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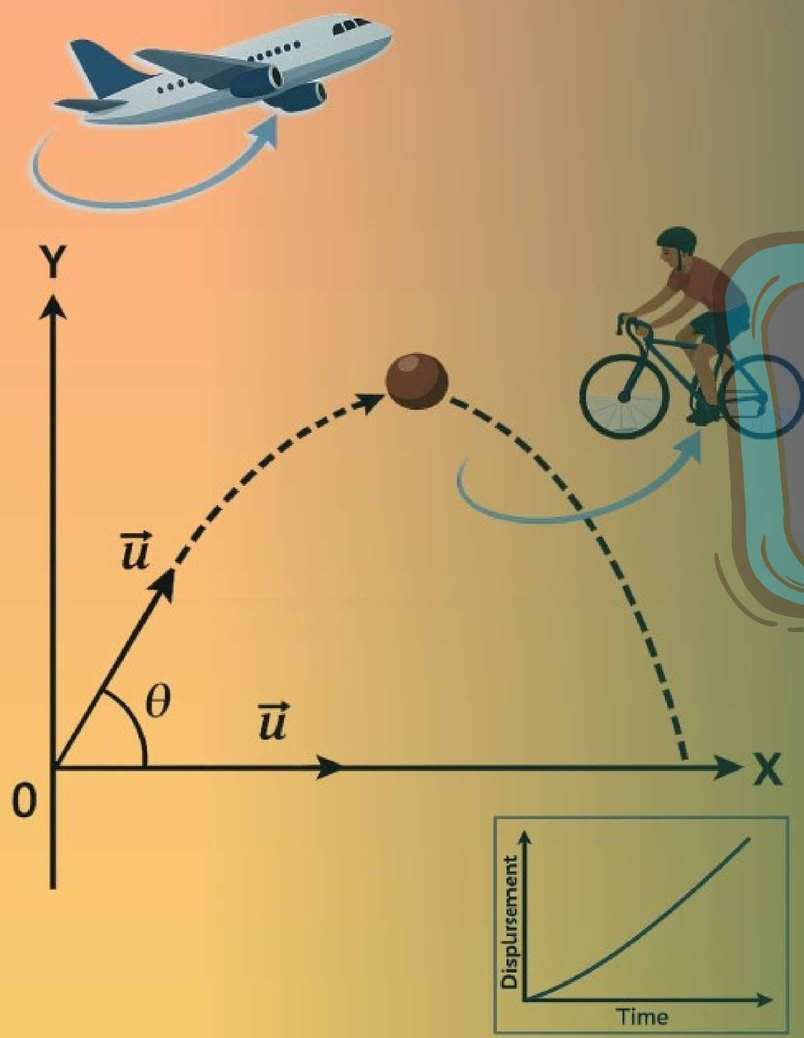

CLASS – 11

PHYSICS

Chapter – 3

Motion in a Plane

Exercise (Q. 1 - 12)

Alok Gaur

EXERCISE

1. State, for each of the following physical quantities, if it is a scalar or a vector :

   volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

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EXERCISE

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1. State, for each of the following physical quantities, if it is a scalar or a vector :

   volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

   Answer : **Scalar :** Mass, volume, density, angular frequency,

EXERCISE

number of moles, speed.

Vector : Acceleration, angular velocity, velocity, displacement.

A scalar quantity is specified by its magnitude. Mass, volume, density, angular frequency, number of moles, speed are some of the scalar physical quantities.

EXERCISE

A vector quantity is specified by its magnitude and the direction associated with it.

Acceleration, angular velocity, velocity, displacement belong to this category

EXERCISE

2. Pick out the two scalar quantities in the following list : force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

EXERCISE

2. Pick out the two scalar quantities in the following list :

force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Answer : Work and current are examples of scalar quantities.

EXERCISE

Work done is said to be the dot product of force and displacement. As the dot product of two quantities is always a scalar, work is considered as a scalar physical quantity.

Current is described by its magnitude. Its direction is not considered. Thus, it is a scalar quantity.

EXERCISE

3. Pick out the only vector quantity in the following list :
Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

EXERCISE

3. Pick out the only vector quantity in the following list :
Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Answer : Impulse

It is given by the product of force and time.

