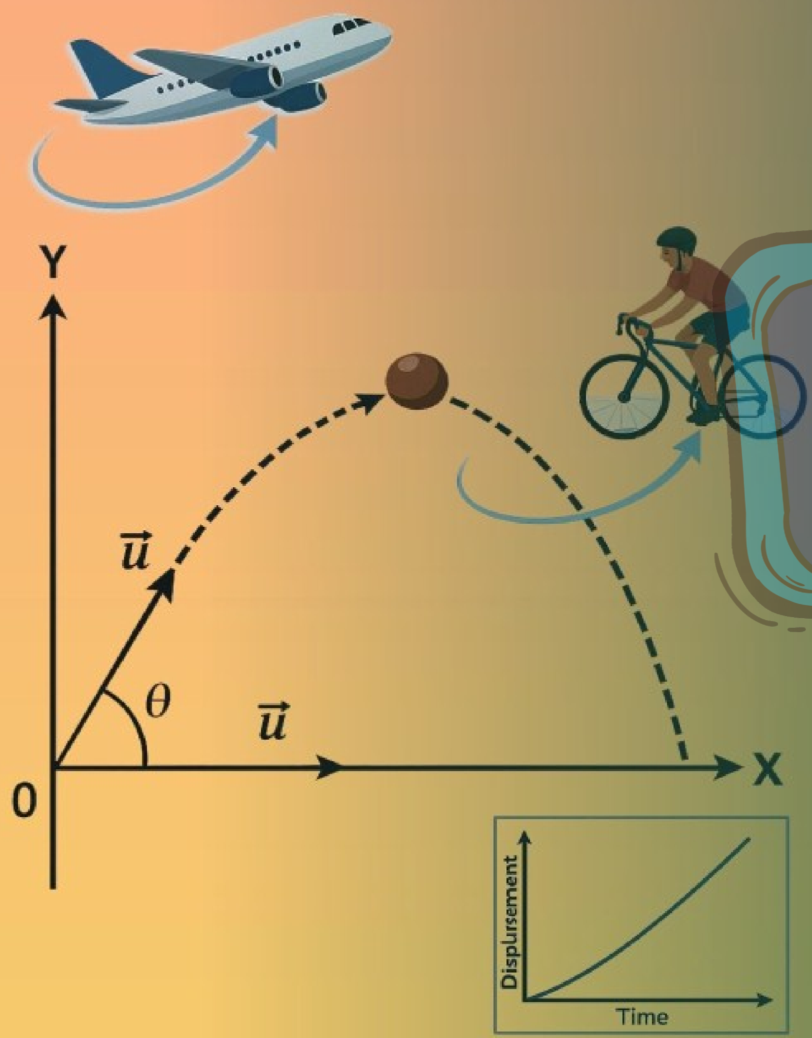

CLASS – 11

PHYSICS

Chapter – 3

Motion in a Plane

Part – 3

Resolution of Vectors

Alok Gaur

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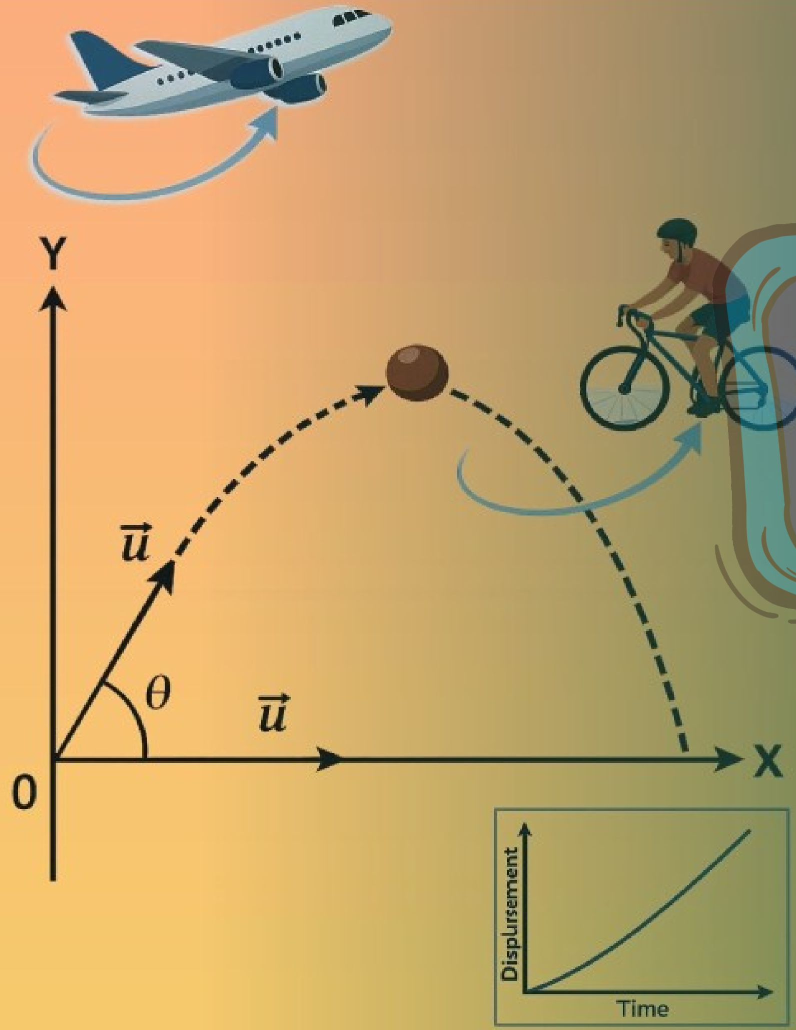
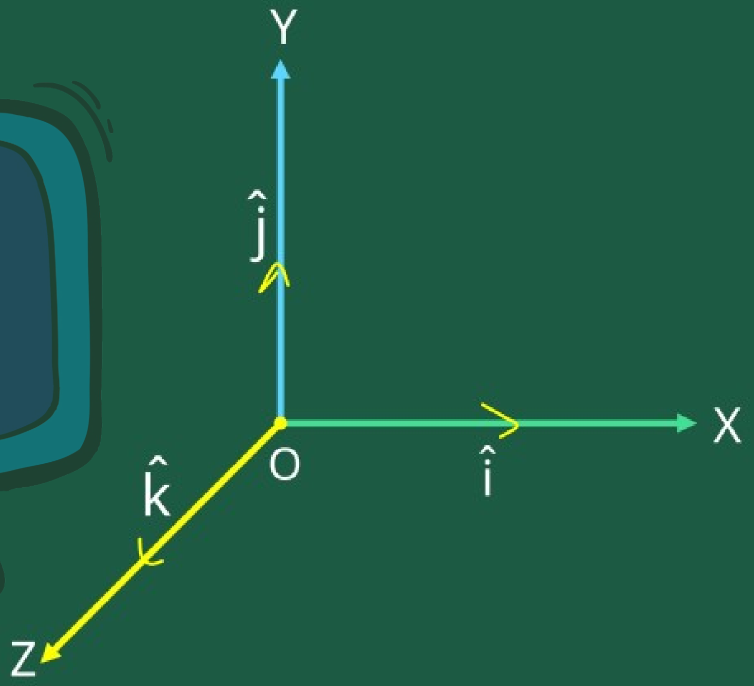
OVERVIEW



1. Scalars and Vectors
2. Addition of Vectors
3. Resolution of Vectors
4. Projectile Motion
5. Uniform Circular Motion

RESOLUTION OF VECTORS

- In this co-ordinate system OX, OY and OZ are three mutually perpendicular axes which meet at one origin O.
- There are the following two methods of resolution of a vector.


