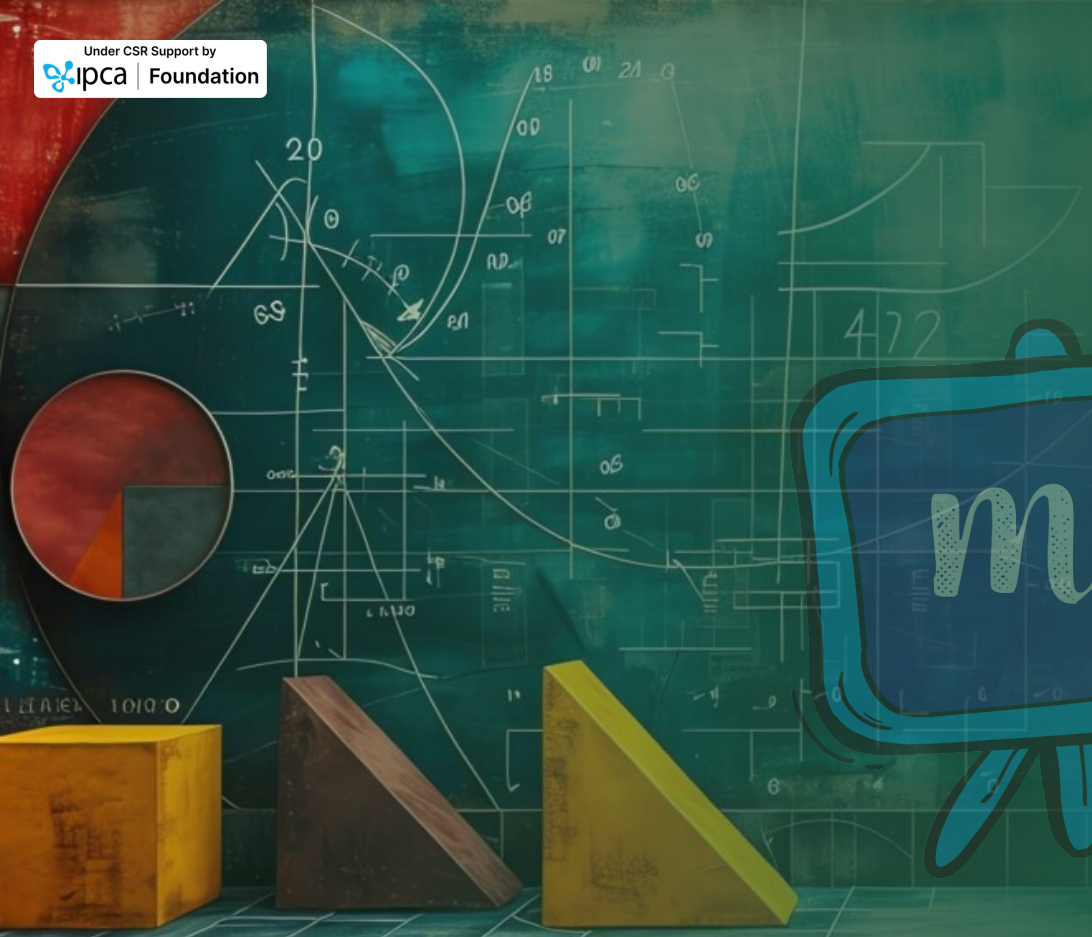
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कक्षा - 11

