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कक्षा - 11

गणित

अध्याय – 4

सम्मिश्र संख्याएँ और द्विघातीय समीकरण

भाग – 9

विविध प्रश्नावली (प्रश्न 7 से 10)

हर्ष गोयल

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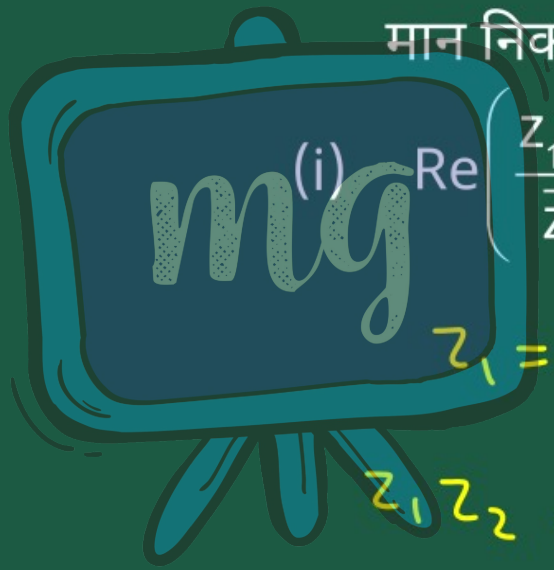


विविध प्रश्नावली

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7. माना $z_1 = 2 - i$, $z_2 = -2 + i$, निम्न का मान निकालिए।



(i) $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

$z_1 = 2-i$, $z_2 = -2+i$

$z_1 z_2 = (2-i)(-2+i)$

$= -4 + 2i + 2i - i^2$

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$$= -4 + 4i + 1$$

$$z_1 z_2 = -3 + 4i$$



$$\therefore \quad z_1 = 2 - i$$

$$\bar{z}_1 = 2 + i$$

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{-3+4i}{2+i}$$

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$$\frac{z_1 z_2}{\bar{z}_1} = \frac{-3+4i}{2+i} \times \frac{2-i}{2-i}$$



$$= \frac{-6+3i+8i+4}{(2)^2-(i)^2}$$

$$= \frac{-6+4+i(3+8)}{4+1}$$

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{-2+11i}{5}$$

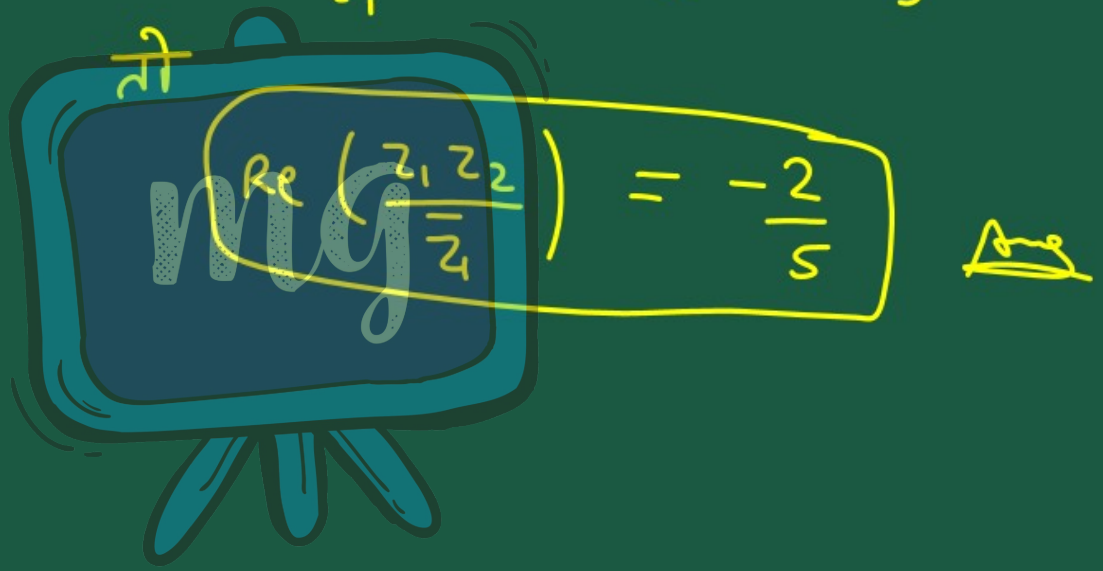
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$$\frac{z_1 z_2}{\bar{z}_1} = \frac{-2}{5} + i\frac{11}{5}$$

तो

$$\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = -\frac{2}{5}$$



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$z \neq 0$

$\text{Im}\left(\frac{1}{z \cdot \bar{z}}\right)$

(ii) $\text{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$



$\because z_1 \bar{z}_1 = |z_1|^2$

$\therefore$ किसी भी समिश्र संख्या का मापांक अऋणेत्तर वास्तविक संख्या होता है

$z_1 \cdot \bar{z}_1 \rightarrow$ वास्तविक संख्या

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$\frac{1}{z_1 \cdot \bar{z}_1}$ → वास्तविक संख्या