EXERCISE

As force is a vector quantity, its product with time gives a vector quantity.

EXERCISE

4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) adding any two scalars

EXERCISE

(a) adding any two scalars

Answer : Not Meaningful,

The addition of two scalar quantities will be meaningful only if they both represent the same physical quantity.

EXERCISE

(b) adding a scalar to a vector of the same dimensions

EXERCISE

(b) adding a scalar to a vector of the same dimensions

Answer : Not Meaningful,

The addition of a vector quantity with a scalar quantity is considered not meaningful.

EXERCISE

(c) multiplying any vector by any scalar

EXERCISE

(c) multiplying any vector by any scalar

Answer : Meaningful,

A scalar can be multiplied with a vector. Force is multiplied with time to give impulse.

EXERCISE

(d) multiplying any two scalars

EXERCISE

(d) multiplying any two scalars

Answer : Meaningful,

A scalar, respective to the physical quantity, can be multiplied with another scalar having the same or different dimensions.

EXERCISE

(e) adding any two vectors

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EXERCISE

(e) adding any two vectors

Answer : Not Meaningful,

The addition of two vector quantities is considered meaningful only if they both represent the same physical quantity.

Force + Force

× Force + Momentum

EXERCISE

(f) adding a component of a vector to the same vector

EXERCISE

(f) adding a component of a vector to the same vector

Answer : Meaningful,

A component of a vector can be added to the same vector as both ave the same dimensions.

EXERCISE

5. Read each statement below carefully and state with reasons, if it is true or false :

(a) The magnitude of a vector is always a scalar

EXERCISE

(a) The magnitude of a vector is always a scalar

Answer : True,

The magnitude of a vector is a number. So, it is a scalar.

EXERCISE

(b) each component of a vector is always a scalar

EXERCISE

(b) each component of a vector is always a scalar ×

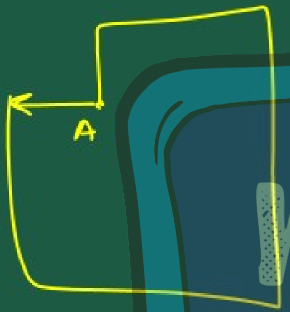
Answer : False,

Each component of a vector is a vector.

EXERCISE

(c) the total path length is always equal to the magnitude of the displacement vector of a particle.

EXERCISE



(c) the total path length is always equal to the magnitude of the displacement vector of a particle.

Answer : False,

The total path length is scalar, whereas displacement is a vector quantity. So, the total path length is greater than the magnitude of displacement.

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EXERCISE

It is equal to the magnitude of displacement only when a particle is moving in a straight line.

EXERCISE

(d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time,

$\text{Avg speed} = \frac{\text{Total distance}}{\text{Total time}}$

$\text{Avg velocity} = \frac{\text{Total Displacement}}{\text{Total time}}$

EXERCISE

(d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time,

Answer : True,

It is because the total path length is always greater than or equal to the

EXERCISE

always greater than or equal to the magnitude of displacement of a particle.

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EXERCISE

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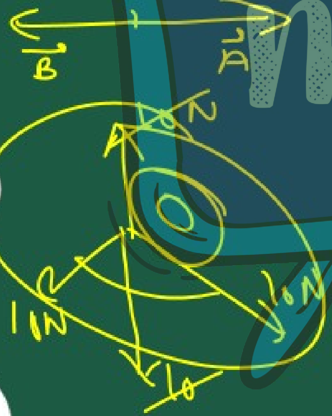
(e) Three vectors not lying in a plane can never add up to give a null vector.

EXERCISE



(e) Three vectors not lying in a plane can never add up to give a null vector.

Answer : True,

Three vectors, which do not lie in a plane, can't be represented by the sides of a triangle taken in the same order.

