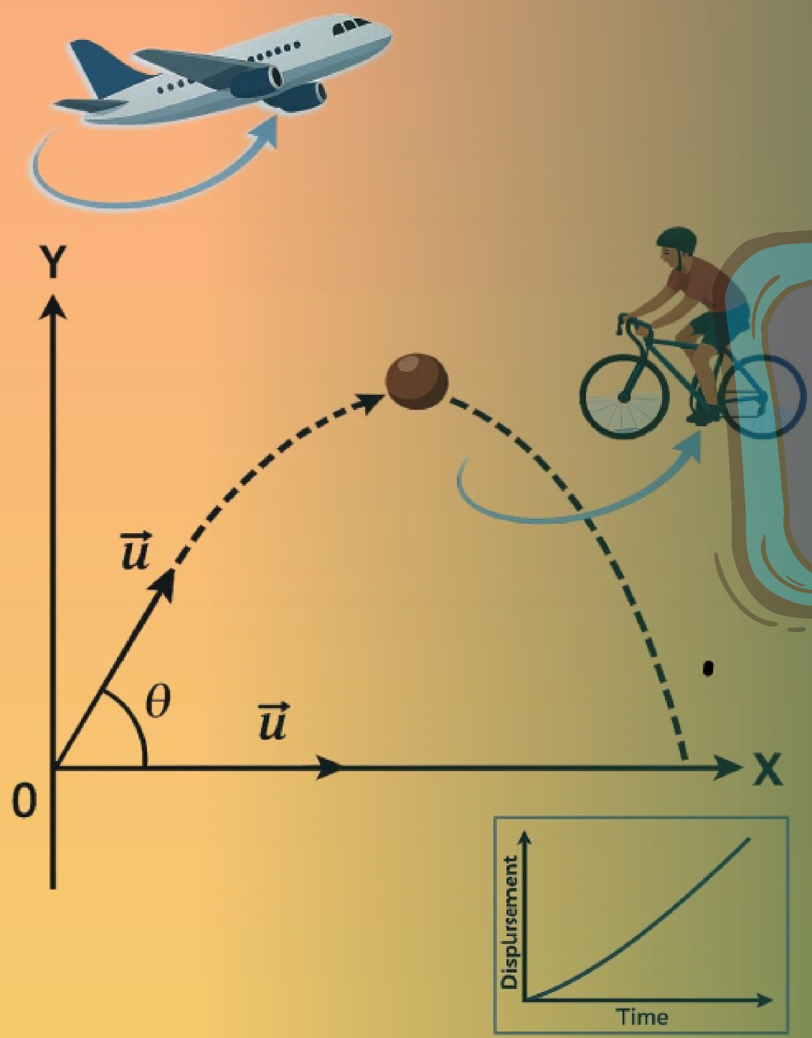
Under CSR Support by ipca | Foundation

missiongyan
LET'S LEARN TOGETHER



CLASS – 11

PHYSICS

Chapter – 3

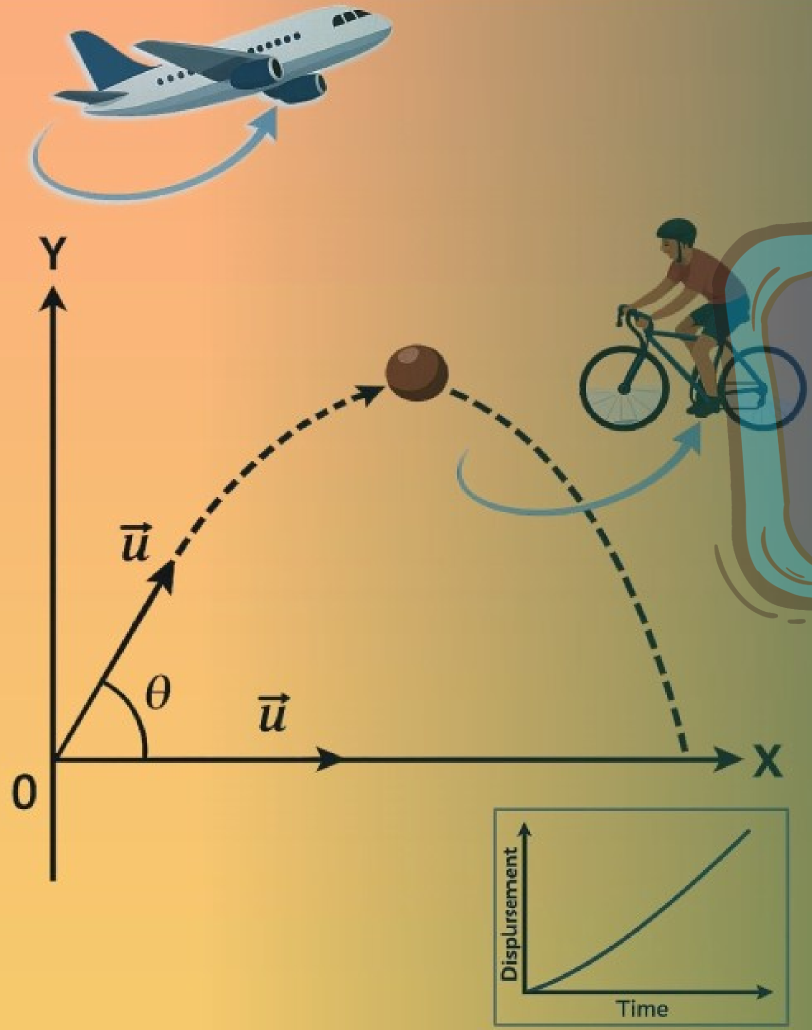
Motion in a Plane

Part – 4

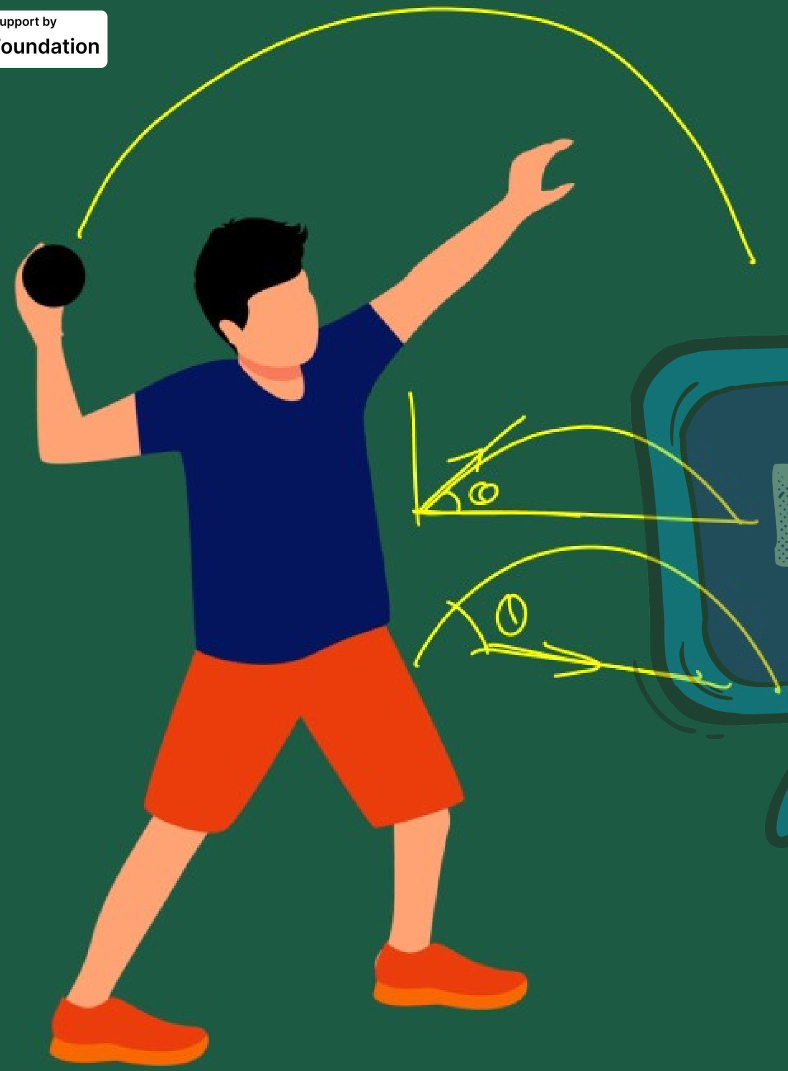
Projectile Motion

Alok Gaur

OVERVIEW



1. Scalars and Vectors
2. Addition of Vectors
3. Resolution of Vectors
4. Projectile Motion
5. Uniform Circular Motion



