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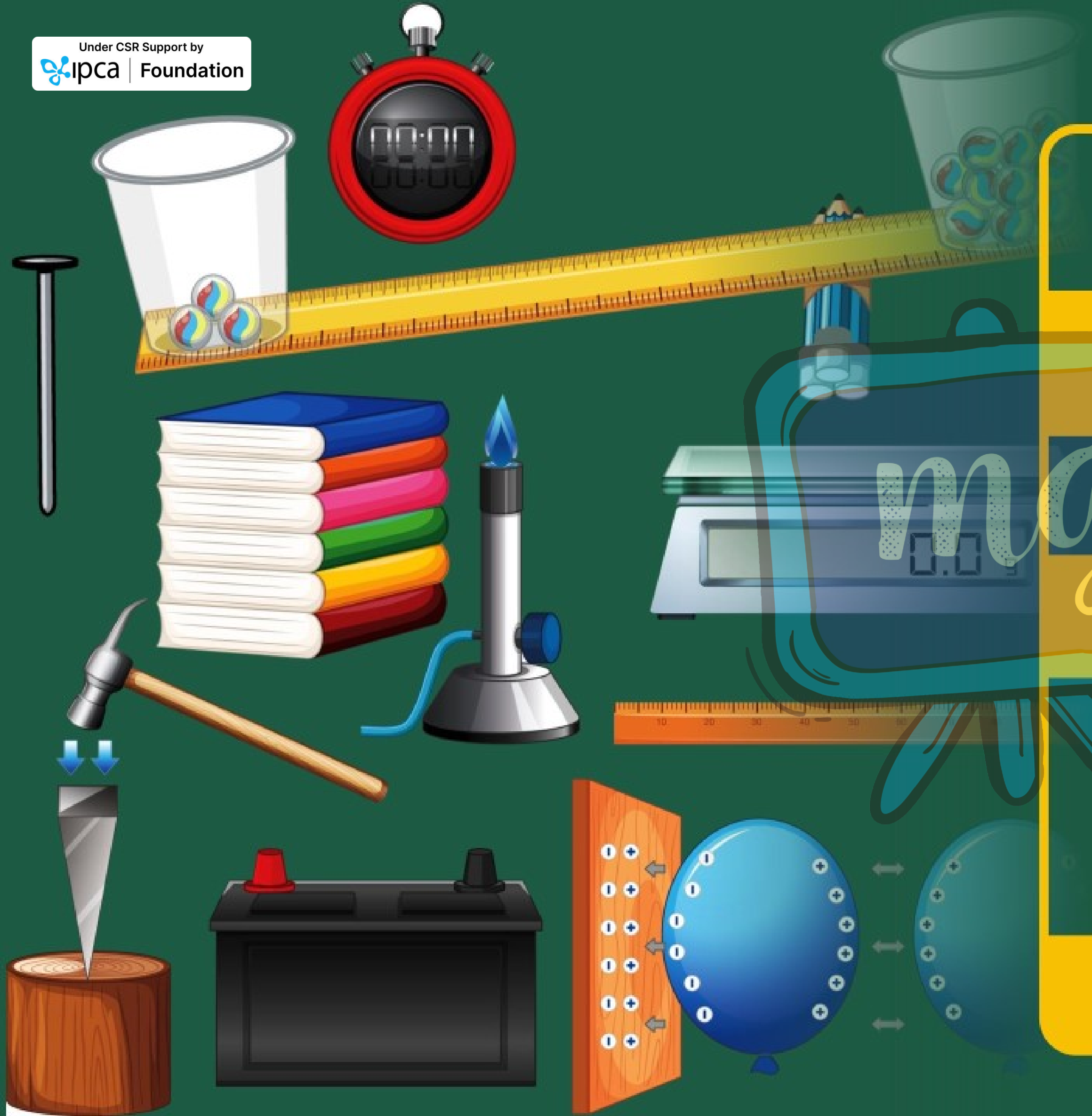