गणित

अध्याय - 2

संबंध एवं फलन

भाग - 5

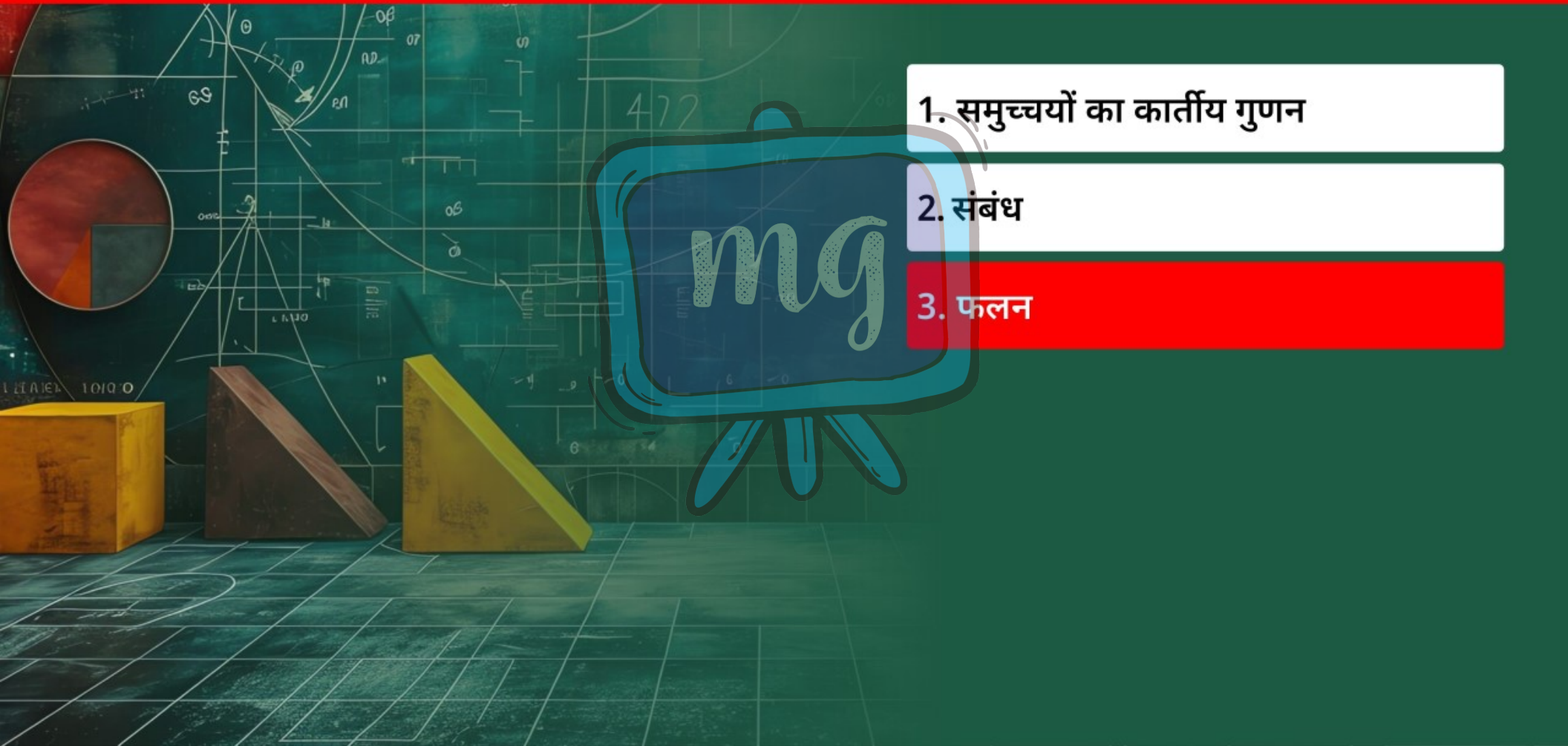
फलन

हर्ष गोयल

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विवरण (Overview)

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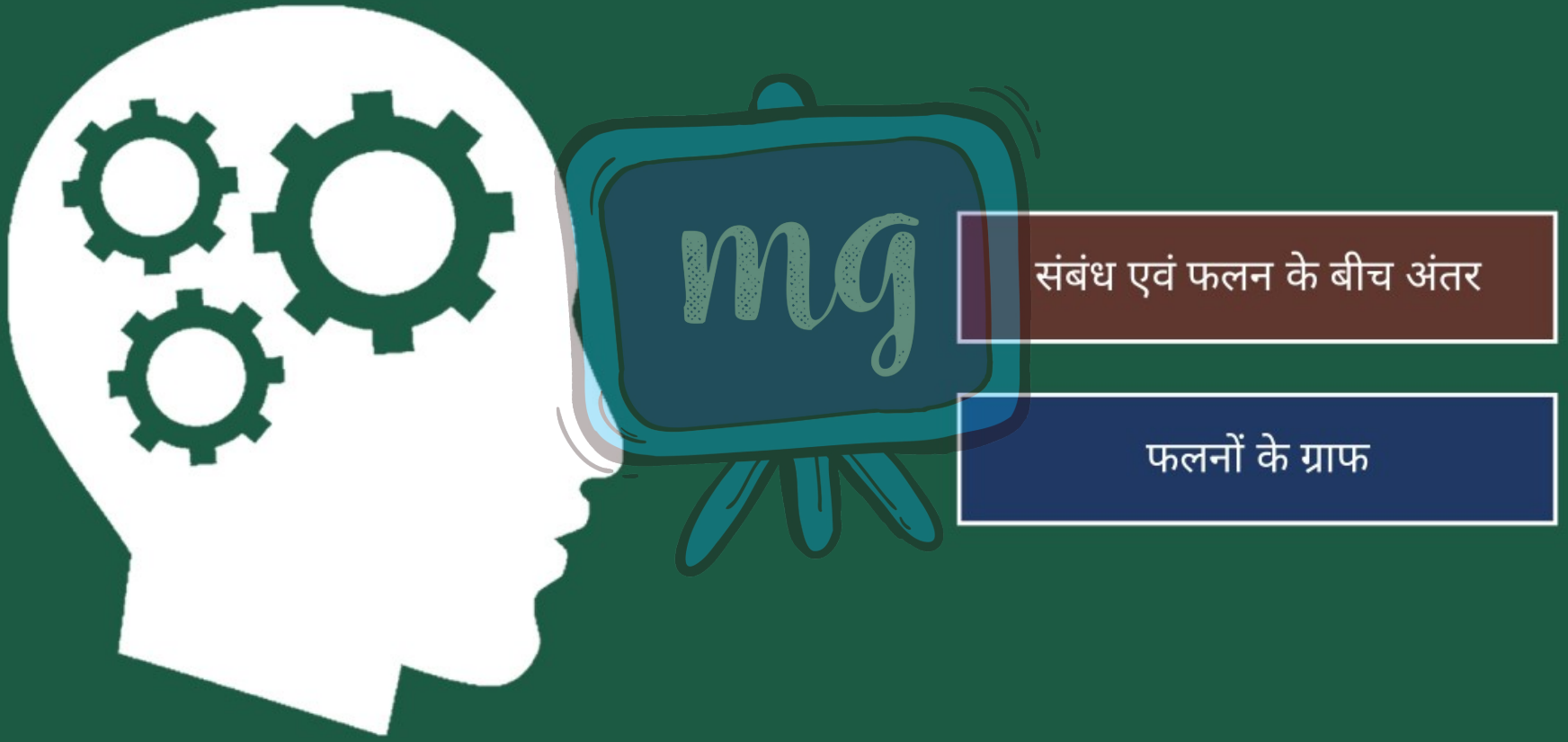


1. समुच्चयों का कार्तीय गुणन
2. संबंध
3. फलन

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योग्यता आधारित शिक्षण (Competency)

