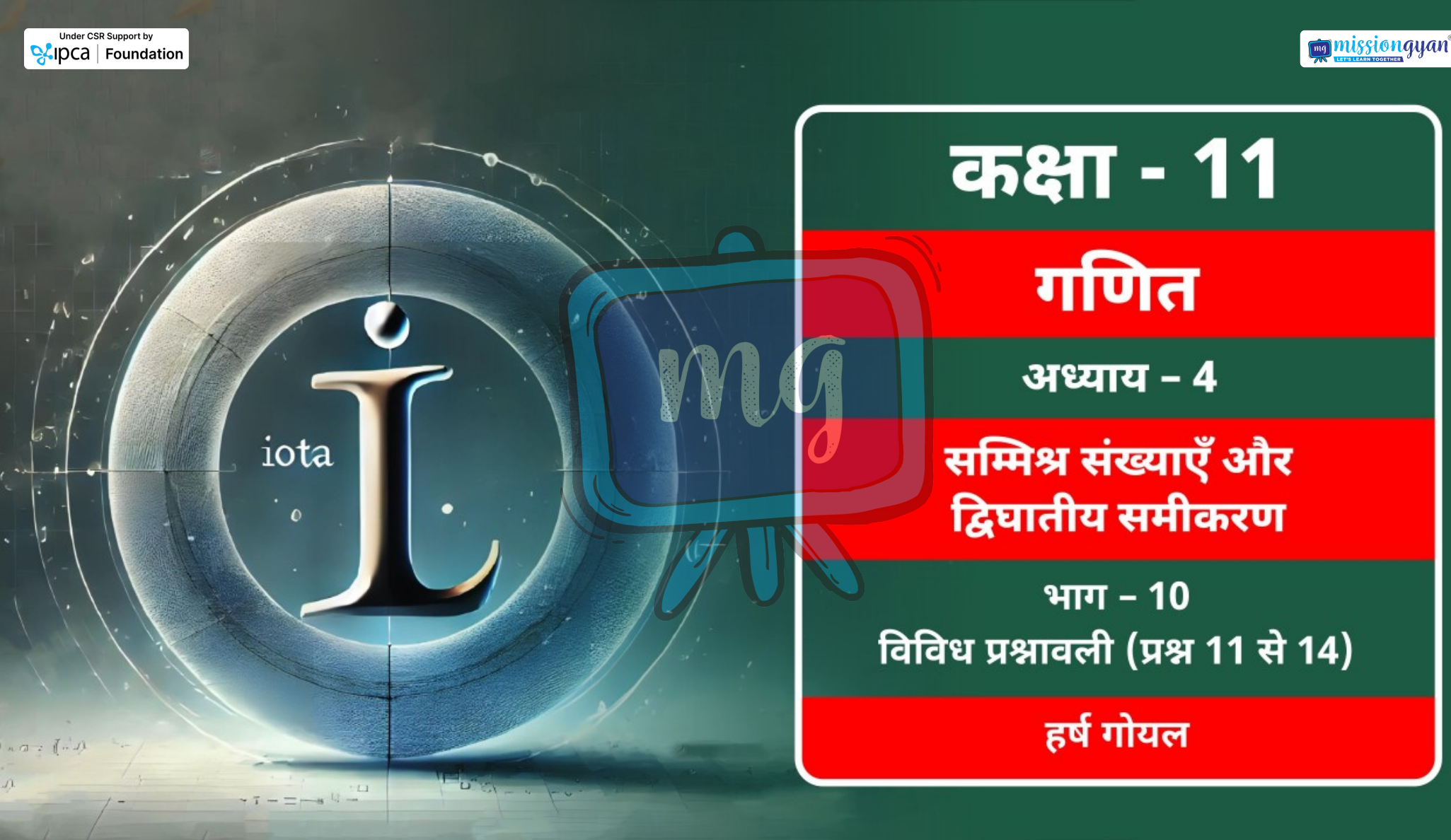
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कक्षा - 11

गणित

अध्याय – 4

सम्मिश्र संख्याएँ और द्विघातीय समीकरण

भाग – 10

विविध प्रश्नावली (प्रश्न 11 से 14)