CLASS – 11

PHYSICS

Chapter – 1

Units and Measurement

Part – 3

Dimensional Analysis and It's Applications

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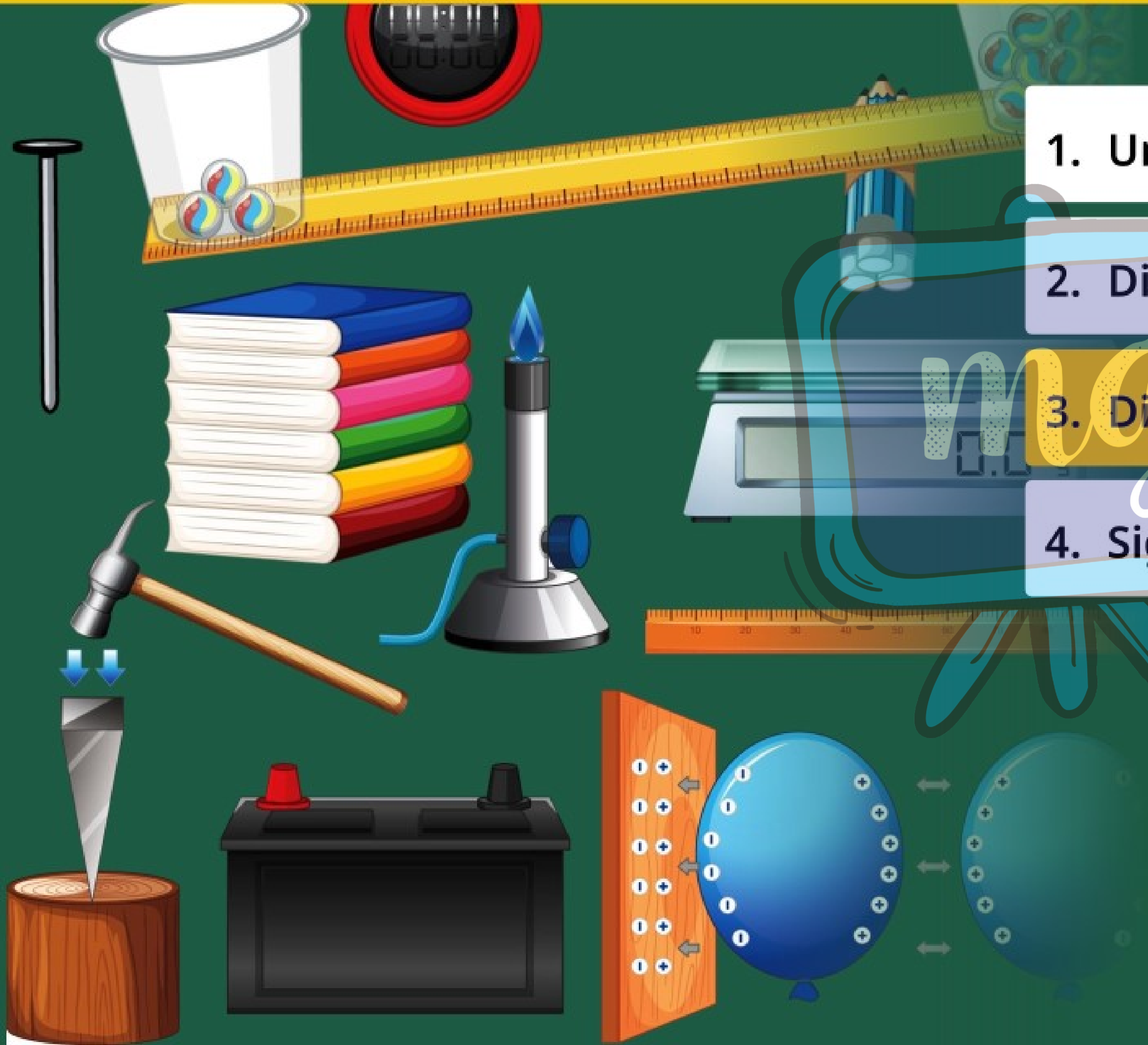
OVERVIEW



1. Unit
2. Dimensions of Physical Quantities
3. Dimensional Analysis and it's Applications
4. Significant Figures

USE OF DIMENSIONS

1. Convert from one unit system to another

$Q = nu$

$nu = \text{constant}$

$n_1 u_1 = n_2 u_2$

$$n_1 \left[M_1^a L_1^b T_1^c \right] = n_2 \left[M_2^a L_2^b T_2^c \right]$$

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

Example :

Convert 1 newton into dyne?

