Speed = Constant}}$



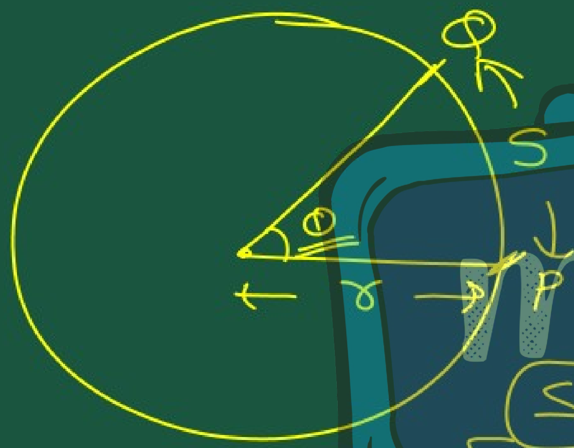
= tangent velocity

ANGULAR DISPLACEMENT



- The change in the angular position of a particle is called angular displacement.

  Angular Displacement $\Delta\theta = \dfrac{\Delta s}{R}$
- Unit : radian
- It has no dimention.
- It is axis vector quantity.



linear Displacement = S

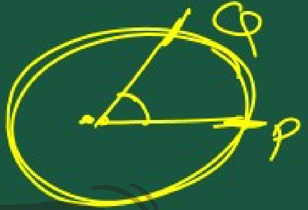
Angular Displacement = θ

Arc = Angle x Radius

$$\boxed{S = r\theta}$$

$$\boxed{\theta = \frac{S}{r}}$$

$\longrightarrow$ displacement



θ_1

θ_2

Change in angle = angular Displacement

$$\boxed{\Delta\theta = \theta_2 - \theta_1}$$

Radian

Velocity = $\dfrac{\text{Displacement}}{\text{Time}}$

Angular vel = $\dfrac{\text{Angular Displacement}}{\text{Time}}$

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \omega = \frac{d\theta}{dt}$$

ANGULAR VELOCITY

- The rate of change in angular displacement with time is called angular velocity.

Angular frequency $\omega =$

Angular Velocity $\omega = \dfrac{\Delta\theta}{\Delta t}$ $\boxed{\dfrac{d\theta}{dt}}$

Unit : $\dfrac{\text{radian}}{\text{sec}}$

Dimension : $[M^0 L^0 T^{-1}]$

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☛ It is axis vector quantity.