$\text{Im}\left(\frac{1}{z_1 \cdot \bar{z}_1}\right) = 0$

अर्थात्

$z_1 = 2 - i$

$\bar{z}_1 = 2 + i$

$\frac{1}{z_1 \cdot \bar{z}_1} = \frac{1}{|z_1|^2} = \left(\frac{1}{\sqrt{(2)^2 + (-1)^2}}\right)^2$

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$$\frac{1}{z_1 \cdot \bar{z}_1} = \frac{1}{4+1}$$

$$= \frac{1}{5}$$

$$\text{Im}\left(\frac{1}{z_1 \cdot \bar{z}_1}\right) = 0$$

Ans

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8. यदि $(x - iy)(3 + 5i)$, $-6 - 24i$ की संयुग्मी है तो वास्तविक संख्याएँ x और y ज्ञात कीजिए।



माना $z = -6 - 24i$

$\bar{z} = -6 + 24i$

प्रश्नानुसार $-6 - 24i$ की संयुग्मी $(x - iy)(3 + 5i)$

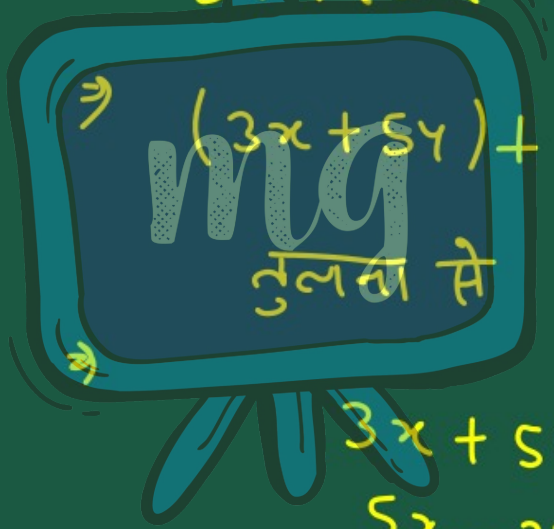
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तो $\Rightarrow (x - iy)(3 + 5i) = -6 + 24i$

$\Rightarrow 3x + i5x - i3y + 5y = -6 + 24i$

$\Rightarrow (3x + 5y) + i(5x - 3y) = -6 + 24i$



$\Rightarrow 3x + 5y = -6 \quad \text{—— (1)}$

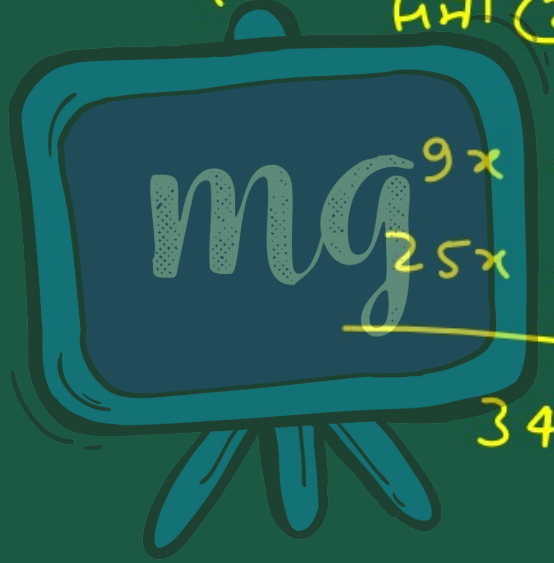
$5x - 3y = 24 \quad \text{—— (2)}$

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समी (1) × 3 से

तथा समी (2) × 5 से



$$9x + 15y = -18$$
$$25x - 15y = 120$$

$$34x = 102$$

$$x = \frac{102}{34}$$

$$x = 3$$

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x का मान समी (1)
में रखने पर