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फलन

$A \longrightarrow B$ संबंध

↓

के प्रत्येक अवयव का मे केवल और केवल एक प्रतिबिंब

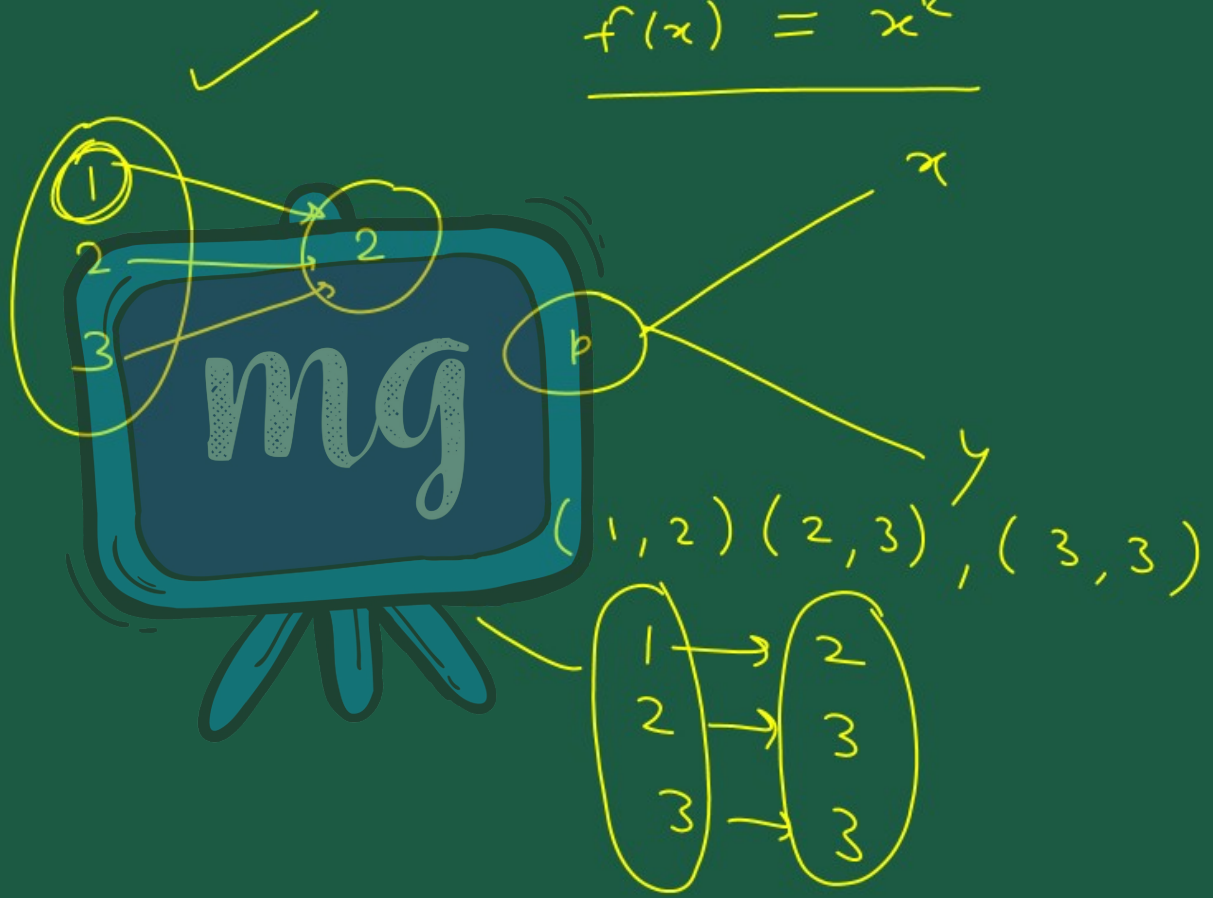
$A = \{1, 2, 3\}$

$B = \{1, 2\}$

$2 \rightarrow 2$

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फलन

एक समुच्चय A से समुच्चय B का संबंध, f एक फलन कहलाता है, यदि समुच्चय A के प्रत्येक अवयव का समुच्चय B में, एक और केवल एक प्रतिबिंब होता है।

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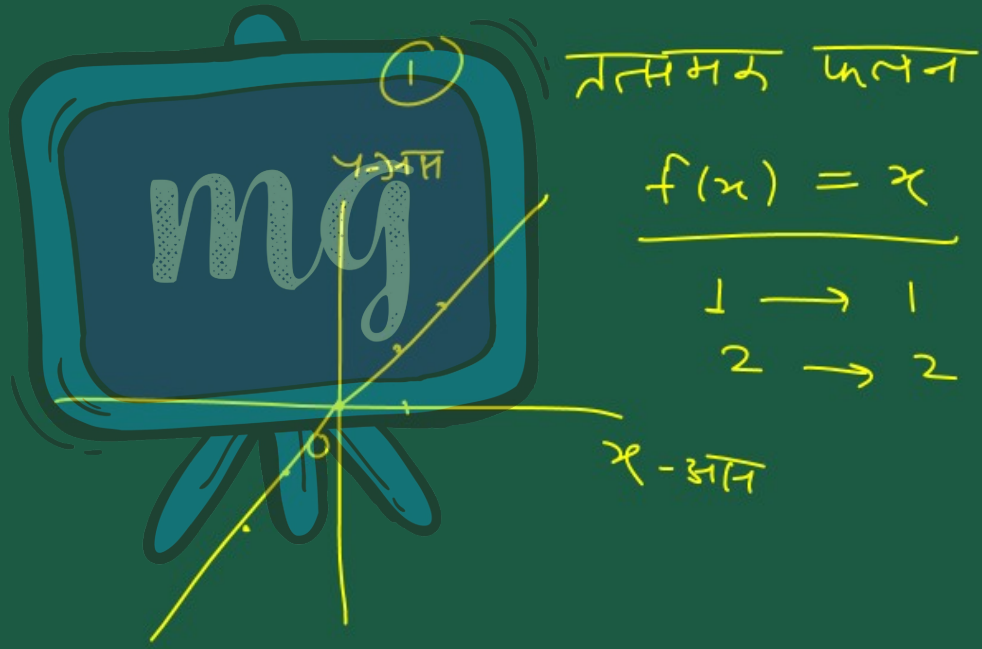


- एक ऐसे फलन को जिसका परिसर वास्तविक संख्याओं का समुच्चय या उसका कोई उपसमुच्चय हो, वास्तविक मान फलन कहते हैं।
- यदि वास्तविक चर वाले किसी वास्तविक मान फलन का प्रांत भी वास्तविक संख्याओं का समुच्चय अथवा उसका कोई उपसमुच्चय हो तो इसे वास्तविक फलन भी कहते हैं।