$$\boxed{S = r\theta}$$

$$V = \frac{ds}{dt}$$

$$V = \frac{d}{dt}(r\theta)$$

$$V = r\frac{d\theta}{dt}$$

$$\boxed{V = r\omega}$$

linear velocity = r × Angular velocity

$$\omega = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{1}{T} = \nu$$

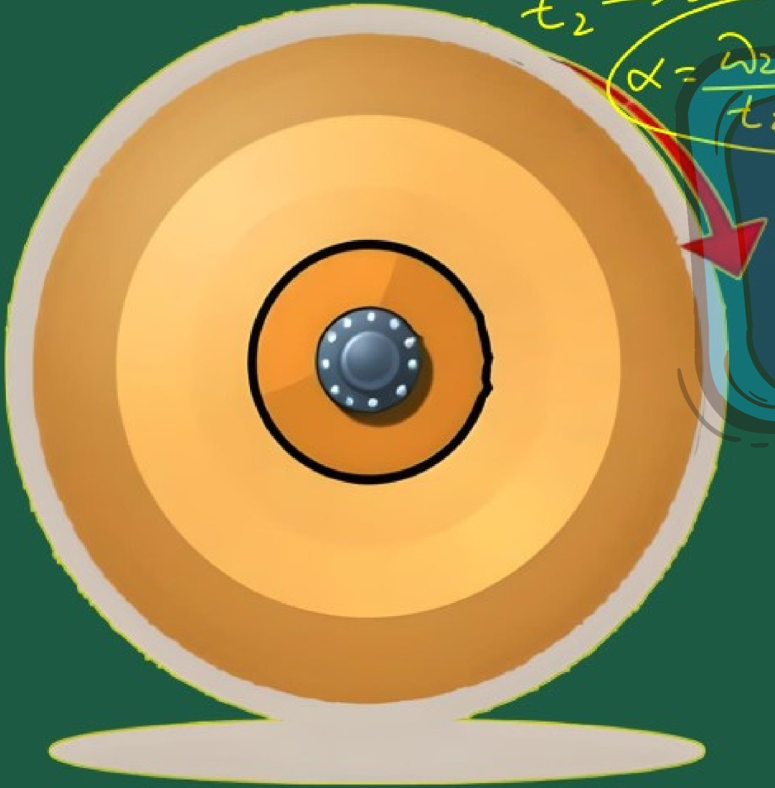
$$\boxed{\omega = 2\pi\nu}$$

ANGULAR ACCELERATION

$t_1 \rightarrow \omega_1$

$t_2 \rightarrow \omega_2$

$\alpha = \dfrac{\omega_2 - \omega_1}{t_2 - t_1}$



- The rate of change in angular velocity with time is called angular acceleration.

Angular acceleration $\alpha = \dfrac{\Delta\omega}{\Delta t}$

Unit : $\dfrac{\text{radian}}{\text{sec}^2}$

Dimension : $[M^0 L^0 T^{-2}]$

Note

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \alpha = \frac{d^2\theta}{dt^2}$$

$\alpha = \frac{d\omega}{dt}$

$\alpha = \frac{d}{dt}\left(\frac{d\theta}{dt}\right)$

* $\boxed{\alpha = \frac{d^2\theta}{dt^2}}$

$v = \frac{dx}{dt}$ $\quad a = \frac{dv}{dt}$ $\quad a = \frac{d^2x}{dt^2}$

$x \rightarrow \theta$

$v \rightarrow \omega$

$a \rightarrow \alpha$

$\alpha = \frac{d}{dt}\left(\frac{v}{r}\right)$

$\alpha = \frac{1}{r}\frac{dv}{dt}$

$\alpha = \frac{a}{r}$

$\boxed{a = r\alpha}$

RELATION BETWEEN A LINEAR VELOCITY (V) AND ANGULAR VELOCITY (ω)

$$\omega = \frac{2\pi}{T}$$

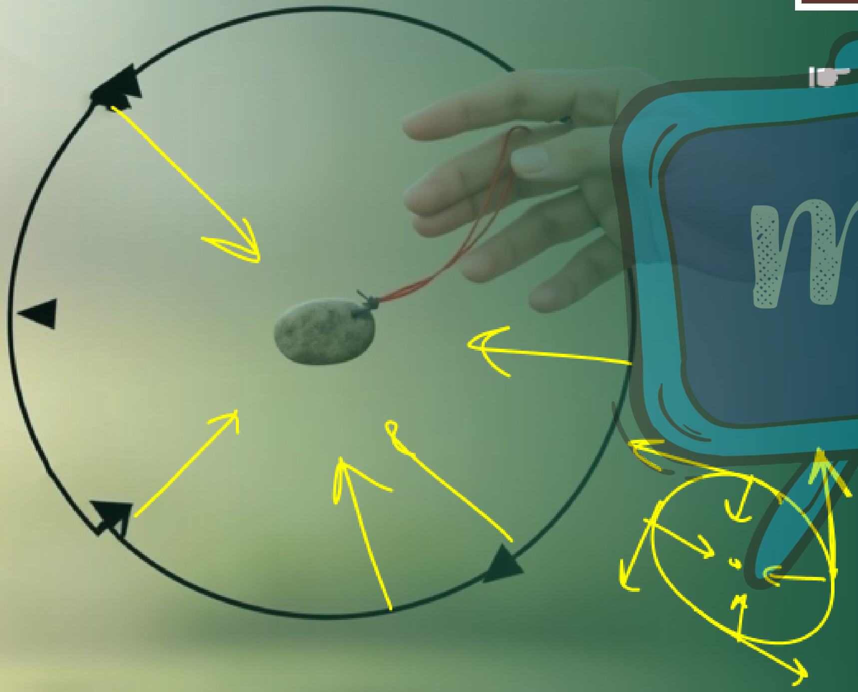
Or

$$\omega = 2\pi f$$

$\omega = \frac{\theta}{T}$

$\frac{1}{T} = f$

CENTRIPETAL FORCE



- The force that makes a particle move in a circular path and which has direction always towards the centre of the circular path is called centripetal force.

