CLASS – 11

PHYSICS

Chapter – 2

Motion in a Straight Line

Part – 4

Uses of Differential and Integral Calculus in Physics

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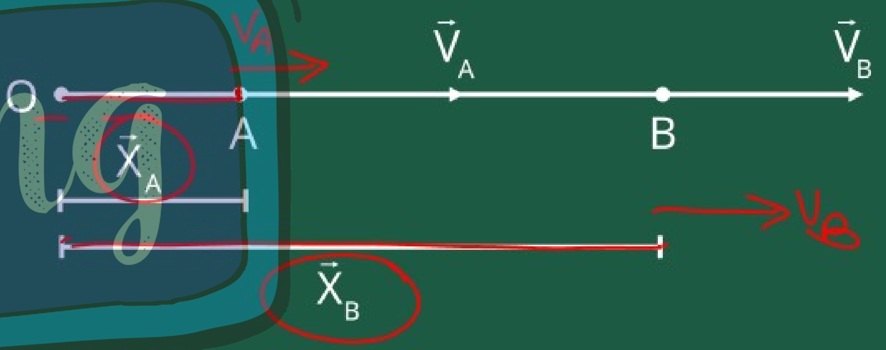
OVERVIEW

1. Instantaneous Velocity and Speed
2. Acceleration
3. Motion Under Gravity and Graphs
4. Uses of Differential and Integral Calculus in Physics

RELATIVE VELOCITY IN ONE DIMENSION



Displacement of B with respect to A	=	Displacement of B as measured from A

$$\Rightarrow \vec{X}_{BA} = \vec{X}_B - \vec{X}_A$$

$$\Rightarrow \frac{dX_{BA}}{dt} = \frac{dX_B}{dt} - \frac{dX_A}{dt}$$

$$\Rightarrow \vec{V}_{BA} = \vec{V}_B - \vec{B}_A$$

Relative = Actual − Reference

A → 20 km/hr

B → 10 km/hr

$V_{AB} = 10 \text{ km/hr}$

$V_{AG} = 20 \text{ km/hr}$

velocity of A w.r.t B

A → V_A

B → V_B

$V_{AB} = V_A - V_B$

← 30 km/hr   → 40 km/hr

70

A → V_A

V_B ← B

$V_{AB} = V_A + V_B$

A: $40\ m/sec$, a_A

B: $60\ m/sec$, a_B

240 m

$a_{AB} = a_A - a_B$

$a_A + a_B$

$t = \frac{\text{distance}}{\text{speed}} = \frac{240}{20} = 12\ sec$

$V_{BA} = V_B - V_A = 60 - 40 = 20$

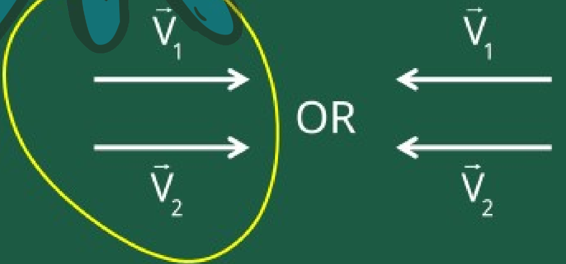
g

g

relative acc. $= g - g = 0$

For same direction :

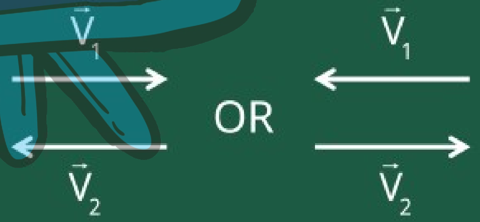
When two particles are moving in the same direction, then magnitude of their relative velocity is equal to the difference between their individual speeds.



$$|\vec{V}_{12}| \text{ or } |\vec{V}_{21}| = v_1 \sim v_2$$

For opposite direction :

When two particles are moving in the opposite directions, then magnitude of their relative velocity is always equal sum of their individual speeds.



$$|\vec{V}_{12}| \text{ or } |\vec{V}_{21}| = v_1 + v_2$$

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Note

When two particles move simultaneously then the concept of relative motion becomes applicable conveniently.

DIFFERENT CALCULUS

$$\frac{dy}{dx}$$

$$* \quad \frac{d(x^n)}{dx} = n x^{n-1}$$

Differential calculus is the branch in which the rate of instant change in a variable quantity relative to another variable quantity is studied.