हर्ष गोयल

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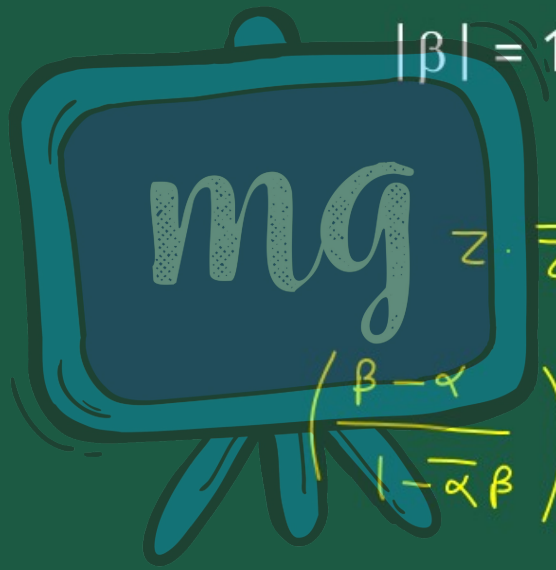


विविध प्रश्नावली

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11. यदि α और β भिन्न सम्मिश्र संख्याएँ हैं जहाँ $|\beta| = 1$ तब $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$ का मान ज्ञात कीजिए।



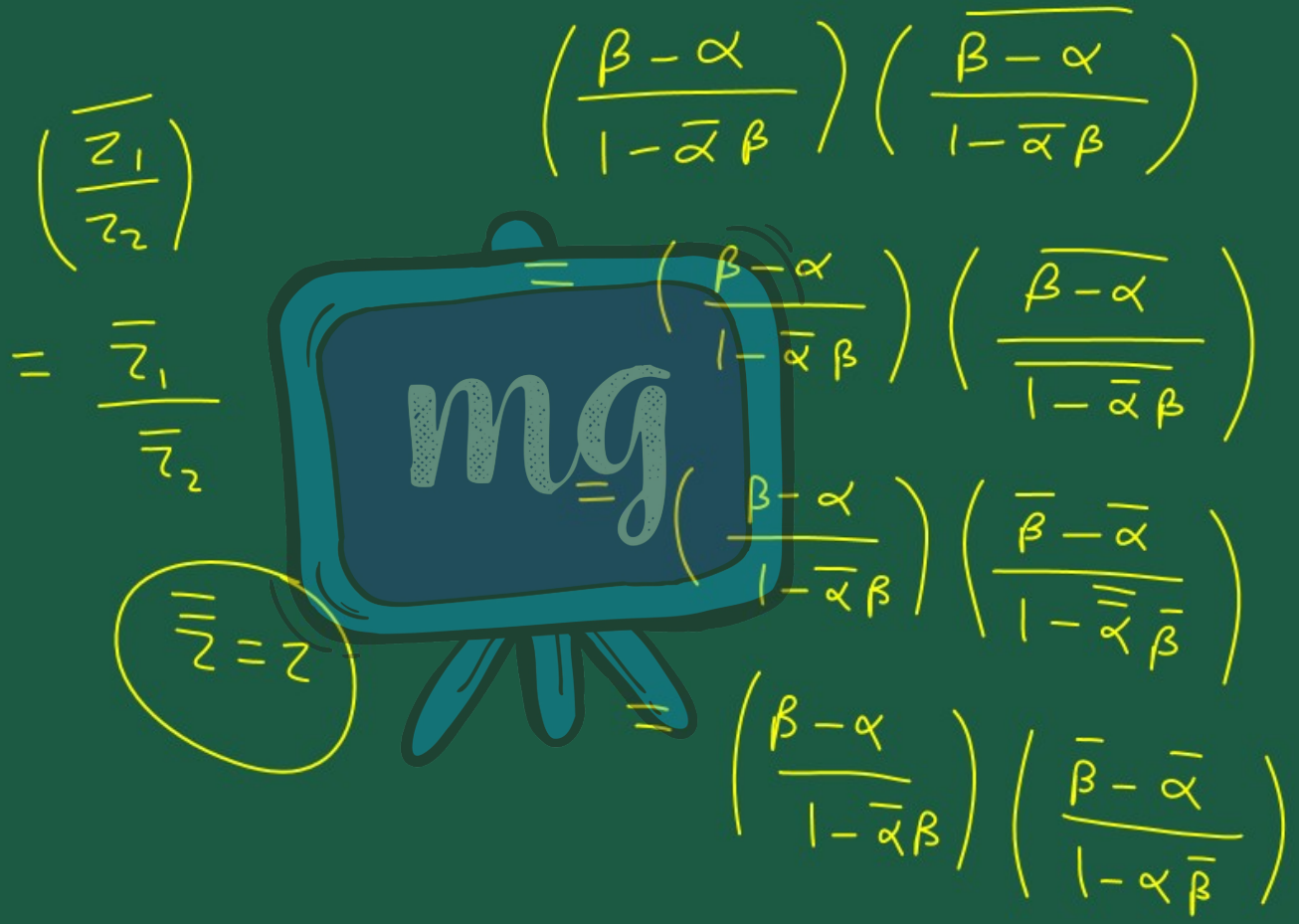
$$z \cdot \bar{z} = |z|^2$$

$$\left(\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right)\left(\frac{\overline{\beta - \alpha}}{1 - \bar{\alpha}\beta}\right) = \left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|^2$$