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कुछ फलन और उनके आलेख

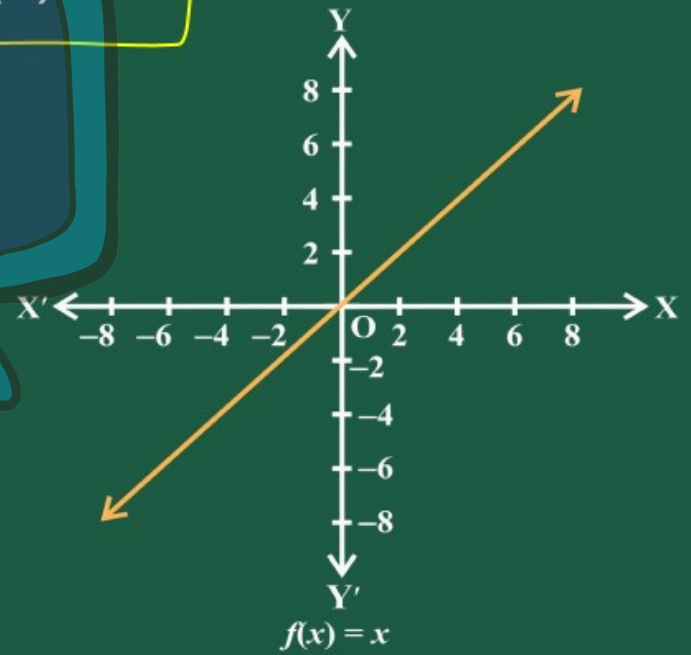


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तत्समक फलन
(Identity function)

$f(x) = x$



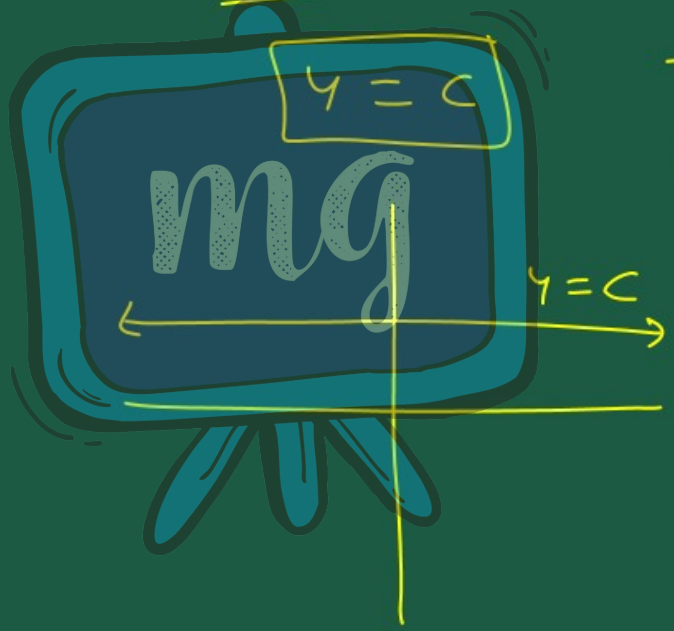
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(2) अचर फलन

$f(x) = y$

$f(x) = c$ (अचर)



0 → c
1 → c
2 → c
⋮
10 → c

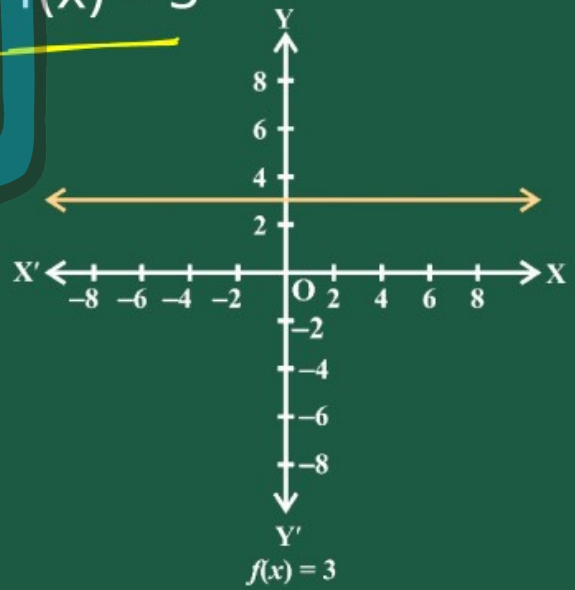
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अचर फलन
(Constant function)

