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कक्षा - 11

गणित

अध्याय – 4

सम्मिश्र संख्याएँ और द्विघातीय समीकरण

भाग – 7

विविध उदाहरण और विविध प्रश्नावली (प्रश्न 1 व 2)

हर्ष गोयल

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विविध उदाहरण

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उदाहरण - 7

$\dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ का संयुग्मी ज्ञात कीजिए।

$a+ib$

माना $z = \dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

$= \dfrac{6+9i-4i-6i^2}{2-i+4i-2i^2}$

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$$= \frac{6+9i-4i+6}{2-i+4i+2}$$

$$= \frac{12+5i}{4+3i}$$

$$= \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{48-36i+20i+15}{16-12i+12i+9}$$

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$$= \frac{48 - 36i + 20i + 15}{16 + 9}$$

$$= \frac{63 - 16i}{25}$$

$$z = \frac{63}{25} - \frac{16}{25}i$$

तो z का संयुग्मी

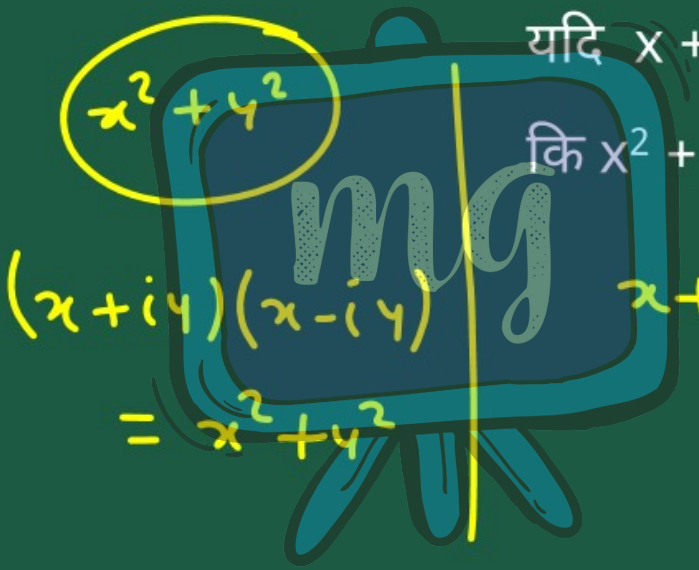
$$\bar{z} = \frac{63}{25} + \frac{16}{25}i$$ Ans

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उदाहरण – 8

यदि $x + iy = \frac{a + ib}{a - ib}$ है तो, सिद्ध कीजिए कि $x^2 + y^2 = 1$



$x^2 + y^2$

$(x+iy)(x-iy)$

$= x^2 + y^2$

$x + iy = \frac{a+ib}{a-ib}$

$= \frac{a+ib}{a-ib} \times \frac{a+ib}{a+ib}$

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$$x + iy = \frac{(a+ib)^2}{a^2-(ib)^2}$$



$$= \frac{a^2+(ib)^2+2abi}{a^2+b^2}$$

$$= \frac{a^2-b^2+i2ab}{a^2+b^2}$$

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$$x+iy = \frac{a^2-b^2}{a^2+b^2} + i\left(\frac{2ab}{a^2+b^2}\right)$$

तो

$$x-iy = \frac{a^2-b^2}{a^2+b^2} - i\left(\frac{2ab}{a^2+b^2}\right)$$

$(a+ib)$

$(a-ib)$

तो

$$(x+iy)(x-iy)$$

$$= \left(\frac{a^2-b^2}{a^2+b^2} + i\,\frac{2ab}{a^2+b^2}\right)\left(\frac{a^2-b^2}{a^2+b^2} - i\,\frac{2ab}{a^2+b^2}\right)$$

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$$x^2 + y^2 = \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 - \left(\frac{2ab}{a^2 + b^2}\, i\right)^2$$



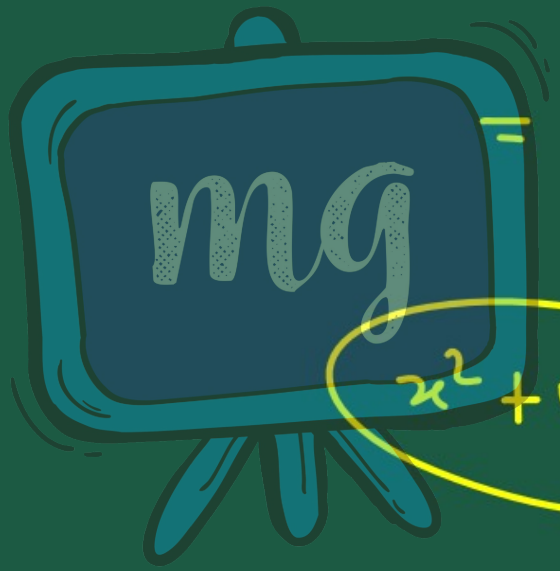
$$= \frac{(a^2 - b^2)^2}{(a^2 + b^2)^2} + \frac{(2ab)^2}{(a^2 + b^2)^2}$$