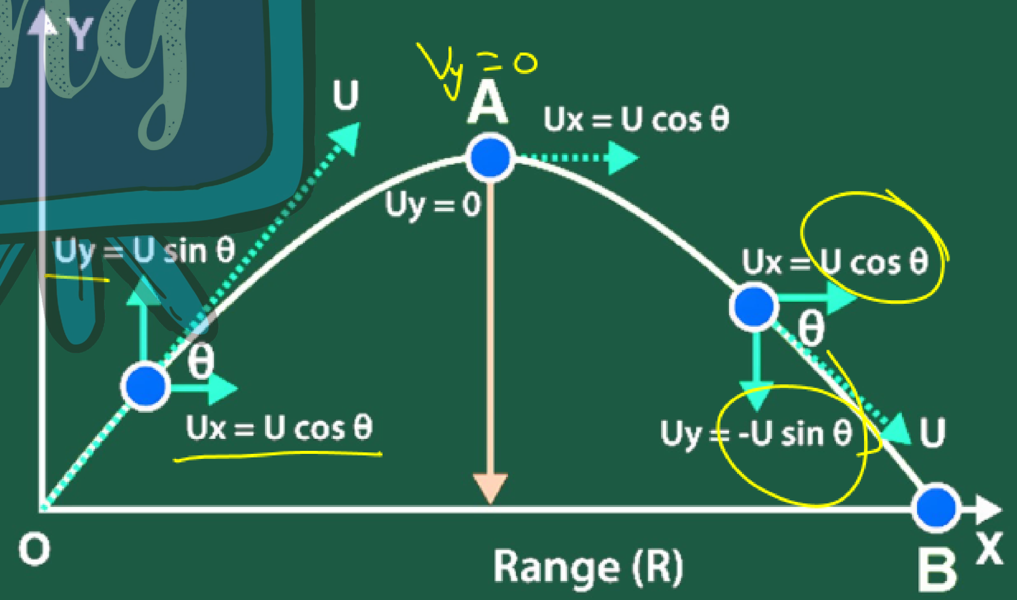
PROJECTILE MOTION

On throwing a body into the space with a definite velocity it moves freely inthe gravitational field of the earth then it is called a projectile and the motionof the projectile is called projectile motion.

- In the range of motion the value of acceleration due to gravity 'g' is constant. and the direction is always vertically downwards.
- Air resistance is negligible.

TRAJECTORY OF PROJECTILE

A body is thrown from an angle θ from the horizon with velocity U



Trajectory of projectile

Let at time t (x, y)

$s = ut + \frac{1}{2}at^2$

$x = u_x t + \frac{1}{2}a_x t^2$

$x = (u\cos\theta)t + \boxed{\frac{1}{2}(0)t^2}$

$t = \frac{x}{u\cos\theta}$

$y = u_y t + \frac{1}{2}a_y t^2$

$y = (\cancel{u}\sin\theta)\frac{x}{\cancel{u}\cos\theta} + \frac{1}{2}(-g)\frac{x^2}{u^2\cos^2\theta}$