Force → MKS → CGS

Force → $[M^1 L^1 T^{-2}]$

$a = 1,\ b = 1,\ c = -2$

M.K.S → C.G.S

$n_1 = 1$ | $n_2 = ?$

$M_1 = 1\,kg$ | $M_2 = 1\,g$

$L_1 = 1\,m$ | $L_2 = 1\,cm$

$T_1 = 1\,sec$ | $T_2 = 1\,sec$

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$$n_2 = 1 \left[\frac{1\,kg}{1\,g}\right]^1 \left(\frac{1\,m}{1\,cm}\right)^1 \left(\frac{1\,sec}{1\,sec}\right)^{-2}$$

$$n_2 = 1 \left[\frac{1000\,g}{1\,g}\right]^1 \left[\frac{100\,cm}{1\,cm}\right]^1 (1)$$

$$n_2 = 1 \times 1000 \times 100 \times 1$$

$$n_2 = 10^5$$

$$1\,N = 10^5\ \text{Dyne}$$

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Example : work M.K.S C.G.S

Convert 1 joule into erg?

$[M^1L^2T^{-2}]$

$a=1, b=2, c=-2$

M.K.S. System	C.G.S. System
M_1 = 1kg	M_2 = 1gm
L_1 = 1 m	L_2 = 1cm
T_1 = 1 sec	T_2 = 1 sec
n_1 = 1 joule	n_2 = ... erg

Dimension of work = $[M^1L^2T^{-2}]$

$a = 1, b = 2, c = -2$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = 1 \times \left[\frac{1\text{kg}}{1\text{gm}}\right]^1 \left[\frac{1\text{m}}{1\text{cm}}\right]^2 \left[\frac{1\text{sec}}{1\text{sec}}\right]^{-2}$$

$$n_2 = \left[\frac{1000\text{gm}}{1\text{gm}}\right]^1 \left[\frac{100\text{cm}}{1\text{cm}}\right]^2$$

$$n^2 = 10^7$$

$$1 \text{ joule} = 10^7 \text{ erg}$$

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$$n_2 = (1000)^1 (100)^2$$

$$= 1000 \times 100 \times 100$$

$$n_2 = 10^7$$

2. Checking the dimensional consistency of equations

☛ We can add or subtract similar physical quantities. This simple principle called the principle of homogeneity of dimension-in an equation is extremely useful in checking the correctness of an equation.

* Same type of quantities can be added or substracted

Force + ____ A

Length + B

P + ____

W + ____

Force

$F + A = B$

$A = F = [M^1 L^1 T^{-2}]$

Example - 1

Let us consider an equation

$$\frac{1}{2}mv^2 = mgh$$

Where m is the mass of the body, V it's velocity, g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct.

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$$\frac{1}{2}mv^2 = mgh$$

L.H.S

$$mv^2 = [M^1][L^1T^{-1}]^2$$

$$mv^2 = [M^1L^2T^{-2}]$$

R.H.S

$$mgh = [M^1][L^1T^{-2}][L^1]$$

$$mgh = [M^1L^2T^{-2}]$$

L.H.S = R.H.S

Dimensionaly Correct

Answer :

The dimension of LHS

$[M] – [LT^{-1}]^2$

$= [M]\ [L^2T^{-2}]$

$= [ML^2T^{-2}]$

The dimension of RHS

$[M]\ [LT^{-2}]\ [L]$

$= [M]\ [L^2T^{-2}]$

$= [ML^2T^{-2}]$

The dimension of LHS and RHS are the same and hence the equation is dimensionally correct.

Example - 2

Let us consider an equation

$$S = ut + \frac{1}{2}at^2$$

Where u is the initial velocity at time t = o, V is the final velocity at time t, a is the acceleration and s is the displacement. Check whether this equation is dimensionally correct.

$$S = ut + \frac{1}{2}at^2$$

S → Distance

u → initial velocity

t → Time

a → Acceleration

$S = [L^1] = [M^0 L^1 T^0]$

$ut = [M^0 L^1 T^{-1}][T^1]$

$ut = [M^0 L^1 T^0]$

$at^2 = [M^0 L^1 T^{-2}][T]^2$

$= [M^0 L^1 T^{-2}][T^2]$

$at^2 = [M^0 L^1 T^0]$

Answer :

The dimension of S

$= [L]$

The dimension of ut

$= [LT^{-1}]\ [T]$

$= [L]$

The dimension of $\frac{1}{2}at^2$

$= [LT^{-3}]\ [T^2]$

$= [L]$

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As each term of this equation has the same dimension. Hence this equation is a dimensionally correct equation.

3. Deducing relation among the physical quantities

- The method of dimensions can some times be used to deduce relation among the physical quantities.
- For this we should know the dependance of the physical quantity on other quantities (upto three physical quantities or linearly independent variables) and consider it.