$$F = ma$$

$$F = \frac{mv^2}{r}$$

$$F = m\omega^2 r$$

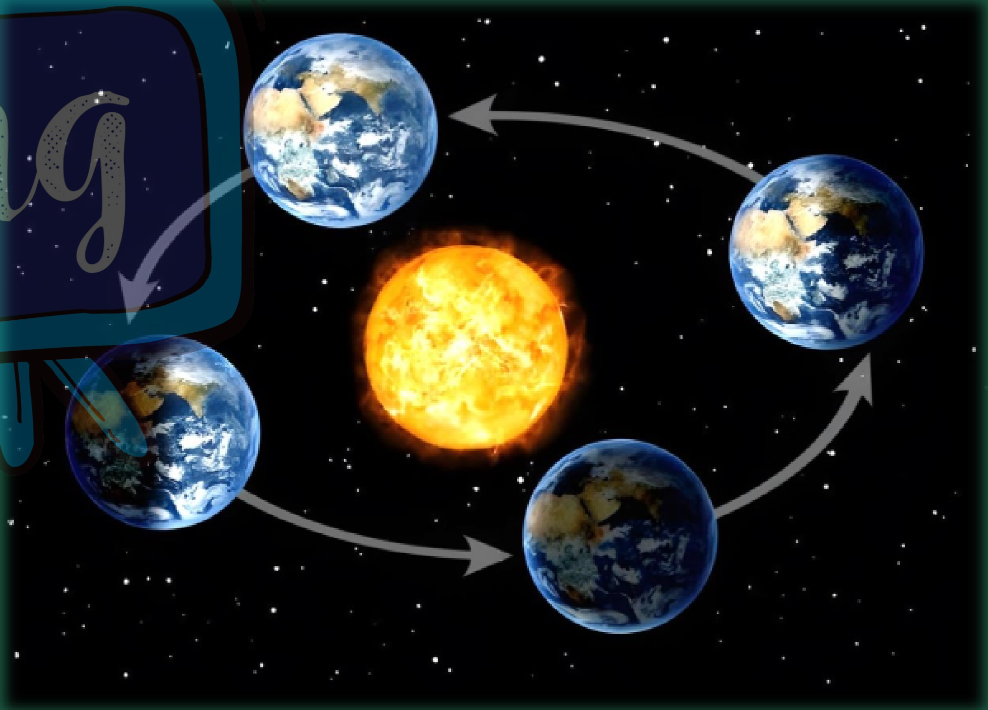
$$F = 4m\pi^2 f^2 r$$

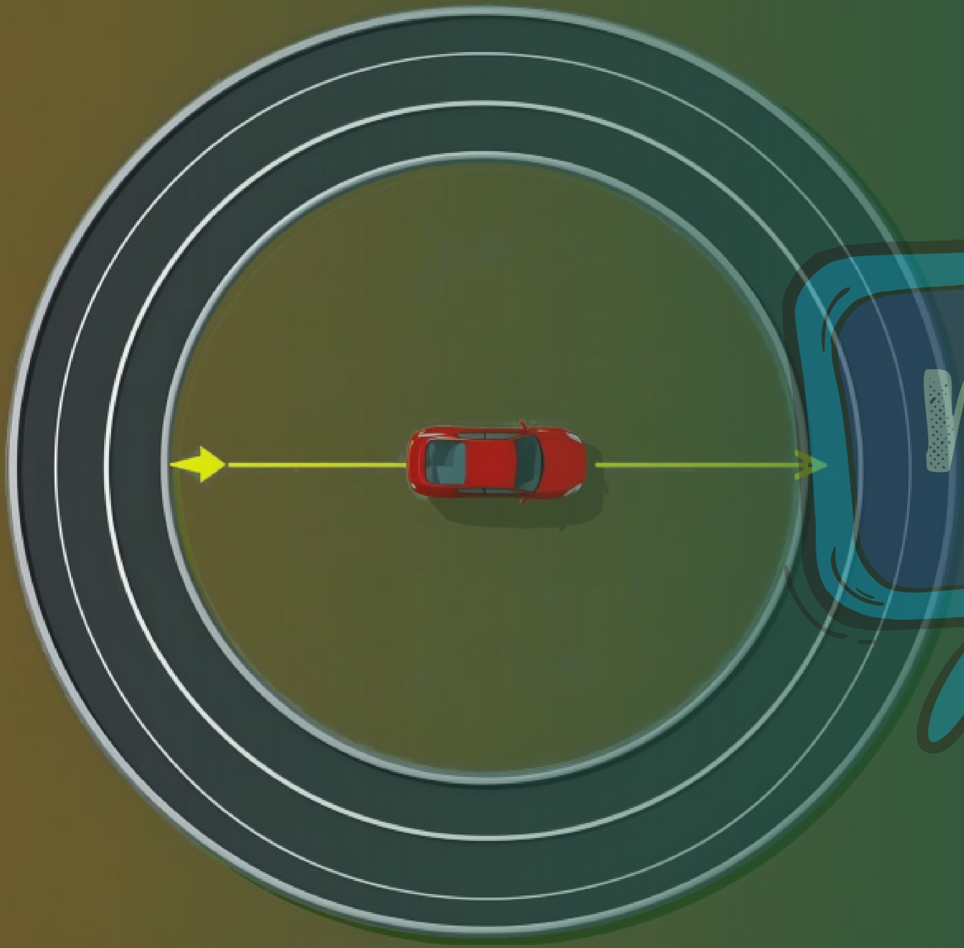
$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{m(r\omega)^2}{r} = \frac{m\cancel{r^2}\omega^2}{\cancel{r}}$$