Y
ĵ
X
O
î
k̂
Z

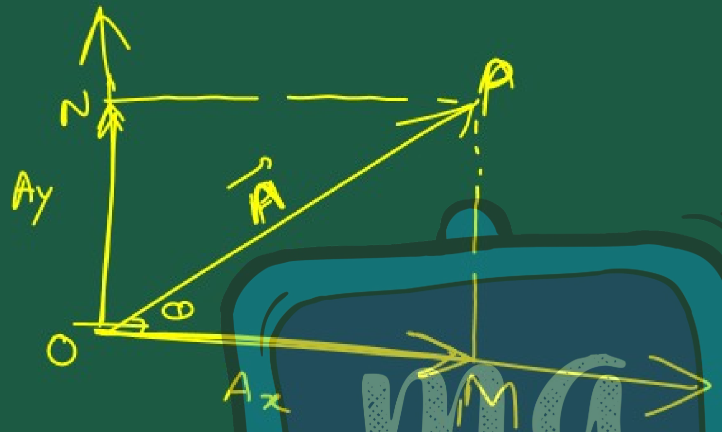
RESOLUTION OF VECTORS

2D Frame

3D Frame

TWO DIMENSIONAL RESOLUTION OF A VECTOR

Suppose that the resolution of a vector $\vec{A}$ is to be done in 2D frame which is shown in figure by OP and is located in the X–Y plane.


N
P
Ay
A
θ
O
Ax
M

$\vec{A} = \vec{A_x} + \vec{A_y}$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

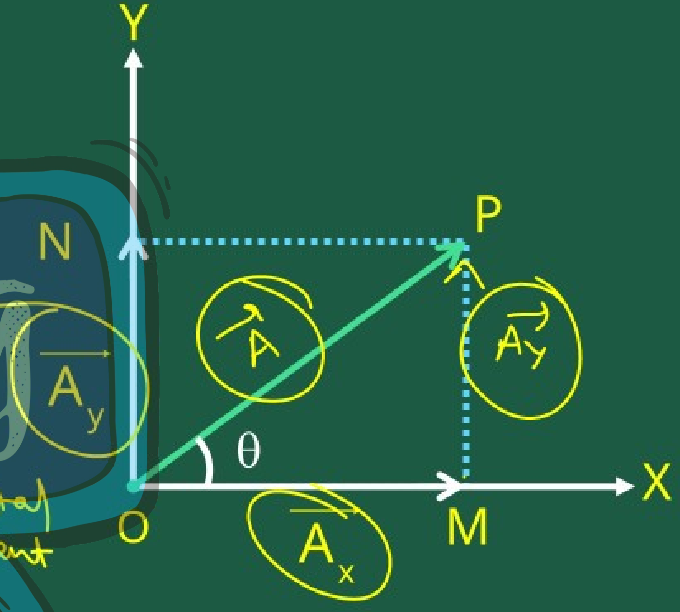
$\vec{A} = 2\hat{i} + 3\hat{j}$

$\cos\theta = \frac{A_x}{A}$

$A_x = A\cos\theta$ → Horizontal Component

$\sin\theta = \frac{A_y}{A}$

$A_y = A\sin\theta$ → Vertical Component



$OM = A_x, \quad ON = A_y$

In the right angled ΔOMP

From the triangle law of vector addition

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{ON}$$

$$\overrightarrow{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$$

If $\hat{i}$ and $\hat{j}$ are the unit vectors along X and Y axes respectively then $\overrightarrow{A_x} = A_x\hat{i}$, $\overrightarrow{A_y} = A_y\hat{j}$ then

$$\overrightarrow{A} = A_x\hat{i} + A_y\hat{j}$$

In ΔOMP

$$\frac{A_x}{A} = \cos\theta \Rightarrow A_x = A\cos\theta \quad ......(i)$$

$$\frac{A_y}{A} = \sin\theta \Rightarrow A_y = A\sin\theta \quad .....(ii)$$

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To find magnitude A of vector $\vec{A}$

On adding equation (i) and (ii) after squaring them

$$A_x^2 + A_y^2 = A^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow A_x^2 + A_y^2 = A^2$$

$$\Rightarrow A = \sqrt{A_x^2 + A_y^2}$$

$\vec{A} = 2\hat{i} + 3\hat{j}$

$A = \sqrt{(2)^2 + (3)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

Magnitude of $\vec{A}$

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To find direction

In ΔOMP

$$\tan\theta = \frac{MP}{OM} = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