$f(x) = C$, जहाँ C एक अचर है।

उदाहरण : $f(x) = 3$



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बहुपद फलन या बहुपदीय फलन

फलन $f : R \to R$, एक बहुपदीय फलन कहलाता है। यदि R के प्रत्येक x के लिए,

$$y = f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_n x^n$$

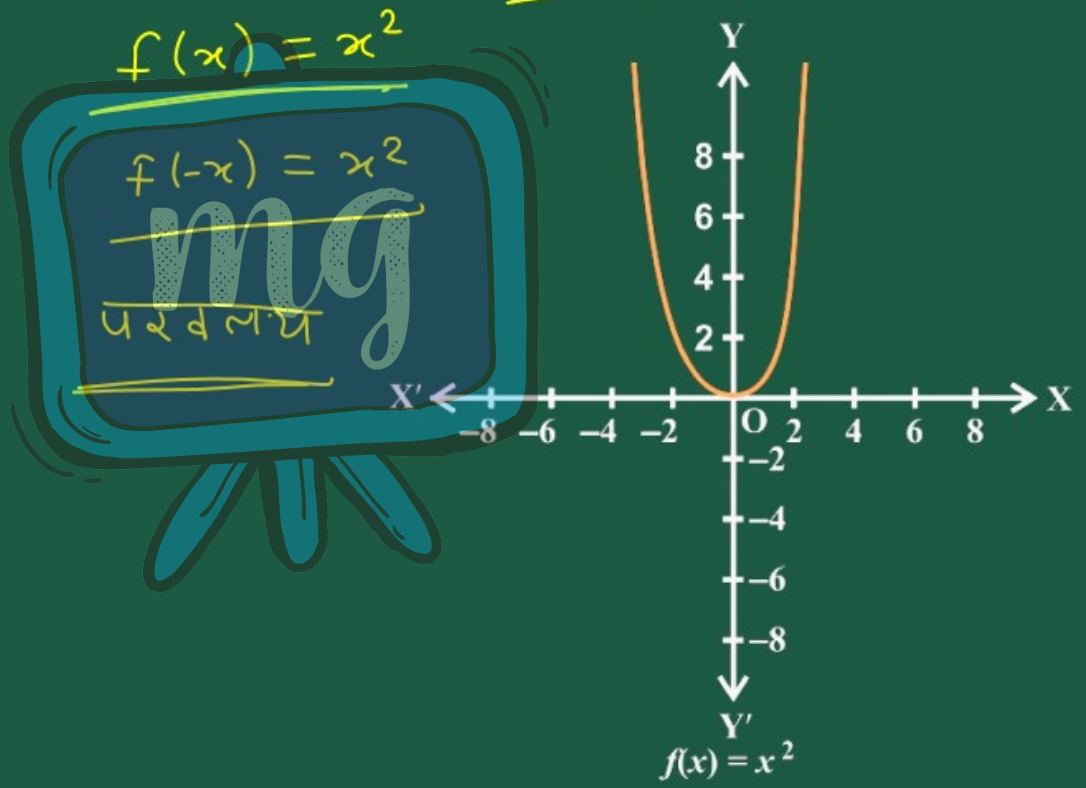
जहाँ n एक ऋणेतर पूर्णांक है तथा

$a_0, a_1, a_2, \ldots, a \in R$

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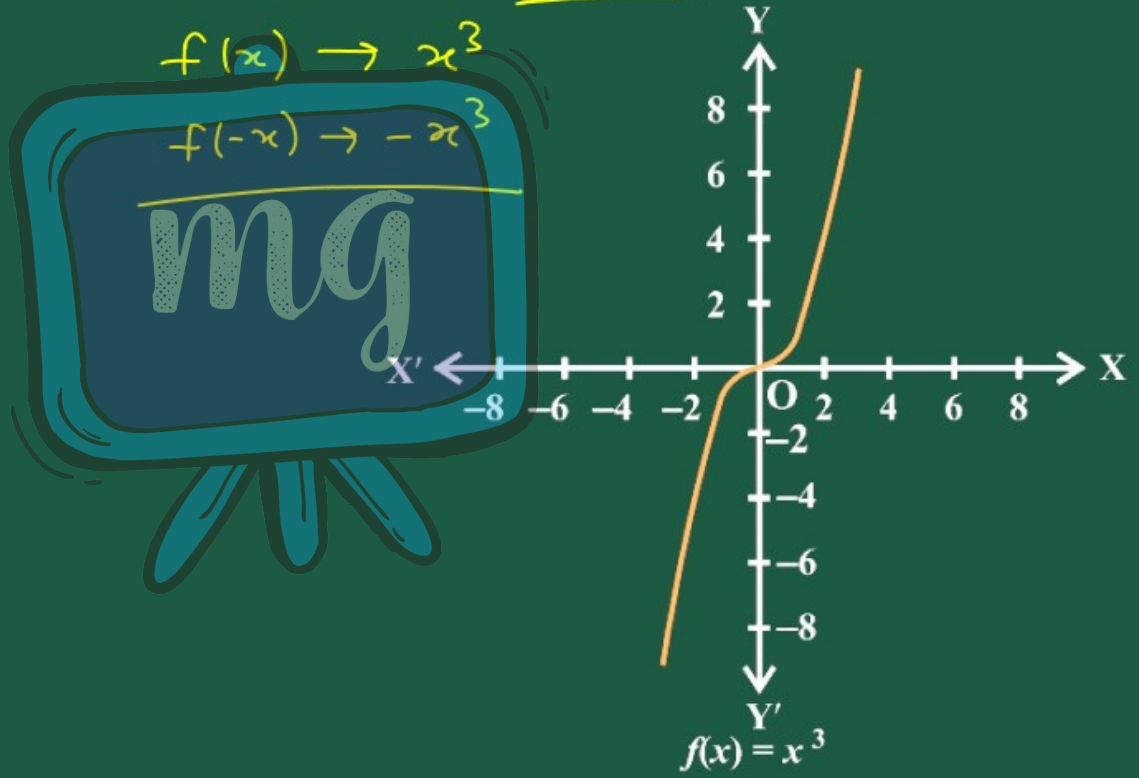
उदाहरण : $f(x) = x^2$



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उदाहरण : $f(x) = x^3$



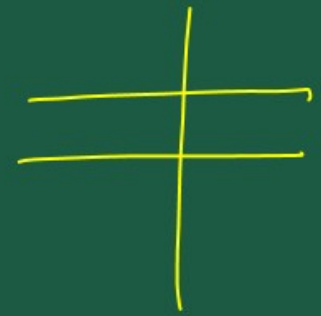
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बहुपद फलनों के प्रकार

(i) अचर बहुपद फलन :

$f(x) = a = ax^0$



(ii) रैखिक बहुपद फलन :