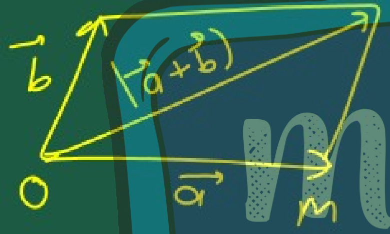
EXERCISE

6. Establish the following vector inequalities geometrically or otherwise :

(a) $|a + b| \leq |a| + |b|$

EXERCISE

(a) $|a + b| \leq |a| + |b|$



Answer : Let $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram OMNP as shown below:

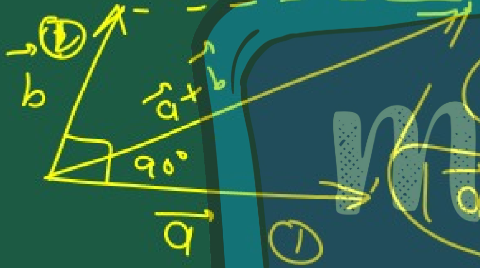
$|\overrightarrow{OM}| = |\vec{a}|$ ....(i)

$|\overrightarrow{MN}| = |\overrightarrow{OP}| = |\vec{b}|$ ....(ii)

$|\overrightarrow{ON}| = |\vec{a} - \vec{b}|$ ....(iii)

$\vec{a}$ $\vec{b}$

$|\vec{a}+\vec{b}| \leq |\vec{a}| + |\vec{b}|$



$|\vec{a}+\vec{b}| = \sqrt{(1)^2+(1)^2+2(1)(1)\cos 90°}$

$|\vec{a}+\vec{b}| = \sqrt{2}$

$|\vec{a}| + |\vec{b}| = 1+1$

$= 2$

EXERCISE

As each side is smaller than the sum of the other two sides in a triangle,

In ΔOMN,

$ON < (OM + MN)$

$|\vec{a} + b| < |\vec{a}| + |b|$ ....(iv)

EXERCISE

If $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then:

$|\vec{a}+\vec{b}| = |\vec{a}|+|\vec{b}|$ ....(v)

Combine equations (iv) and (v)

$|\vec{a}+\vec{b}| \leq |\vec{a}|+|\vec{b}|$

EXERCISE

(b) $|a + b| \geq ||a| - |b||$

$|\vec{b}| = 1$, $|\vec{a}| = 2$

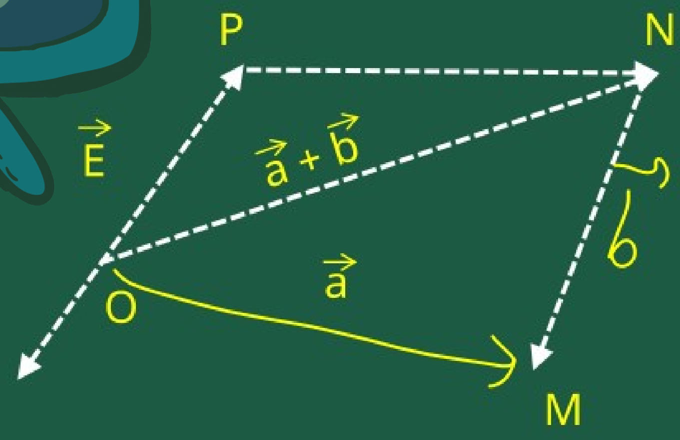
$|\vec{a} + \vec{b}| = \sqrt{(2)^2 + (1)^2 + 2(1)(2)\cos 90^\circ}$

$= \sqrt{5} = 2.2$

$||\vec{a}| - |\vec{b}|| = 2 - 1 = 1$

(b) $|a + b| \geq ||a| - |b||$

Answer : Let $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown below:



EXERCISE

$|\overrightarrow{OM}| = |\vec{a}|$ ....(i)

$|\overrightarrow{MN}| = |\overrightarrow{OP}| = |\vec{b}|$ ....(ii)

$|\overrightarrow{ON}| = |\vec{a} + \vec{b}|$ ....(iii)

As each side is smaller than the sum of the other two sides in a triangle,

InΔOMN,

$ON + MN > OM$

$ON + OM > MN$

$|ON| > |OM - OM| \, (\because OP = MN)$

EXERCISE

If $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then:

$|a+b| = ||a|-|b||$..........(v)

Combine equations (iv) and (v)

$|\vec{a}+b| \geq ||\vec{a}|-|b||$

EXERCISE

(c) $|a - b| \leq |a| + |b|$

$|a-b| = \sqrt{4+1-2(2)(1)\cos 90^\circ}$

$= \sqrt{5} \leq 2.2$

$|\vec{a}| + |\vec{b}| = 2+1 = 3$

EXERCISE

(c) $|a - b| \leq |a| + |b|$

Answer : Let $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram PQRS:

$|\overrightarrow{OR}| = |\overrightarrow{PS}| = |\vec{b}|$ ....(i)

$|\overrightarrow{OP}| = |\vec{a}|$ ....(ii)

As each side is smaller than the sum of the other two sides in a triangle,

In ΔOPS,

EXERCISE

$OS < OP + PS$

$|\vec{a} - b| < |\vec{a}| + |-b|$

$|\vec{a} - b| < |\vec{a}| + |b|$ ....(iii)

If the two vectors act in a straight line but in opposite directions, then:

$|\vec{a} - b| = |\vec{a}| + |b|$

Combine equations (iii) and (iv)

$|\vec{a} - b| \leq |\vec{a}| + |b|$

EXERCISE

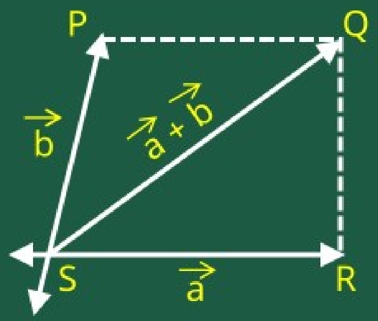
(d) $|a - b| \geq ||a| - |b||$

When does the equality sign above apply?

(d) $|a - b| \geq ||a| - |b||$

When does the equality sign above apply?

Answer : Let $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram PQ



EXERCISE

$OS + PS > OP$ ....(i)

$OS > OP - PS$ ....(ii)

$|\vec{a} - b| < |\vec{a}| - |b|$ ....(iii)

The L.H.S is always positive and R.H.S can be positive or negative.

To make both quantities positive, take modulus on both sides.

$\left\| \vec{a} - \vec{b} \right\| < \left\| \vec{a} \right| - \left| \vec{b} \right\|$

$\left| \vec{a} - \vec{b} \right| < \left\| \vec{a} \right| - \left| \vec{b} \right\|$ ....(iv)

EXERCISE

If the two vectors act in a straight line but in the same direction:

$$\left|\vec{a}-\vec{b}\right| = \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \qquad \text{..........(v)}$$

Combine equation (iv) and equation (v) $\left|\vec{a}-\vec{b}\right| \geq \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$

EXERCISE

7. Given a + b + c + d = 0, which of the following statements are correct :

(a) a, b, c, and d must each be a null vector

$-\vec{d} = \vec{a} + \vec{b} + \vec{c}$

$\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$

EXERCISE

(a) a, b, c, and d must each be a null vector

Answer : Incorrect

To make $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$, it is not necessary to have all four vectors as null vectors. There are many other combinations which will give the sum zero.

EXERCISE

(b) The magnitude of (a + c) equals the magnitude of (b + d)

EXERCISE

(b) The magnitude of (a + c) equals the magnitude of (b + d)

Answer : Correct

$$\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$$

$$\vec{a}+\vec{c}=-(\vec{b}+\vec{d})$$

Take modulus on both sides:

$$|\vec{a}+\vec{c}|=|-(b+d)|=|(b+d)|$$

So, the magnitude of $(\vec{a}+\vec{c})$ is the same as the magnitude of $(\vec{b}+\vec{d})$

EXERCISE

(c) The magnitude of a can never be greater than the sum of the magnitudes of b, c, and d

EXERCISE

(c) The magnitude of a can never be greater than the sum of the magnitudes of b, c, and d,

Answer : Correct

$\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$

$\vec{a} = -(\vec{b} + \vec{c} + \vec{d})$

Take modulus on both sides:

$\vec{a} = |(\vec{b} + \vec{c} + \vec{d})|$

$a \leq |b| + |c| + |d|$ ....(i)

EXERCISE

$(\vec{b}+\vec{c}+\vec{d})$ is the sum of vectors $\vec{b}$, $\vec{c}$ and $\vec{d}$. The magnitude of $(\vec{b}+\vec{c}+\vec{d})$ is less than, or equal to the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$. So, the magnitude of $\vec{a}$ cannot be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$. Equation (i) shows that the magnitude of $\vec{a}$ is equal to or less than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$

EXERCISE

(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear?

EXERCISE

(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear?