$$F_c = mr\omega^2$$

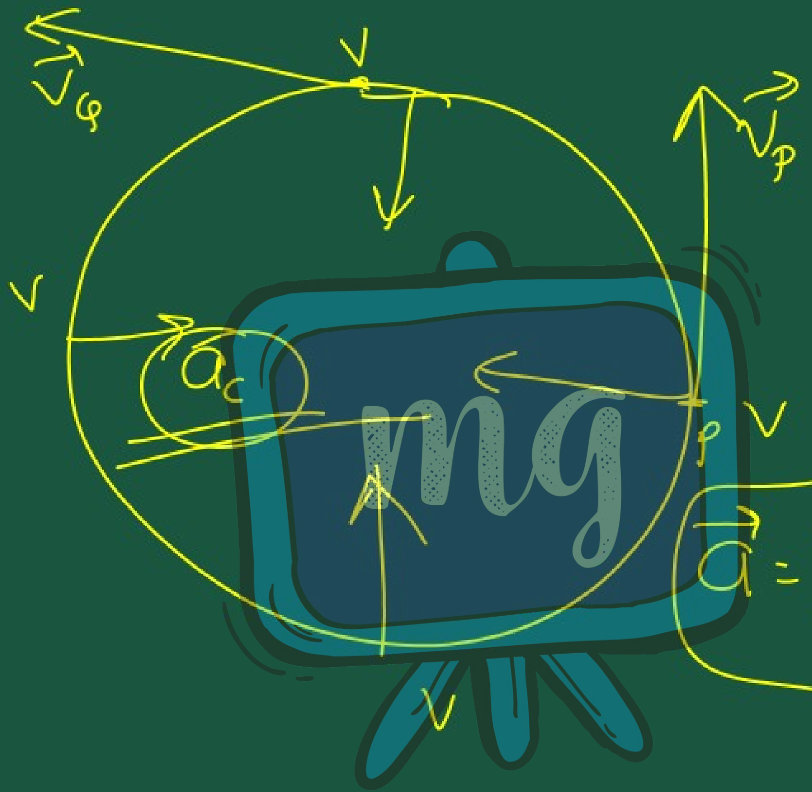
- Example : revolution of the earth around the sun.





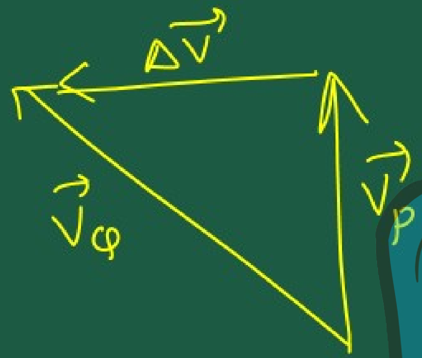
CENTRIPETAL ACCELERATION

- In uniform circular motion, the speed of the particle remains unchanges then the direction of acceleration is perpendicular to the direction of velocity with along the radius of the circle towards the centre. This type of acceleration is called centripetal acceleration.