$\vec{A} = 2\hat{i} + 3\hat{j}$

$\theta = \tan^{-1}\left[\frac{3}{2}\right]$

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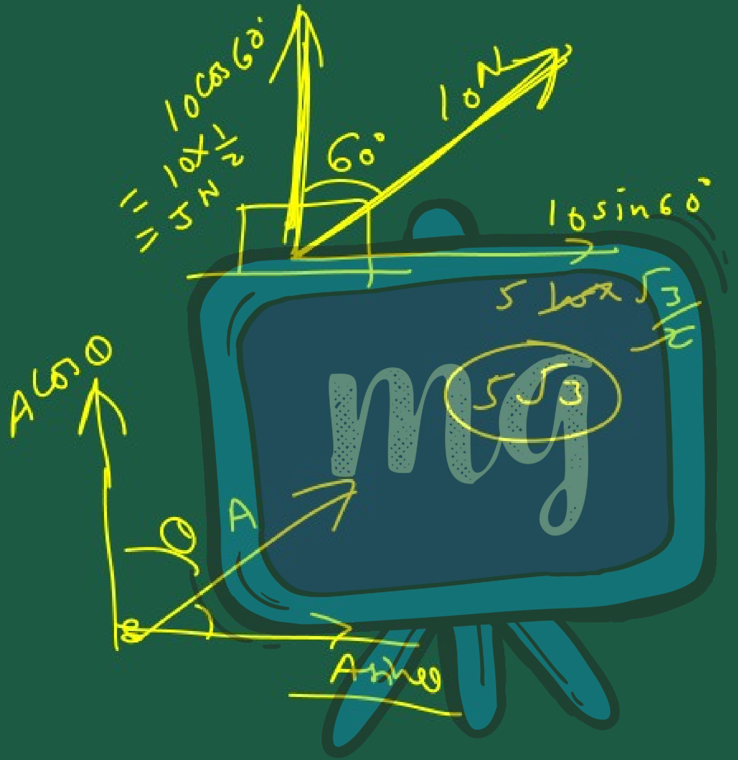


THREE DIMENSIONAL RESOLUTION OF A VECTOR

Suppose a vector $\vec{A}$ is to be resolved in three dimensional frame of reference.