$$= \frac{a^4 + b^4 - 2a^2b^2 + 4a^2b^2}{(a^2 + b^2)^2}$$

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$$x^2 + y^2 = \frac{a^4 + b^4 + 2a^2b^2}{(a^2 + b^2)^2}$$



$$= \frac{(a^2 + b^2)^2}{(a^2 + b^2)^2}$$

$$x^2 + y^2 = 1$$

Hence Proved

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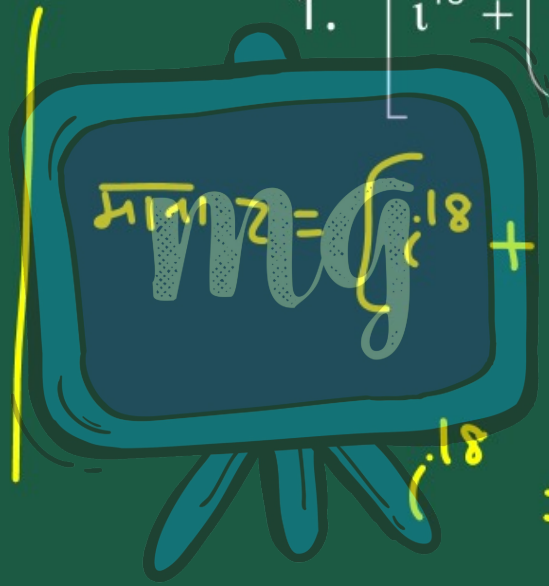


विविध प्रश्नावली

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1. $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$ का मान ज्ञात कीजिए।



$\frac{18}{4}$

$= \left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3 - (1)$

$i^{18} = (i^4)^4 \cdot i^2$

$= (1)^4 \cdot (-1)$

$= -1$

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$$\left(\frac{1}{i}\right)^{25} = \frac{1}{i^{25}}$$

$$\frac{25}{4} = \frac{1}{(i^4)^6\, i}$$

$$= \frac{1}{(1)^6\, i}$$

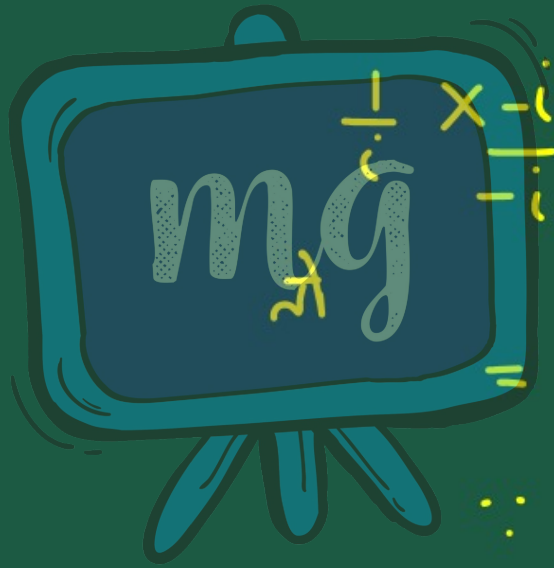
$$= \frac{1}{i}$$

i^{18} तथा $\left(\frac{1}{i}\right)^{25}$ का मान ज्ञात कीजिए।

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$$z = \left(-1 + \frac{1}{i}\right)^3$$



$$\frac{1}{i} \times \frac{-i}{-i} = \frac{-i}{1} = -i$$

$$= (-1-i)^3$$

$$\because (z_1 - z_2)^3$$

$$= z_1^3 - z_2^3 - 3z_1^2 z_2 + 3z_1 z_2^2$$

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$$(-1-i)^3 = (-1)^3 - (i)^3 - 3(-1)^2(i) + 3(-1)(i)^2$$

$$= -1 + i - 3i + 3$$

$$= -1 + 3 + i(1-3)$$

$$= 2 + i(-2)$$

$$= \underline{2-2i}$$

Ans

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2. किन्हीं दो सम्मिश्र संख्याओं z_1 और z_2 के लिए, सिद्ध कीजिए:

$\text{Re}(z_1 z_2) = \text{Re}z_1 \text{Re}z_2 - \text{Im}z_1 \text{Im}z_2$



माना $z_1 = a + ib$

तथा $z_2 = c + id$

$z_1 \cdot z_2 = (a+ib)(c+id)$

$= ac + iad + ibc - bd$

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$$z_1 \cdot z_2 = \underline{ac - bd} + i(ad + bc)$$

$$\therefore Re(z_1 z_2) = ac - bd \quad \text{—— (1)}$$



आता $Re(z_1)\,Re(z_2) - Im(z_1)\,Im(z_2)$

$= a \cdot c - b \cdot d$

$= ac - bd$ —— (2)

वरील (1) व (2) वरून

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अत:

$$Re(z_1 z_2) = Re(z_1)\,Re(z_2) - Im(z_1)\,Im(z_2)$$

इति