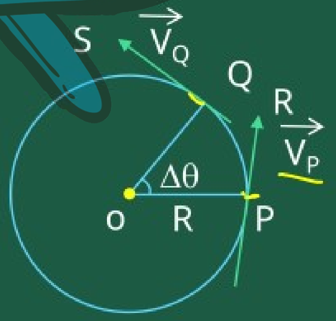


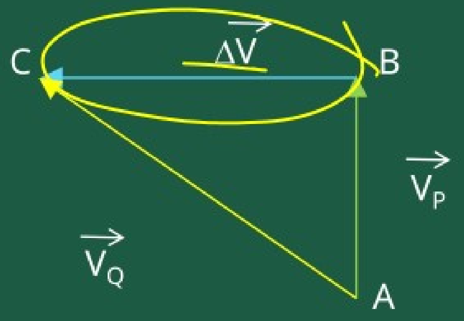
$$\Delta \vec{v} = \vec{v}_\varphi - \vec{v}_p$$

$$\vec{a} = \frac{\vec{v}_\varphi - \vec{v}_p}{\Delta t}$$



- Suppose a particle is moving on a circular path of radius R. the magnitude of the velocity of the particle is contant, while the direction changes with time.





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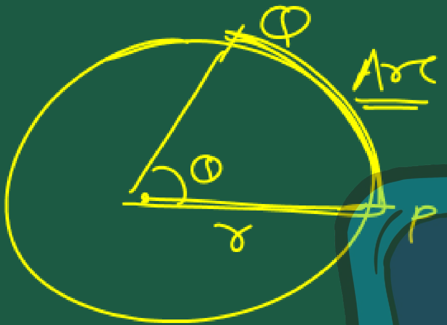
- Suppose at time t the particle is at point P and the velocity $\overrightarrow{V_p}$ and while at time $t + \Delta t$ the particle is at point Q and velocity is $\overrightarrow{V_Q}$.

$$\text{Acceleration } \vec{a} = \frac{\overrightarrow{V_Q} - \overrightarrow{V_P}}{\Delta t} \qquad .....(i)$$



distance = speed x time

$PQ = V\Delta T$

From the triangle law of vector addition

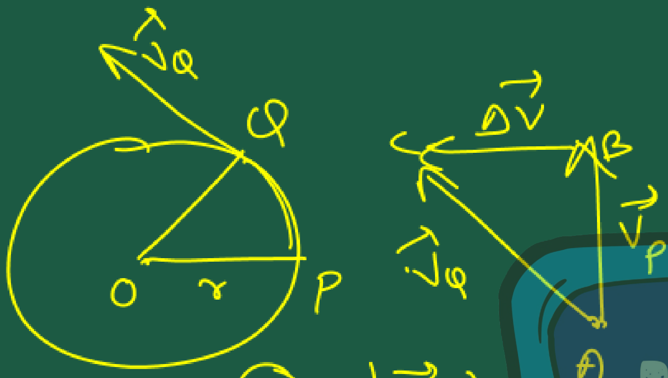
$$\vec{V_Q} = \vec{V_P} + \vec{\Delta V}$$

$$\vec{\Delta V} = \vec{V_Q} - \vec{V_P} \qquad \text{......(ii)}$$

From equation (i) and (ii)