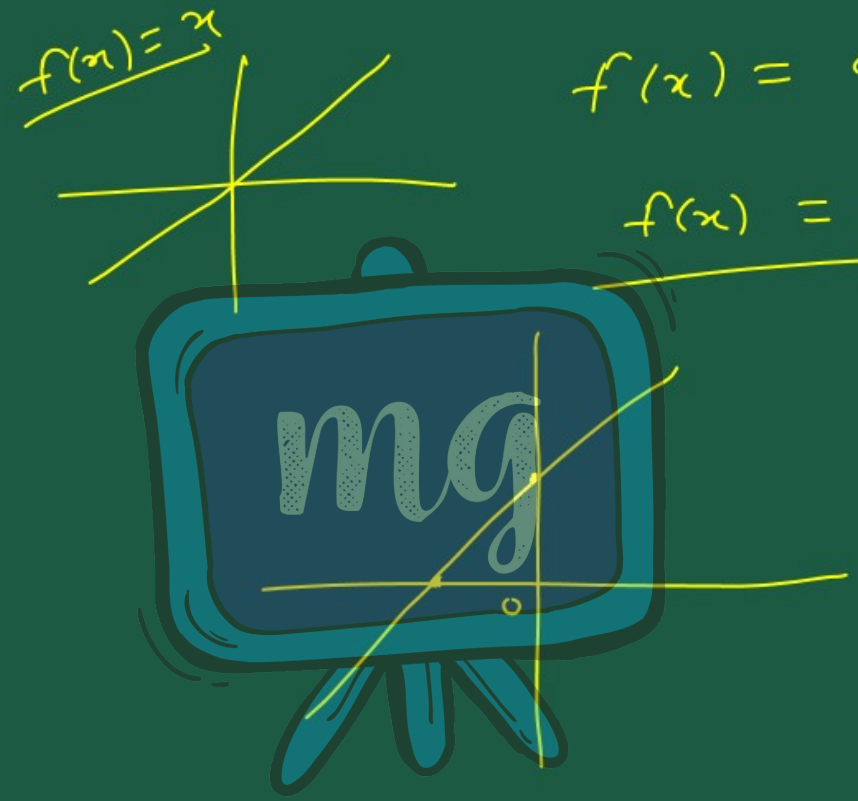
$f(x) = ax + b$

$f(x) = x$



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$f(x) = ax + b$

$f(x) = x + 2$

$x = 0 \rightarrow 2$

$x = 1 \rightarrow 3$

$x = -1 \rightarrow 1$

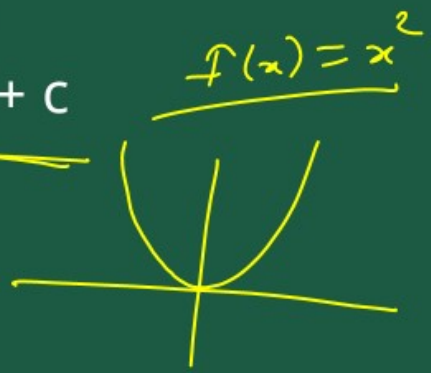
$x = -2 \rightarrow 0$

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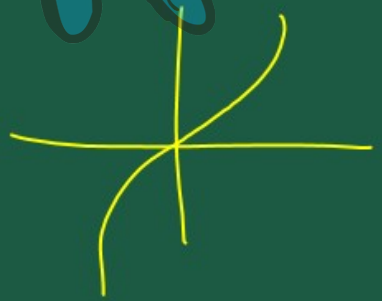
(iii) द्विघात बहुपद फलन :

$f(x) = ax^2 + bx + c$



(iv) घन बहुपद फलन :

$f(x) = ax^3 + bx^2 + cx + d$

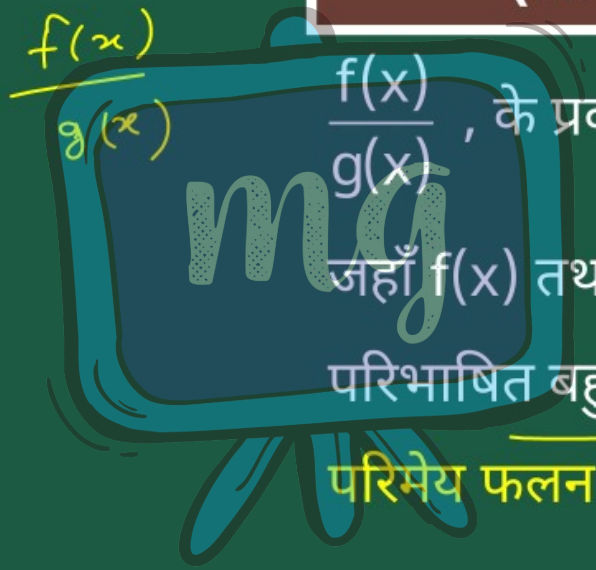


$f(x) = x^3$

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परिमेय फलन
(Rational functions)



$\frac{f(x)}{g(x)}$, के प्रकार के फलन

जहाँ f(x) तथा g(x) एक प्रांत में, x के परिभाषित बहुपदीय फलन हैं, जिसमें $g(x) \neq 0$

परिमेय फलन कहलाते हैं।

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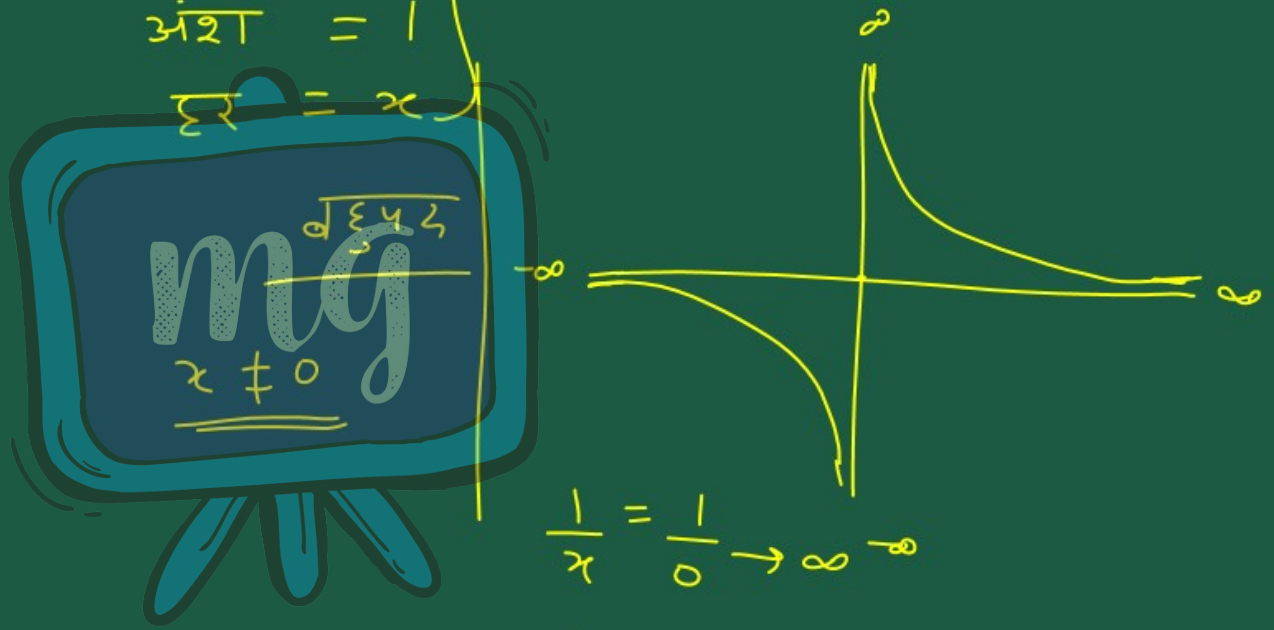
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$f(x) = \frac{1}{x}$