Example :

$T = ?$

Consider a simple pendulam having a bob attached to a string, that oscilates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulam depends on it's length (l), mass of the bob (m) and acceleration due to gravity (g) derive the expression for it's time period using method of dimensions.

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$$T \propto m^a l^b g^c$$

$$T = k m^a l^b g^c$$

$$[M^0 L^0 T^1] = k [M^1]^a [L^1]^b [L T^{-2}]^c$$

$$[M^0 L^0 T^1] = k [M^a][L^b][L^c T^{-2c}]$$

$$[M^0 L^0 T^1] = k [M^a L^{b+c} T^{-2c}]$$

$a = 0$, $b + c = 0$, $-2c = 1$

$c = -\frac{1}{2}$

$b - \frac{1}{2} = 0$

$b = \frac{1}{2}$

$$T = k m^0 l^{1/2} g^{-1/2}$$

$$T = k \frac{l^{1/2}}{g^{1/2}}$$

$$T = k \left(\frac{l}{g}\right)^{1/2}$$

$$T = k \sqrt{l/g}$$

$k = 2\pi$

$$T = 2\pi \sqrt{l/g}$$

Answer :

The dependance of time period T on the quantities l, g and m as a product may be written as

$$T = K\, l^a\, g^b\, m^c$$

Where K is dimensionless constant and a, b and c are the exponents.

By considering dimensions on both sides, we have

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$$[L^0M^0T^1] = [L^1]^a\,[L^1T^{-2}]^b\,[m^1]^c$$

$$= L^{a+b}\,T^{-2b}\,m^c$$

On equating the dimensions on both sides, we have

$$a + b = 0$$

$$-2b = 1$$

$$c = 0$$

So that $b = \frac{-1}{2}$ $\quad a = \frac{1}{2}$ $\quad c = 0$

Then $T = K [l]^{1/2} [g] [m]^0$

$$T = K\sqrt{\frac{l}{g}}$$

Actually $K = 2\pi$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

LIMITATIONS OF DIMENSIONAL EQUATIONS

- They do not give knowledge that tells about the quantity being scalar or vector.
- By this method those formulae cannot be derived in which logarithm, exponents and trigonometric functions are used also their correctness can not be tested.

$\log(x)$

$\sin\theta$

- In dimensional equations, the information about pure number and dimensionless constants in the formulae of physical quantity can not be obtained and by the dimensional analysis method the value of constants can not be determined.

- If a physical quantity depends upon more than three quantities then relationship cannot be established between them because by three fundamental quantities M_1L_1T only three equations can be established.

Example : $T = \frac{hrdg}{2\cos\theta}$

LEARNING OUTCOMES

1. Convert unit systems
2. Checking formula or equation
3. Deducing relation

ASSESSMENT

1

The vander wall's equation for a gas is –

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

Determine the dimensions of a and b, hence write the S.I. units of a and b respectively –

A $[M^0L^0T^3]$ $[M^1L^2T^{-2}]$

B $[M^0L^0T^5]$ $[M^1L^6T^{-2}]$

C $[M^1L^5T^{-2}]$ $[M^0L^3T^0]$

D $[M^2L^5T^{-2}]$ $[M^0L^6T^0]$

ASSESSMENT

2 (a) 9.8 m/s^2 = ...n_2........... cm/s^2

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$n_1 = 9.8$ | $n_2 = ?$

$M_1 = 1\,kg$ | $M_2 = 1\,g$

$L_1 = 1\,m$ | $L_2 = 1\,cm$

$T_1 = 1\,sec$ | $T_2 = 1\,sec$

$[M^0 L^1 T^{-2}]$

$a = 0, b = 1, c = -2$

$n_2 = 980$

ASSESSMENT

2 (b) $1\ gm/cm^3 = \ldots\ldots\ldots\ldots\ kg/m^3$

C.G.S $\longrightarrow$ M.K.S

$n_1 = 1$, $n_2 = ?$

$M_1 = 1\,g$ | $M_2 = 1\,kg$

$L_1 = 1\,cm$ | $L_2 = 1\,m$

$T_1 = 1\,sec$ | $T_2 = 1\,sec$

$[M^1 L^{-3} T^0]$ $a = 1$, $b = -3$, $c = 0$

$$n_2 = 1\left(\frac{1\,g}{10^3\,g}\right)^1 \left(\frac{1\,cm}{10^2\,cm}\right)^{-3} \left(\frac{1\,sec}{1\,sec}\right)^0$$

$$n_2 = 1 \times 10^{-3} \times 10^6 \times 1$$

$$n_2 = 10^3$$