$$= 1$$

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दिया है

$|\beta| = 1$

$$= \frac{\beta \cdot \bar{\beta} - \bar{\alpha}\beta - \alpha\bar{\beta} + \alpha\bar{\alpha}}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + \alpha\bar{\alpha}\beta\bar{\beta}}$$

$$= \frac{|\beta|^2 - \bar{\alpha}\beta - \alpha\bar{\beta} + |\alpha|^2}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2|\beta|^2}$$

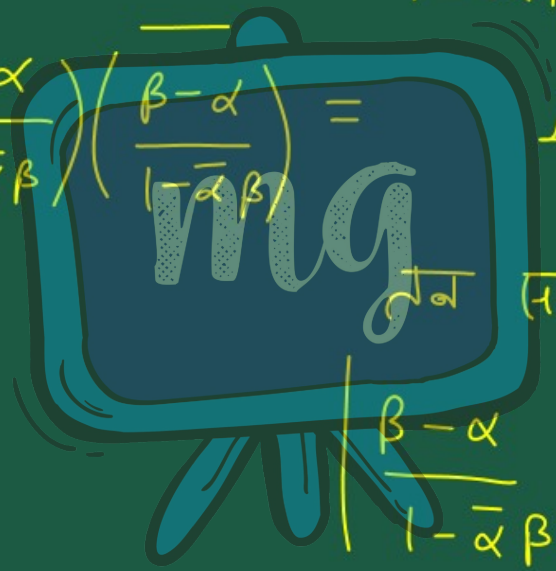
$$= \frac{(1)^2 - \bar{\alpha}\beta - \alpha\bar{\beta} + |\alpha|^2}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2(1)^2}$$

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$$= \frac{1 - \bar{\alpha}\beta - \alpha\bar{\beta} + |\alpha|^2}{1 - \bar{\alpha}\beta - \alpha\bar{\beta} + |\alpha|^2}$$

$$\left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\overline{\beta-\alpha}}{\overline{1-\bar{\alpha}\beta}}\right) = 1$$



अब यही ① से

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = 1$$

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$$\Rightarrow \left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| = 1$$

Ans



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12. समीकरण $|1-i|^x = 2^x$ के शून्येत्तर पूर्णांक मूलों की संख्या ज्ञात कीजिए।



$$|1-i|^x = 2^x$$

$$\Rightarrow \left(\sqrt{(1)^2+(-1)^2}\right)^x = 2^x$$

$$\Rightarrow \left(\sqrt{1+1}\right)^x = 2^x$$

$$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$$

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$\Rightarrow \left((2)^{\frac{1}{2}}\right)^{x} = 2^{x}$

$\Rightarrow 2^{x/2} = 2^{x}$

घातों से

$\frac{x}{2} = x$

$x = 2x$

$2x - x = 0$

$x = 0$

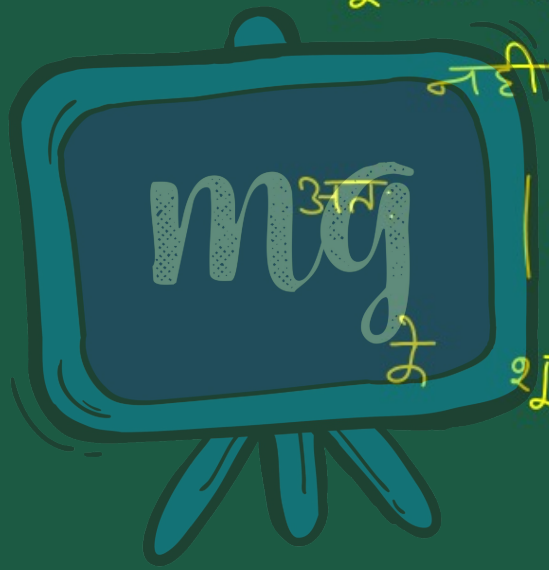
यह तभी संभव है जब

$x = 0$

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अतः कोई भी शून्येत्तर पूर्णांक मूल विद्यमान मान नहीं है।