अंश = 1

हर = x

$x \neq 0$



$\frac{1}{x} = \frac{1}{0} \to \infty$

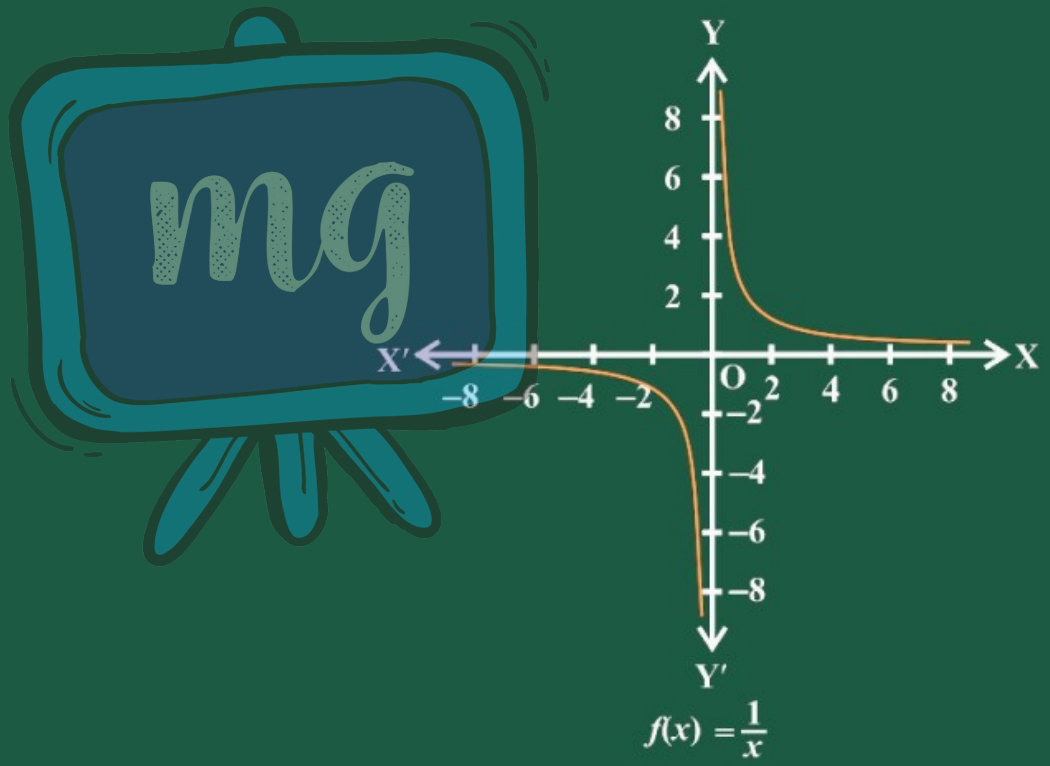
$x \to \infty \quad \frac{1}{\infty} \to 0$

$x \to -\infty$

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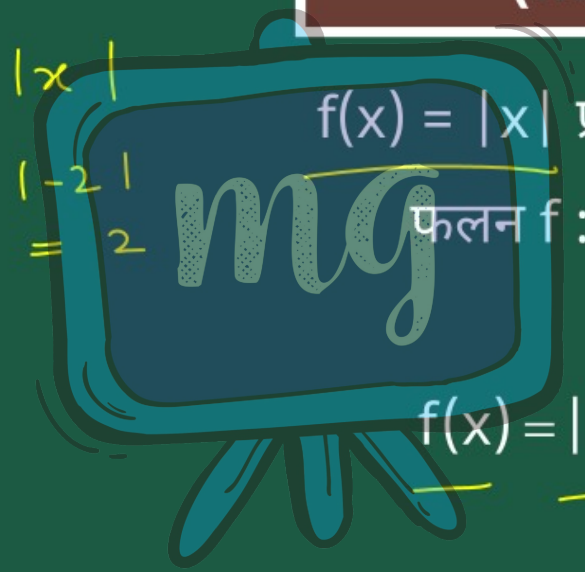
उदाहरण : $f(x) = \frac{1}{x}, x \in \mathbf{R} - \{0\}$



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मापांक फलन
(Modulus function)



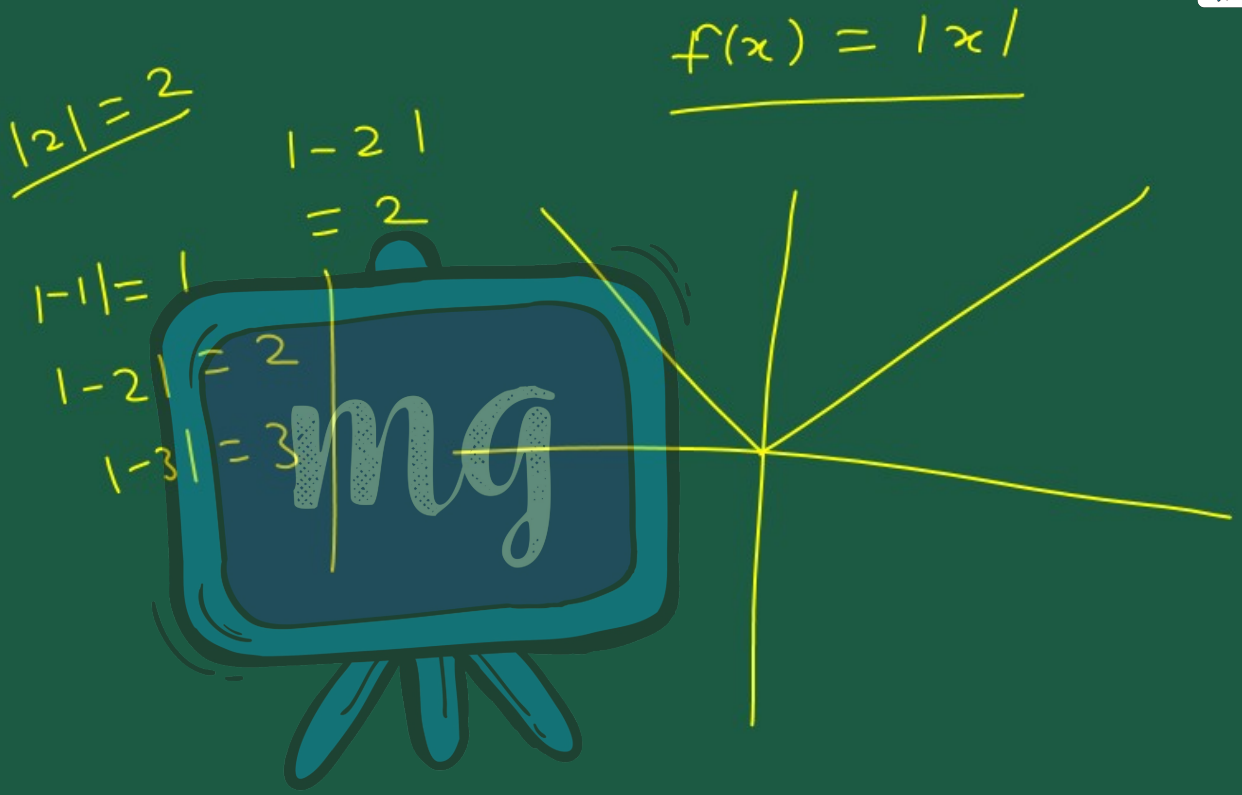
$f(x) = |x|$ प्रत्येक $x \in R$ द्वारा परिभाषित