$\frac{dy}{dx} \rightarrow$ differential co-efficient of y relative to x

Instantaneous velocity $= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

(c) Motion with the initial velocity of the particle (negative).

$$V = \frac{dx}{dt}$$

Acceleration $a = \frac{dv}{dt}$

Or $a = \frac{d^2x}{dt^2}$

- Similarly from the second law of motion –

$$\vec{F} = \frac{\overrightarrow{dp}}{dt}$$

Momentum $\Delta\vec{p} = \vec{F}.\Delta t$

Or $\quad a = \dfrac{d^2x}{dt^2}$

FORMULAE OF DIFFERENTIAL CALCULUS

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = n.x^{n-1}$$

$$\frac{d}{dx}(\text{long}_e x) = \frac{1}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}\left(a^x\right) = a^x \log_e a$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

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$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx}(\text{constant}) = 0$$

$$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$= \frac{1}{2}x^{-1/2}$$

$$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

USES OF DIFFERENTIAL MATHEMATICS IN PHYSICS

Dependance of position and time

$$\vec{v} = \frac{d\vec{x}}{dt}$$

Here $\frac{dx}{dt}$ is called the first differential co-efficient

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

Example - 1

The displacement x (in meter) of a particles of mass 'm' (in kg) moving in one dimension under the action of a force is related to time 't' (in sec) by $t = \sqrt{x} + 3$.

The displacement of the particle when it's velocity it zero will be?

$$\sqrt{x} = t - 2$$

$$x = (t-2)^2$$

$$x = t^2 - 4t + 4$$

$$v = \frac{dx}{dt}$$

$$v = 2t - 4 + 0$$

$$v = 2t - 4 = 0$$

$$2t = 4$$

$$t = 2 \text{ sec}$$

Solution

$$t = \sqrt{x} + 3$$

$$x = (t-3)^2$$

$$x = t^2 + 9 - 6t$$

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}(t^2 + 9 - 6t)$$

$$v = 2t - 6$$

$$\because v = 0$$

$$0 = 2t - 6$$

$$2t = 6$$

$$t = 3sec$$

$$\because x = (t-3)^2$$

$$x = 0$$

Example - 2

The motion of a particle along a straight line is described by equation $x = 8 + 12t - t^3$ where x in metre and t in second.

The retardation of the particle when it's velocity becomes zero is?

$$V = \frac{dx}{dt}$$

$$V = 0 + 12 - 3t^2$$

$$V = 12 - 3t^2 = 0$$

$$3t^2 = 12$$

$$t^2 = 4$$

$$t = 2 \text{ sec}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(12 - 3t^2)$$

$$= 0 - 6t$$

$$a = -6t = -6 \times 2 = -12 \text{ m/s}^2$$

Solution

$$x = 8 + 12t - t^3$$

$$v = \frac{dx}{dt} = 12 - 3t^2$$

$$\because v = 0$$

$$0 = 12 - 3t^2$$

$$t^2 = 4$$

$$t = 2 \text{ sec}$$

$$a = \frac{dv}{dt} \quad \text{or} \quad a = \frac{d^2x}{dt^2}$$

$$a = \frac{d}{dt}\left(12 - 3t^2\right)$$

$$a = -6t$$

$$\because t = 2\,sec$$

$$a = -12\,m/sec^2$$

$\int dx$

$\int dy$

$\int dx$

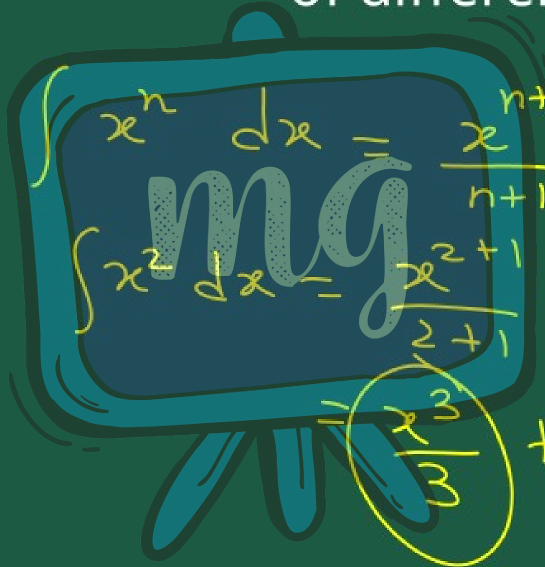
INTEGRATION

The integration of a function f(x) with respect to x is a function g(x), the differential co-efficient of which with respect to x is f(x).