$$\vec{a} = \frac{\vec{\Delta V}}{\Delta t} \qquad \text{......(iii)}$$

$BC = \Delta V$, $PQ = V\Delta t$, $AB = V$, $OP = R$



In ΔPOQ and ΔBAC

$$\frac{BC}{PQ} = \frac{AB}{OP}$$

$$\frac{\Delta V}{V\Delta t} = \frac{V}{R}$$

$$\frac{\Delta V}{\Delta t} = \frac{V^2}{R}$$

$$\because \; a = \frac{\Delta V}{\Delta t}$$

$$\frac{\Delta V}{V\Delta t} = \frac{|\vec{V}_p|}{r}$$

$$\frac{a}{V} = \frac{V}{r}$$

Centripetal Acceleration

$$a_c = \frac{V^2}{r}$$

$$F_c = m a_c$$

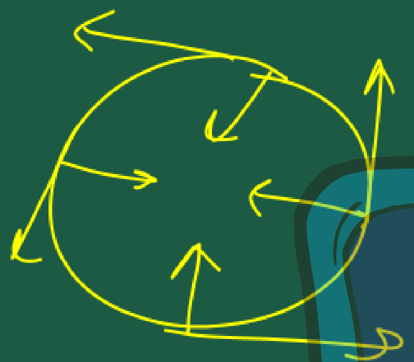
$$F_c = \frac{mV^2}{r}$$

$$a = \frac{V^2}{R}$$

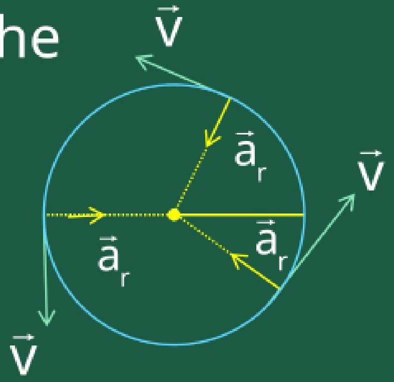
$$\boxed{a_r = \frac{V^2}{R}}$$

$$\because \quad V = R\omega$$

$$\boxed{a_r = \omega^2 R}$$



- The direction of velocity of a particle moving in uniform circular motion is tangential to the circle and the direction of acceleration is always perpendicular to the direction of velocity and towards the centre of the circle.



Example :

An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 seconds.

(a) What is the angular speed and the linear speed of the motion.

(b) Is the acceleration vector a constant vector? what is it's magnitude.

$7 \text{ rev.} = 100 \text{ sec}$

$7 \text{ rev} \Rightarrow 100 \text{ sec}$

$1 \longrightarrow \dfrac{7 \text{ rev}}{100 \text{ sec}}$

$f = \nu = \dfrac{7 \text{ rev}}{100}$

$\omega = 2\pi f$

$= \dfrac{2\pi \times 7}{100}$

$= \dfrac{7\pi}{50} = \dfrac{\cancel{7} \times 22}{\cancel{7} \times 50}$

$= \dfrac{11}{25} \text{ Rad/sec}$

Answer :

$r = 12\text{cm},\ T = \dfrac{100}{7}\text{sec}$

$\omega = \dfrac{2\pi}{T} = \dfrac{2 \times 3.14}{100/7} = 0.44\,\text{rad/s}$

Linear velocity $v = r\omega$

$v = 12 \times 10^{-2} \times 0.44$

(a)

$V = r\omega$

$V = 12 \times 10^{-2} \times \dfrac{11}{25}$

$= \underline{\underline{0.44}}$

(b) The direction of acceleration at every point of the circle being towards the centre the direction of acceleration keeps changing continuously.

$a = \frac{v^2}{r}$

$= \omega^2 r$

$= \left(\frac{11}{25}\right)^2 \times 12 \times 10^{-2}$

$$a_r = \omega^2 r = (0.44)^2 \times 12 \times 10^{-2}$$

$$= 2.3 \times 10^{-2} m/s^2$$

LEARNING OUTCOME

To study for Uniform Circular Motion

ASSESSMENT

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1 In Uniform Circular Motion –

A. Both velocity and acceleration change

B. Bothe velocity and acceleration are constant

C. Velocity remains constant and acceleration changes

D. Acceleration remains constant and velocity changes

ASSESSMENT

2 A particle moves in a circular path with uniform speed. The acceleration of the particle is –

A along the circumference of circle

B along the tangent

C along the radius

D zero

ASSESSMENT

3 Two particles of same mass are moving with same speed in circular paths of radius r_1 and r_2 . The ratio of their centripetal forves wil be –

$F_1 = \frac{mv^2}{r_1}$

$F_2 = \frac{mv^2}{r_2}$

$\frac{F_1}{F_2} = \frac{r_2}{r_1}$

A $\frac{r_2}{r_1}$

B $\frac{r_1}{r_2}$

C $\left(\frac{r_2}{r_1}\right)^2$

D $\left(\frac{r_1}{r_2}\right)^2$

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ASSESSMENT

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4 What is the angular velocity of the second hand of a clock?

$\omega = \frac{2\pi}{60} = \frac{\pi}{30}$ Rad/sec