* $y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$

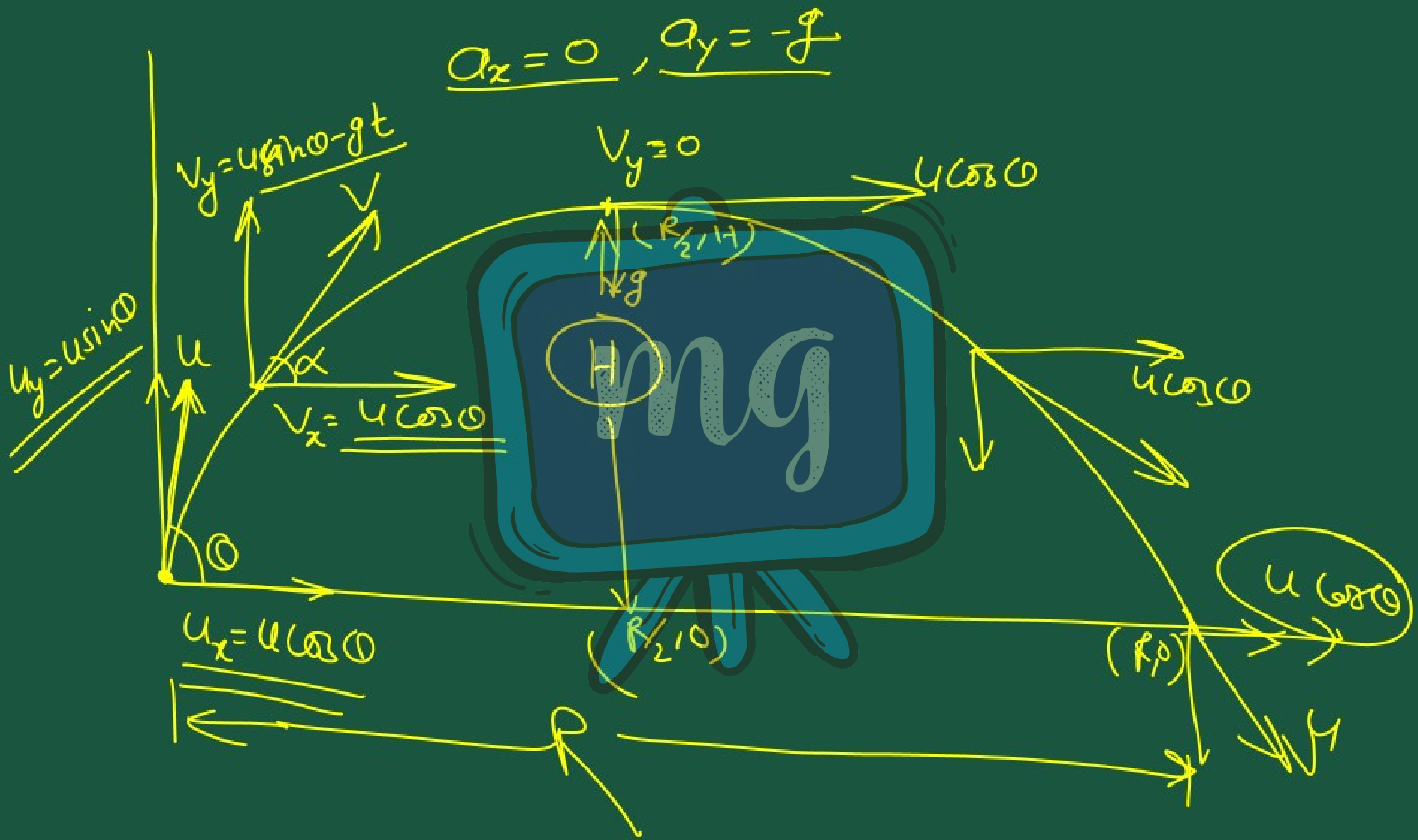
Let $\tan\theta = a$, $\frac{g}{2u^2\cos^2\theta} = b$

$y = ax - bx^2 \rightarrow$ Equation of a Parabola

Trajectory (Path)