$\int f(d)dx = g(x)$

This process to finding the integration of function f(x) is called integration.

- ☛ Integration is the opposite process of differentiantion.



$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^2 \, dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3} + c$$

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FORMULAE OF INTEGRATION

$$\int 5\,dx =$$

$$5\int x^0\,dx$$

$$= 5\left[\frac{x^{0+1}}{0+1}\right]$$

$$= 5\left[\frac{x^1}{1}\right]$$

$$= 5x$$

(i) $\int \text{constant}(dx) = \text{constant}(x)$

(ii) $\int x^n dx = \dfrac{x^{n+1}}{n+1}$

(iii) $\int \dfrac{1}{x} dx = \log_e x$

(iv) $\int \sin x\, dx = -\cos x$

(v) $\int \cos x\, dx = \sin x$

APPLICATION OF INTEGRATION IN PHYSICS

Positive $x = \int v\,dt$

Velocity $v = \int a\,dt$

work $w = \int \vec{f}.\vec{dx}$

$x \xrightarrow{\text{diff.}} v$

$v \xrightarrow{\text{diff.}} a$

$a \xrightarrow{\text{int.}} \Delta v$

$v \xrightarrow{\text{int.}} \Delta x$

Example :

The velocity $\vec{v}$ of a particle depends upon time 't' as follows –

$$\vec{v} = 3t^2\hat{i} + 2t\hat{j} + 5\hat{k}$$

If this particle is located at origin at t = 0 then find its location at t = 2

$$\Delta x = \int v\, dt$$

Solution

$$x = \frac{3t^3}{3}\hat{i} + \frac{2t^2}{2}\hat{j} + 5t\hat{k}$$

$$x = t^3\hat{i} + t^2\hat{j} + 5t\hat{k}$$

$$\vec{v} = 3t^2\hat{i} + 2t\hat{j} + 5\hat{k}$$

$$\vec{v} = \frac{d\vec{x}}{dt}$$

$$\int d\vec{x} = \int \vec{v}\,dt$$

$$x = \int \left(3t^2 i + 2t\vec{j} + 5k\right)dt$$

$$x = t^3 i + t^2 j + 5t\hat{k}$$

$$t = 2$$

$$x = 8\hat{i} + 4\hat{j} + 10\hat{k}$$

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$$W = \int F\,dx$$

$$W = \int -kx\,dx$$

$$W = -k\frac{x^2}{2}$$

Example :

In one dimensional simple periodic motion force can be represented as F = −kx, where k is force constant and x indicate displacement. Obtain the expression for work done by this force in displacement from initial position x_i to final position x_f.

Solution

$$w = \int_{x_i}^{x_f} F\,dx$$

$$\because Fc - kx$$

$$w = \int_{x_i}^{x_f} -Kx\,dx$$

$$w = -k\int_{x_i}^{x_f} x\,dx$$

$$w = -K\left[\frac{x^2}{2}\right]_{xi}^{xf}$$

$$w = -\frac{K}{2}\left[x_f^2 - x_i^2\right]$$

$$w = \frac{K}{2}\left[x_i^2 - x_f^2\right]$$

LEARNING OUTCOMES

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1. To calculate velocity and acceleration by differential calcus
2. To calculate position and velocity by integrating
3. To calculate work by integrating

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ASSESSMENT

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1 Motion of particle is given by equation $s = (3t^3 + 7t^2 + 14t + 8)$ m. The value of acceleraton of the particle at t = 1sec is–

A 10 m/s^2

B 32 m/s^2

C 23 m/s^2

D 16 m/s^2

$v = \frac{ds}{dt} = 9t^2 + 14t + 14$

$a = \frac{dv}{dt} = 18t + 14$

$a = 18t + 14$

$a = 18(1) + 14$

$= 32$

2 Find the value of $\int_{\infty}^{R} \frac{GMm}{x^2} dx$

A $\frac{-GM}{x^2}$

B $\frac{-GM}{R^2}$

C $\frac{-GMm}{R}$

D $\frac{-GMm}{x}$

$GMm \int_{\infty}^{R} x^{-2} dx$

$= GMm \left(\frac{x^{-2+1}}{-2+1} \right)_{\infty}^{R}$

$= GMm \left(\frac{x^{-1}}{-1} \right)_{\infty}^{R}$

$= -GMm \left(\frac{1}{x} \right)_{\infty}^{R}$

$= -GMm \left[\frac{1}{R} - \frac{1}{\infty} \right]$

$= -\frac{GMm}{R}$