Answer : Correct

For, $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

$\vec{a}+(\vec{b}+\vec{c})+\vec{d}=0$

The resultant sum of the vectors $\vec{a},(b+\vec{c})$ and d is zero only if $(\vec{b}+\vec{c})$ lie

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EXERCISE

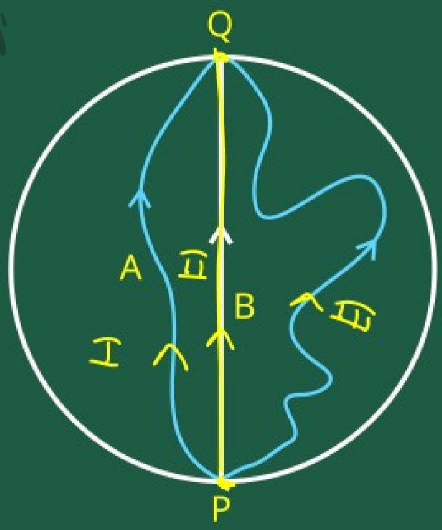
in the same plane as $\vec{a}$ and $\vec{d}$.

If $\vec{a}$ and $\vec{d}$ are collinear, then $(\vec{b}+\vec{c})$ is in the line of $\vec{a}$ and $\vec{d}$. This is true in this case and the vector sum of all the vectors will be zero.

EXERCISE

8. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?

EXERCISE



EXERCISE

Answer : The magnitudes of displacements are equal to the diameter of the ground.

Radius of the ground = 200 m

Diameter of the ground

$= 2 \times 200 = 400$m

So, the magnitude of the displacement for each girl is 400 m which is equal to the actual length of the path skated by girl B .

EXERCISE

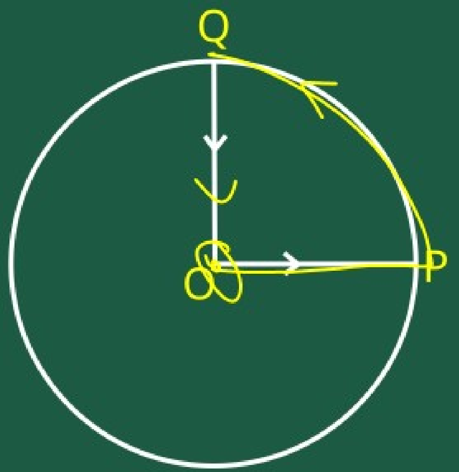
9. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. If the round trip takes 10 min, what is the

(a) net displacement

EXERCISE

(a) net displacement

Answer : The cyclist comes to the starting point after cycling for 10 minutes. So, his net displacement is zero.



EXERCISE

(b) average velocity, and

$\frac{\text{Displacement}}{\text{Time}} = 0$

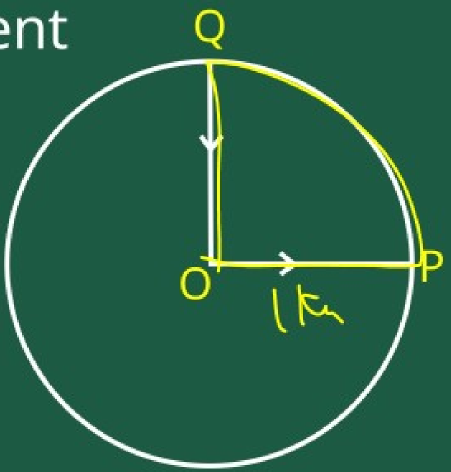
EXERCISE

(b) average velocity, and

Answer :

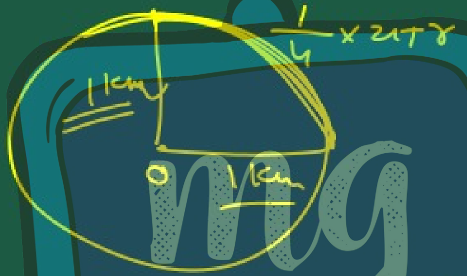
$$\text{Average velocity} = \frac{\text{Net displacement Total time}}{\text{Total time}}$$

As the net displacement of the cyclist is zero, his average velocity is also zero.