$\downarrow$

Parabolic


ax = 0 , ay = -g
Vy = u sinθ - gt
V
Vy = 0
u cosθ
(R/2, H)
g
H
uy = u sinθ
u
α
Vx = u cosθ
θ
ux = u cosθ
(R/2, 0)
u cosθ
(R, 0)
u cosθ
R
V

Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER

Horizontal component $u_x = u \cos\theta$

Vertical component $u_y = u \cos\theta$

Acceleration $a_x = 0$

$a_y = -g$

From the second equation of motion

$$s = ut + \frac{1}{2}at^2$$

Under CSR Support by ipca | Foundation

X-axis

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$x = (u\cos\theta)t$$

$$t = \frac{x}{u\cos\theta}$$

Y-axis

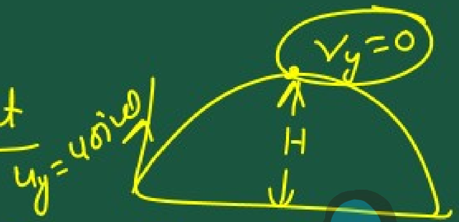
$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = x \tan\theta - \frac{g x^2}{2u^2 \cos^2\theta}$$

$$y = u\sin\theta \left(\frac{x}{u\cos\theta} \right) - \frac{1}{2} g \left(\frac{x^2}{u^2 \cos^2\theta} \right)$$

This equation expresses the equation of parabola. So, the path of the projectile is parabolic.

Maximum height



$v_y = 0,\ u_y = u\sin\theta,\ a_y = -g,\ y = H$

$$v_y^2 = u_y^2 + 2a_y s$$

$$0 = (u\sin\theta) + 2(-g)H$$

$$2gH = u^2\sin^2\theta$$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

$$\boxed{H = \frac{u_y^2}{2g}}$$

If $H \to max$

$\sin^2\theta = max = 1$

$\theta = 90^\circ$

$$\boxed{H_{max} = \frac{u^2}{2g}}$$

MAXIMUM HEIGHT OF PROJECTILE

$$v_y = u_y + a_y t$$

At maximum height

$$v_y = 0$$

$$0 = u\sin\theta - gt$$

$$gt = u\sin\theta$$

$$g = \frac{u\sin\theta}{t}$$

Maximum height of the projectile

$$Y = H = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$H = u\sin\theta\left(\frac{u\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{u^2\sin^2\theta}{g^2}\right)$$

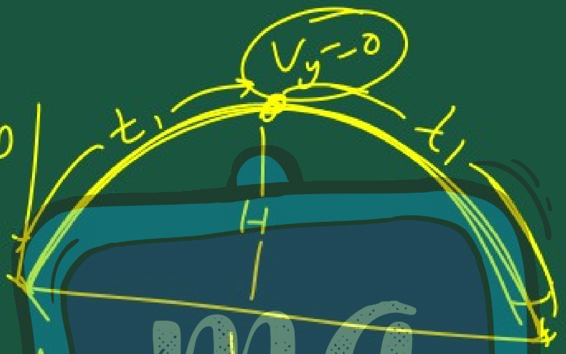
$$H = \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}$$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

Time of flight

$V_y = 0,\ u_y = u\sin\theta$

$a_y = -g,\ t = t_1$



$V_y = u_y + a_y t$

$0 = u\sin\theta - g t_1$

$g t_1 = u\sin\theta$

$$\boxed{t_1 = \frac{u\sin\theta}{g}}$$

$T = 2t_1$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

If $\theta = \frac{\pi}{2}$

$$H_{max} = \frac{u^2}{2g}$$

TIME OF FLIGHT OF PROJECTILE

The total time taken by a projectile in coming back to the same plane from the total time of flight

$$T = 2t$$

$$T = 2\left(\frac{u\sin\theta}{g}\right)$$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