अतः $|1-i|^x = 2^x$

शून्येत्तर पूर्णांक मूलों की संख्या शून्य है।

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13. यदि $(a + ib)(c + id)(e + if)(g + ih) = A + iB$

है तो दर्शाइए कि

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

$A + iB$

↓

$A^2 + B^2$

$(a+ib)(c+id)(e+if)(g+ih)$

इसका परिमाण

$= A + iB$

(1)

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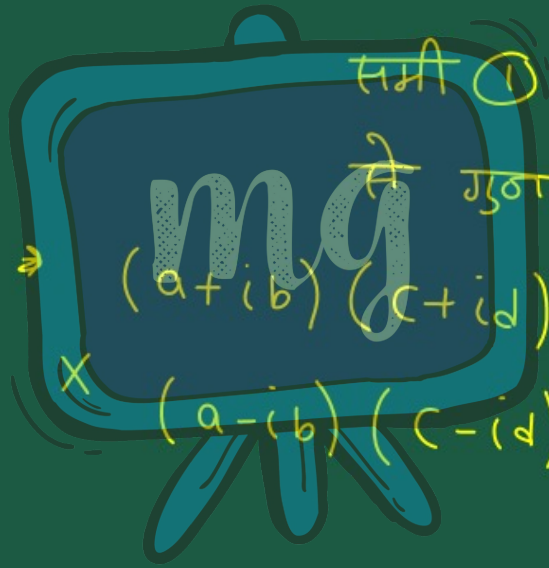
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$$(a-ib)(c-id)(e-if)(g-ih) = A-iB$$

समी (1) को समी (2) ——— (2)

से गुणा करने पर

$$\Rightarrow (a+ib)(c+id)(e+if)(g+ih) \times (a-ib)(c-id)(e-if)(g-ih) = A+iB \times A-iB$$



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$$\Rightarrow (a^2-(ib)^2)(c^2-(id)^2)(e^2-(if)^2)$$

$$(g^2-(ih)^2) = A^2-(iB)^2$$

$$\Rightarrow (a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2)$$

$$= A^2+B^2$$

इतिसिद्धम



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14. यदि $\left(\frac{1+i}{1-i}\right)^m = 1$, तो m का न्यूनतम धनात्मक पूर्णांक मान ज्ञात कीजिए।



$\left(\frac{1+i}{1-i}\right)^m = 1$ —— (1)

$\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

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$$\frac{1+i}{1-i} = \frac{(1+i)^2}{(1)^2-(i)^2}$$

$$= \frac{1+(i)^2+2(1)(i)}{1+1}$$

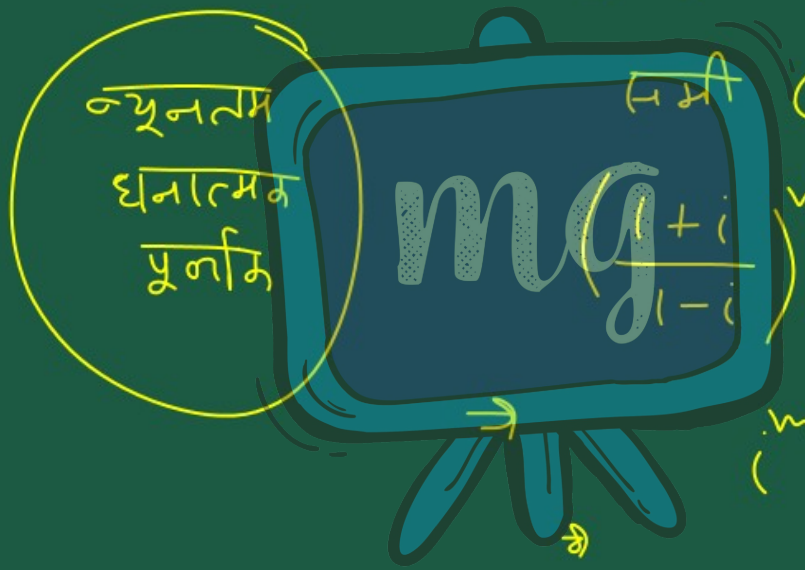
$$= \frac{1-1+2i}{2}$$

$$= \frac{2i}{2}$$

$$= i$$

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$\rightarrow \quad \frac{1+i}{1-i} = i$

न्यूनतम धनात्मक पूर्णांक

तभी ① में रखने पर

$\left(\frac{1+i}{1-i}\right)^m = 1$

$i^m = 1$

$i^m = (i)^4$ तुलना से

$m = 4$

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अत: $\boxed{m = 4}$ पर

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