$3(3) + 5y = -6$

$9 + 5y = -6$

$5y = -6 - 9$

$5y = -15$

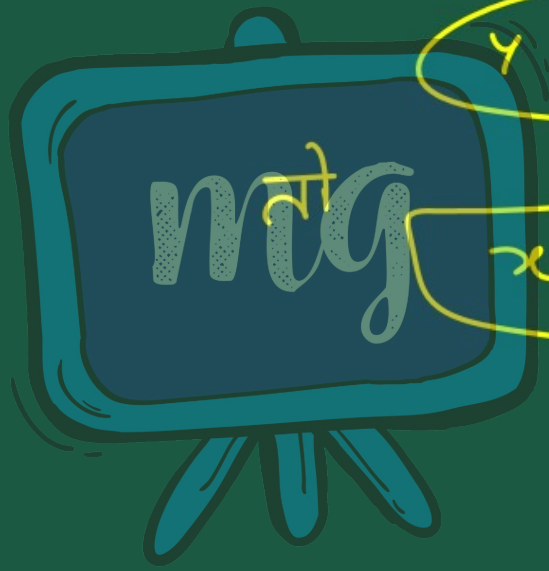
$y = -15$
$\overline{5}$

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$y = \frac{-15}{5}$

$y = -3$



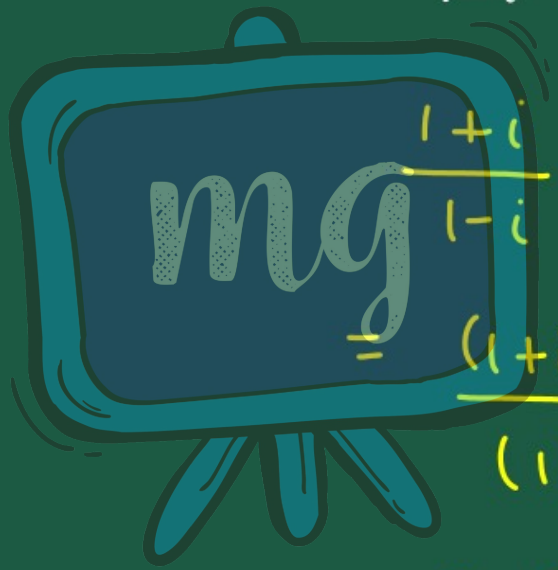
$x = 3$ तथा $y = -3$

Ans

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9. $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ का मापांक ज्ञात कीजिए–



$$\frac{1+i}{1-i} - \frac{1-i}{1+i}$$

$$= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$$

$$= \frac{1+2i-1-(1-2i-1)}{(1)^2-(i)^2}$$

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$$= \frac{2i - 2 + 2i + 2}{1 + 1}$$

$$= \frac{4i}{2}$$

$$\frac{1+i}{1-i} - \frac{1-i}{1+i} = 2i$$

$$\Rightarrow \left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| = |2i| = |0 + 2i|$$

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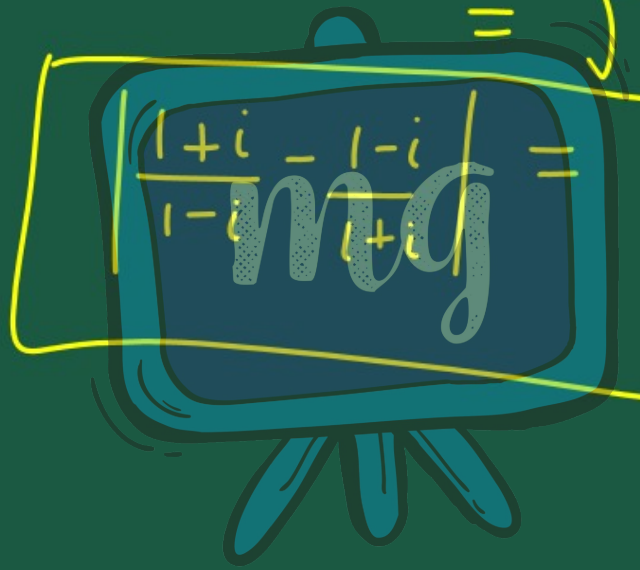
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$$= \sqrt{(0)^2 + (2)^2}$$

$$= \sqrt{0+4}$$

$$\left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| = 2$$

Ans

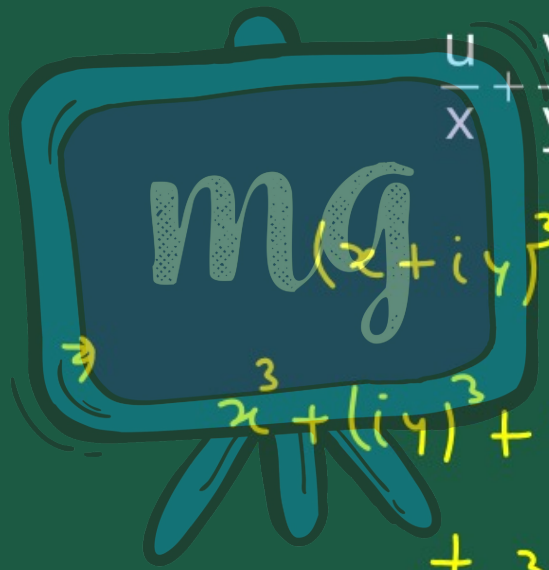


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10. यदि $(x + iy)^3 = u + iv$ तो दर्शाइए कि

$$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$



$$(x+iy)^3 = u+iv$$

$$x^3 + (iy)^3 + 3x^2(iy) + 3x(iy)^2 = u+iv$$

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$\Rightarrow \quad x^3 - iy^3 + 3x^2yi - 3xy^2 = u + iv$

$\Rightarrow \quad (x^3 - 3xy^2) + i(3x^2y - y^3) = u + iv$

तुलना से

$x^3 - 3xy^2 = u \quad \text{—— (1)}$

तथा

$3x^2y - y^3 = v \quad \text{—— (2)}$

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समी ① से

$\frac{u}{x} + \frac{v}{y}$

$x^3 - 3xy^2 = u$

$\Rightarrow x(x^2 - 3y^2) = u$

$\Rightarrow x^2 - 3y^2 = \frac{u}{x}$ — ③

पुनः समी ② से

$3x^2y - y^3 = v$

$\Rightarrow y(3x^2 - y^2) = v$



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$$\Rightarrow \quad 3x^2 - y^2 = \frac{v}{y} \quad \text{— (4)}$$

समी (3) + समी (4)

$$mg\left(\frac{u}{x} + \frac{v}{y}\right) = x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

इति