If $\theta = \frac{\pi}{2}$

$$T_{max} = \frac{2u}{g}$$

HORIZONTAL RANGE

$$R = u_x T + \frac{1}{2} a_x T^2$$

$$R = u\cos\theta \left(\frac{2u\sin\theta}{g} \right) \qquad \because a_x = 0$$

$$R = \frac{u^2}{g} \left(2\sin\theta\cos\theta \right)$$

$$\boxed{R = \frac{u^2}{g}\sin 2\theta} = \frac{2u_x u_y}{g}$$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$R = (u\cos\theta)T - \frac{1}{2}(0)T^2$$

$$R = (u\cos\theta)\frac{2u\sin\theta}{g}$$

$$R = \frac{u^2(2\sin\theta\cos\theta)}{g}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

$$\boxed{R = \frac{2u_x u_y}{g}}$$

$$R \rightarrow \max \Rightarrow \sin 2\theta = 1$$

$$2\theta = 90^\circ \Rightarrow \boxed{\theta = 45^\circ}$$

If $\theta = \frac{\pi}{4}$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

$$\boxed{H_{max} = \frac{u^2}{2g}}$$

$$R_{max} = 2H_{max}$$

RELATION BETWEEN R_{max} AND H_{max}

$$H_{max} = \frac{u^2}{2g}$$

$$R_{max} = \frac{u^2}{g}$$

$$\boxed{H_{max} = \frac{R_{max}}{2}}$$

Proof it :

$$\tan\theta = \frac{4H}{R}$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 (2\sin\theta\cos\theta)}{g}$$

$$\frac{H}{R} = \frac{\tan\theta}{4} \Rightarrow \boxed{\tan\theta = \frac{4H}{R}}$$

$$\frac{H}{R} = \frac{\dfrac{\cancel{u^2}\sin^2\theta}{2\cancel{g}}}{\dfrac{\cancel{u^2}\,2\sin\theta\cos\theta}{\cancel{g}}}$$

$$\frac{H}{R} = \frac{\sin\theta}{4\cos\theta}$$

$$\tan\theta = \frac{4H}{R}$$

Proof it :

$$\boxed{8H = gT^2}$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$T^2 = \left(\frac{2u\sin\theta}{g}\right)^2$$

$$\frac{H}{T^2} = \frac{g}{8} \Rightarrow \boxed{8H = gT^2}$$

$$8 \times \frac{u^2 \sin^2\theta}{2g} = 8H$$

$$gT^2 = g \times \left(\frac{2u\sin\theta}{g}\right)^2$$

$$= g \times \frac{4u^2\sin^2\theta}{g^2}$$

$$= \frac{4u^2\sin^2\theta}{g}$$

$$gT^2 = 8\,\frac{u^2\sin^2\theta}{2g}$$

$$gT^2 = 8H$$

Proof it :

$$\tan\theta = \frac{gT^2}{2R}$$

$$T^2 = \left(\frac{2u\sin\theta}{g}\right)^2$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{u^2(2\sin\theta\cos\theta)}{g}$$

$$\Rightarrow \frac{T^2}{R} = \frac{2\tan\theta}{g} \Rightarrow \boxed{\tan\theta = \frac{gT^2}{2R}}$$

$$gT^2 = g\left(\frac{2u\sin\theta}{g}\right)^2$$

$$gT^2 = \cancel{g}\left(\frac{4u^2\sin^2\theta}{g^{\cancel{2}}}\right)$$

$$gT^2 = \cancel{4}\,\frac{\cancel{u^2}\sin^2\theta}{\cancel{g}}$$

$$2R = 2\times\frac{u^2\,2\sin\theta\cos\theta}{g}$$

$$2R = \frac{\cancel{4}\cancel{u^2}\,\cancel{\sin\theta}\cos\theta}{\cancel{g}}$$

$*$ $$\frac{gT^2}{2R} = \tan\theta$$

Example : 1

$R_{45-\theta}$

$R_{45+\theta}$

Galileo, in his book twonew sciences, tated that "for elevation which exceed or fall short of 45° by equal amounts, the ranges are equal" prove this statement.

Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER

$$R = \frac{u^2 \sin 2\theta}{g}$$

$\theta_1 = 45 - \theta$

$$R_1 = \frac{u^2 \sin 2(45-\theta)}{g}$$

$$= \frac{u^2 \sin [90 - 2\theta]}{g}$$

$$\boxed{R_1 = \frac{u^2 \cos 2\theta}{g}}$$

$\theta_2 = 45 + \theta$

$$R_2 = \frac{u^2 \sin 2(45+\theta)}{g}$$

$$= \frac{u^2 \sin (90 + 2\theta)}{g}$$

$$R_2 = \frac{u^2 \cos 2\theta}{g}$$

$R_1 = R_2$

$\theta, 90 - \theta$

$45 - \theta, 45 + \theta$

Range same

Answer :

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\theta = 45^\circ + \alpha \quad , \quad \theta = 45^\circ - \alpha$$

$$R = \frac{u^2 \sin 2(45^\circ + \alpha)}{g}$$

$$R = \frac{u^2 \sin(90^\circ + 2\alpha)}{g} \Rightarrow \frac{u^2 \cos 2\alpha}{g} \quad \ldots(i)$$

$$R' = \frac{u^2 \sin 2(45° - \alpha)}{g}$$

$$R' = \frac{u^2 \sin(90° - 2\alpha)}{g} \Rightarrow \frac{u^2 \cos 2\alpha}{g} \quad ...(ii)$$

So, $R = R'$

Example : 2

A cricket ball is thrown at a speed of $28 ms^{-1}$ in a direction 30° above the horizontal. Calculate –

(a) The maximum height

(b) The time taken by the ball to return to the same level

$u = 28$ m/sec

$\theta = 30°$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{(28)^2}{2\times 10}\left(\frac{1}{4}\right)$$

$$= \frac{\cancel{28}^{14}\times\cancel{28}^{7}}{\cancel{2}\times 10\times\cancel{4}}$$

$$= \frac{98}{10} = 9.8 \text{ m}$$

(c) The distance from the thrower to the point where the ball return to the same level.

$$T = \frac{2u\sin\theta}{g}$$

$$= \frac{\cancel{2} \times 28 \times \frac{1}{\cancel{2}}}{10}$$

$$= \underline{2.8 \text{ sec}}$$

$$R = \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

$$= \frac{28 \times \overset{14}{\cancel{28}} \times \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{\sqrt{3}}{\cancel{2}}}{10}$$

$$= \underline{14 \times 2.8 \times \sqrt{3}}$$

Answer :

$\theta = 30^\circ$, $u = 28\,m/s$

(a)

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$H = \frac{(28)^2 (\sin 30^\circ)^2}{2 \times 9.8} = 10m$$

(b)

$$T = \frac{2u\sin\theta}{g}$$

$$T = \frac{2 \times 28 \times \sin 30^\circ}{9.8}$$

$$T = 2.9\,\text{sec}$$

(c)

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{(28)^2 \sin 60^\circ}{9.8}$$

$$R = \frac{28 \times 28 + \sqrt{3}}{9.8 \times 2} = 6.9\text{m}$$

LEARNING OUTCOME

To study projectile motion

ASSESSMENT

$u\cos\theta$

$\theta = 90^\circ$

g

1 The direction of velocity and acceleration at the peak of a projectile path are –

A Mutually parallel

B Mutually opposite

C Mutually at 45° angle

D Mutually perpendicular

ASSESSMENT

2 A ball is thrown at an angle θ from the horizon with a velocity. The value of θ for it's horizontal range to be maximum will be –

A 30°

B 0°

C 45°

D 60°

Under CSR Support by ipca | Foundation

ASSESSMENT

3 | Give two examples of projectile motion.

football

Baseball

Cricket six

Under CSR Support by ipca | Foundation

ASSESSMENT

missiongyan LET'S LEARN TOGETHER

4

A cricketer can throw a ball to a maximum horizontal distance of 100m. How high above the ground can the cricketer throw the same ball?

100m

$$R_{max} = 2H_{max}$$

$$H_{max} = \frac{R_{max}}{2} = \frac{100}{2} = \boxed{50\,m}$$