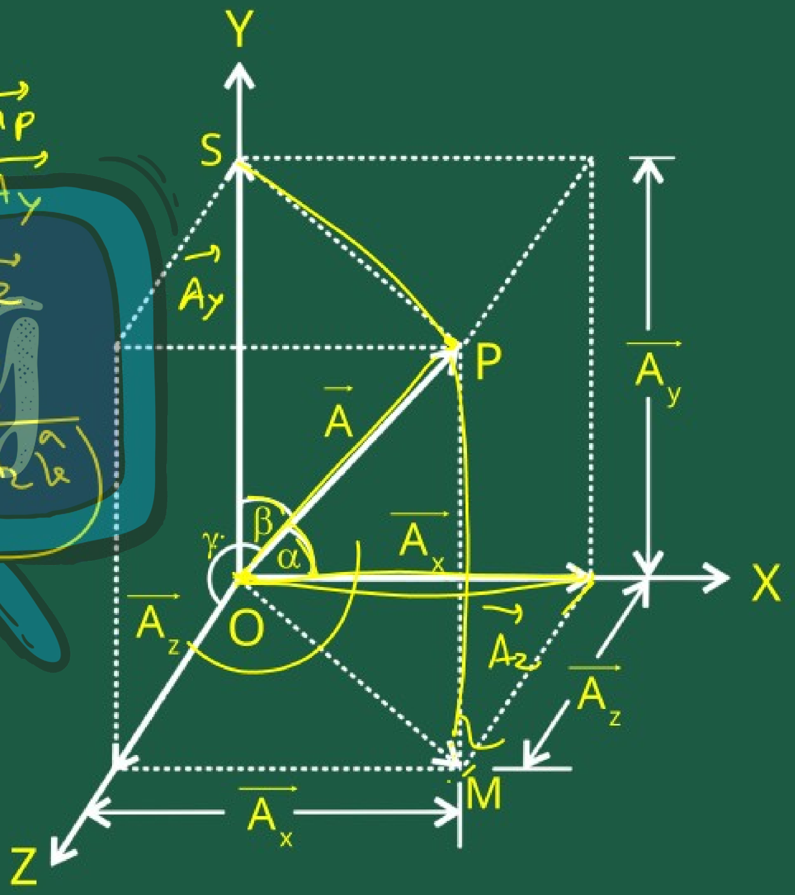
$$\vec{OP} = \vec{OM} + \vec{MP}$$

$$\vec{A} = \vec{OM} + \vec{A_y}$$

$$\vec{OM} = \vec{A_x} + \vec{A_z}$$

$$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$



In ΔOMP

From the triangle law of vector addition

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} \qquad ....(i)$$

From ΔONM

$$\overrightarrow{OM} = \overrightarrow{ON} + \overrightarrow{NM}$$

from equation (i)

$$OP = ON + NM + MP$$

$$A = A_x + A_y + A_z$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

To find magnitude A of vector

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

To find direction

In ΔOSP

$$\cos\beta = \frac{A_y}{A} \Rightarrow A_y = A\cos\beta \quad \text{....(ii)}$$

Similarly

$$A_x = A\cos\alpha \quad \text{.....(iii)}$$

$$A_z = A\cos\gamma \quad \text{.....(iv)}$$

where cosα cosβ and cosγ are called the direction cosines of the vector.

On keeping value of A_x, A_y and A_z in equation

$$A^2 = A_x^2 + A_y^2 + A_z^2$$

$$A^2 = A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma$$

$$A^2 = A^2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

The sum of the squares of direction cosines of a vector is always one (unit).

MOTION IN A PLANE

Vector representation of displacement

$$\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1$$

$$\overrightarrow{\Delta r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$\overrightarrow{\Delta r} = \overrightarrow{\Delta x} + \overrightarrow{\Delta y}$$

$$\left|\overrightarrow{\Delta r}\right| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

VELOCITY

$$\vec{V}_{avg} = \frac{\overrightarrow{\Delta r}}{\Delta t}$$

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

$$\vec{V}_{avg} = V_x\hat{i} + V_y\hat{j}$$

$$V_{avg} = \sqrt{V_x^2 + V_y^2}$$

ACCELERATION

$$\vec{a}_{avg} = \frac{\overrightarrow{\Delta v}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$\vec{a}_{avg} = a_x\,\hat{i} + a_y\,\hat{j}$$

$$a = \sqrt{a_x^2 + a_y^2}$$

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LEARNING OUTCOMES

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1. To study of resolutions of vectors in 2D and 3D
2. To study for motion in plane

ASSESSMENT

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1 The angle by vector $\vec{A} = \hat{i} + \hat{j}$ with the x-axis in 2-D and 3-D

A. 90°

B. 45°

C. 22.5°

D. 30°

$\vec{A} = \hat{i} + \hat{j}$

$\tan\theta = \frac{A_y}{A_x} = \frac{1}{1}$

$\tan\theta = 1$

$\theta = 45^\circ$

ASSESSMENT

2 If $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ then the cosines of the vector $\vec{A}$ are –

A $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, -\frac{5}{\sqrt{45}}$

B $\frac{4}{\sqrt{45}}, 0, \frac{4}{\sqrt{45}}$

C $\frac{1}{\sqrt{45}}, \frac{2}{\sqrt{45}}, \frac{3}{\sqrt{45}}$

D $\frac{3}{\sqrt{45}}, \frac{2}{\sqrt{45}}, \frac{5}{\sqrt{45}}$

$\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$A = \sqrt{(2)^2 + (4)^2 + (-5)^2}$

$A = \sqrt{4+16+25}$

$A = \sqrt{45}$

$A = 3\sqrt{5}$

$\cos\alpha = \frac{A_x}{A} = \frac{2}{\sqrt{45}}$

$\cos\beta = \frac{A_y}{A} = \frac{4}{\sqrt{45}}$

$\cos\gamma = \frac{A_z}{A} = \frac{-5}{\sqrt{45}}$