EXERCISE

(c) average speed of the cyclist?



$$\text{Distance} = 1 + \frac{1}{4} \times 2\pi(1) + 1$$

$$= 2 + \frac{\pi}{2}$$

$$= \frac{4+\pi}{2} \text{ km}$$

EXERCISE

(c) average speed of the cyclist?

Answer :

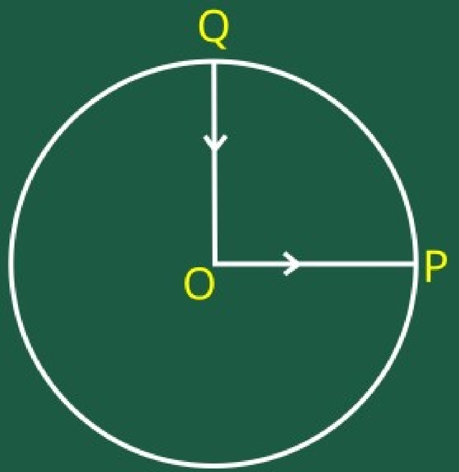
$$\text{Average speed} = \frac{\text{Total path length}}{\text{Total time}}$$

Total path length

= OP + PQ + QO

$$= 1 + \frac{1}{4}(2\pi \times 1) + 1$$

$$= 2 + \frac{1}{2}\pi 3.570\text{km}$$



EXERCISE

Time taken = 10 min

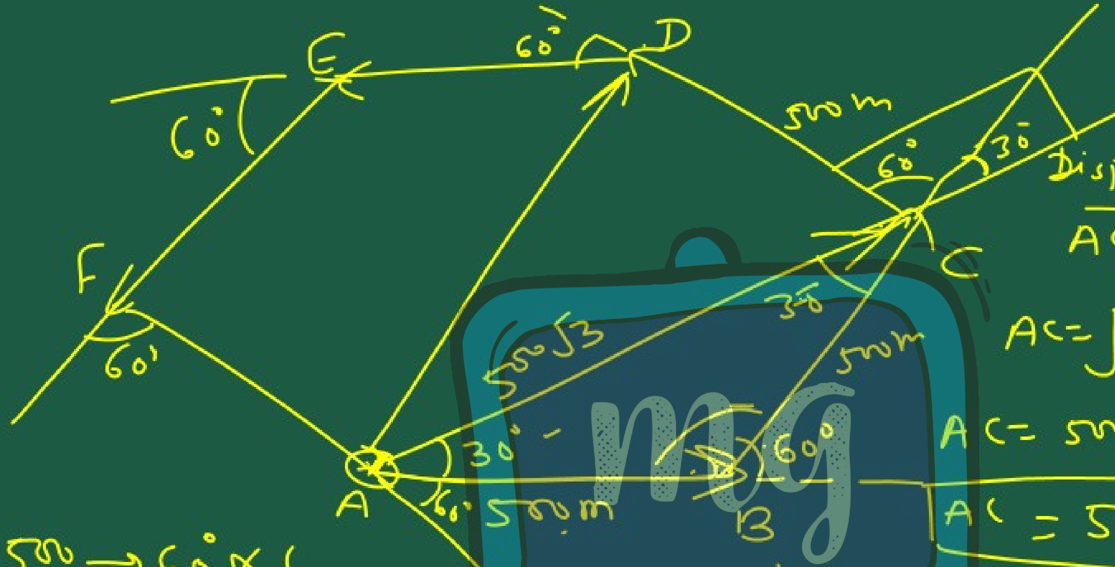
$= \frac{10}{60}$

$= \frac{1}{6} h$

$\therefore$ Average speed $= \frac{3.570}{1} = 21.42 km / h$

EXERCISE

10. On an open ground, a motorist follows a track that turns to his left by an angle of 600 after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.



Displacement at 8th turn

$\vec{AC} = \vec{AB} + \vec{BC}$

$AC = \sqrt{(500)^2 + (500)^2 + 2(500)(500)\cos 60^\circ}$

$AC = 500\sqrt{1+1+1}$

$AC = 500\sqrt{3}\ m$

$|\vec{AD}| = \sqrt{(500\sqrt{3})^2 + (500)^2 + 2(500\sqrt{3})(500)\cos 90^\circ}$

$= 500\sqrt{3+1}$

$= 500 \times 2 = 1000\ m$

$500 \rightarrow 60^\circ \times 6 = 360^\circ$

3rd, 6, 8

1500 m → 1000 m

3000 m → 0

4000 m → $500\sqrt{3}$ m → 866 m

EXERCISE

Answer : The path is a regular hexagon with side 500 m.

Let the motorist start from P.

The motorist takes the third turn at S.

$\therefore$ Magnitude of displacement = PS

= PV + VS

= 500 + 500

= 1000 m

Total path length = PQ + QR + RS

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EXERCISE

= 500 + 500 + 500

= 1500 m

The motorist takes the sixth turn at P, which is the starting point.

$\therefore$ Magnitude of displacement = 0

Total path length

= PQ + QR + RS + ST + TU + UP

= 500 + 500 + 500 + 500 + 500 + 500

= 3000 m

EXERCISE

The motorist takes the eight turns at R.

$\therefore$ Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + (PQ)(QR)\cos 60^0}$$

$$= \sqrt{500^2 + 500^2 + (500)(500)\cos 60^0}$$

$$= \sqrt{250000 + 250000 + \left(500000 \times \frac{1}{2}\right)}$$

$= 866.03m$

$500\sqrt{3}\,m$

EXERCISE

$$\beta = \tan^{-1}\left(\frac{500 \times \sin 60°}{500 + 500 \times \cos 60°}\right)$$

$\beta = 30$

Thus, the magnitude of displacement is 866.03 m at an angle of 30° with PR.

Total path length = Circumference of the hexagon + PQ + QR

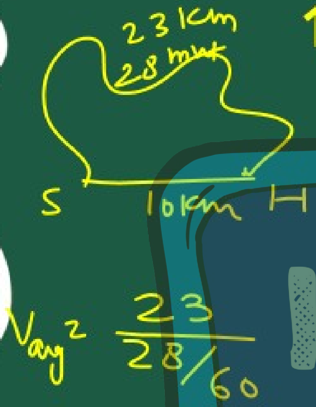
$= 6 \times 500 + 500 + 500$

$= 4000$ m

EXERCISE

Turn	Magnitude of Displacement	Total Path Length
Third	1000	1500
Sixth	0	3000
Eighth	866.03; 30°	4000

EXERCISE



11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is

(a) the average speed of the taxi

EXERCISE

(a) the average speed of the taxi

Answer :

Total distance travelled = 23 km

Total time taken = 28 min = $\frac{28}{60}$ h

Average speed of the taxi = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

EXERCISE

(b) the magnitude of average velocity?

Are the two equal?

EXERCISE

(b) the magnitude of average velocity? Are the two equal?

Answer : Distance between the hotel and the station $= 10\,km =$ Displacement of the car.

$$\therefore \text{Average velocity} = \frac{10}{\frac{28}{60}}$$

$= 21.43 km/h$

$$H = \frac{u^2 \sin^2\theta}{2g} = 25$$

$$\sin^2\theta = \frac{25 \times 20}{40 \times 40} = 5/16$$

$$\sin^2\theta = 5/16$$

$$\sin\theta = \frac{\sqrt{5}}{4}$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - 5/16 = \frac{11}{16}$$

$$\cos\theta = \frac{\sqrt{11}}{4}$$

$$R = \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

$$= \frac{1600 \times 2 \times \frac{\sqrt{5}}{4} \times \frac{\sqrt{11}}{4}}{10}$$

$$= 20\sqrt{55}\ m$$

$$= 20 \times 7.5 = 150\,m$$

EXERCISE

12. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

Answer : Speed of the ball, 40 ms^{-1}

Maximum height, h = 25 m

In projectile motion, the maximum height reached, by a body projected

EXERCISE

at an angle θ is:

$$h = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{40^2 \sin^2\theta}{2 \times 9.8}$$

$$\sin^2\theta = 0.30625$$

$$\sin\theta = 0.5534$$

$$\theta = \sin^{-1}(0.5534)$$

$$\theta = 33.60°$$

EXERCISE

The horizontal range is

$$R = \frac{(40)^2 \times \sin 2 \times 33.60}{9.8}$$

$$R = \frac{1600 \times \sin 67.2}{9.8}$$

$$R = \frac{1600 \times 0.922}{9.8}$$

$$R = 150.53\text{m}$$