फलन $f : R \to R$, मापांक फलन कहलाता है।

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

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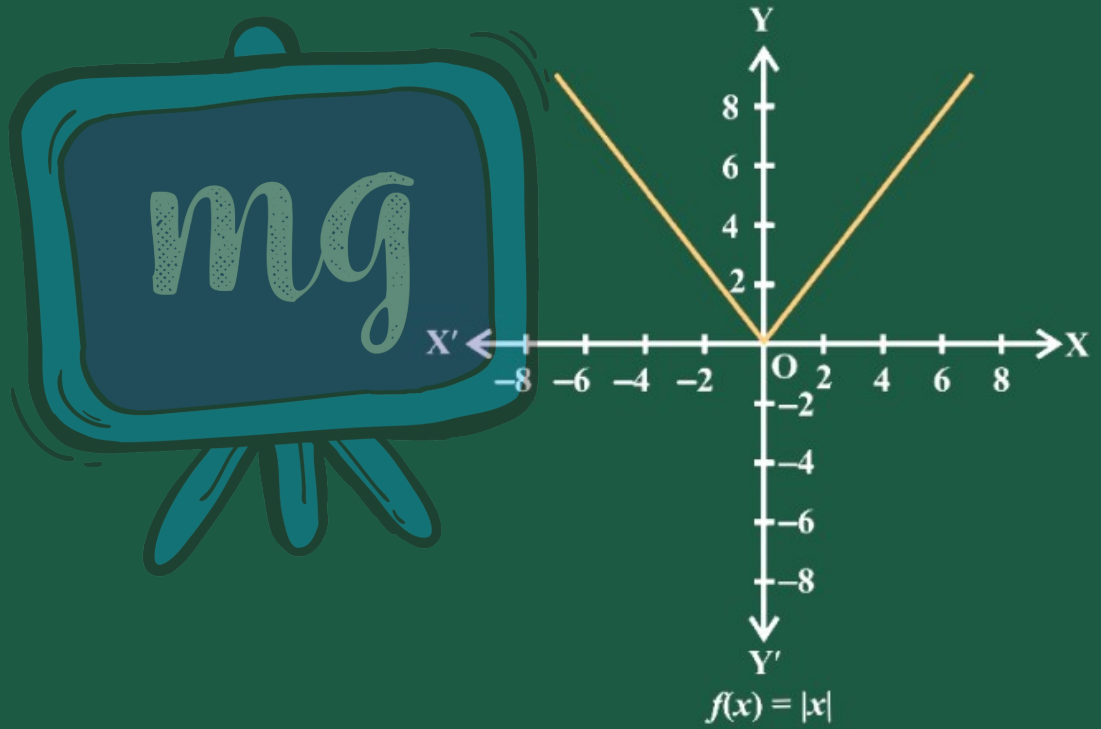
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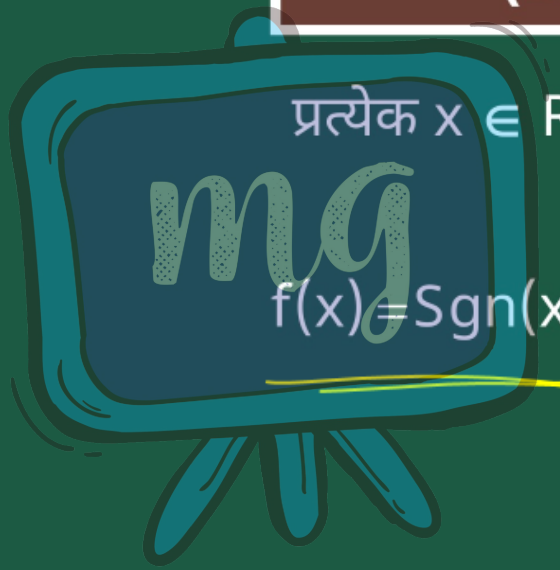
$f(x) = |x|$



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चिह्न फलन
(Signum function)



प्रत्येक $x \in R$, के लिए,

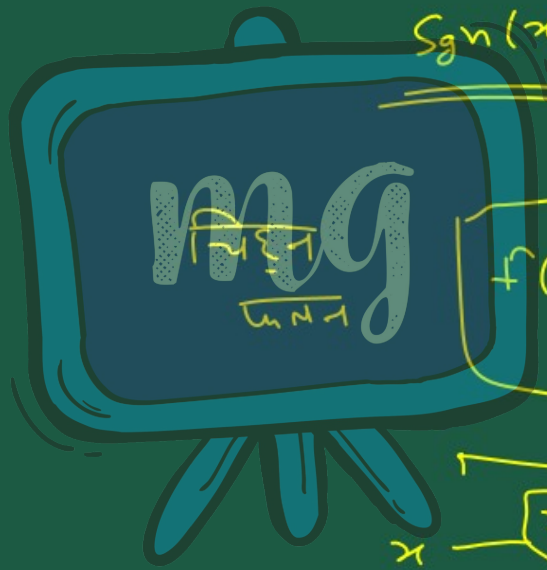
$$f(x) = Sgn(x) = \begin{cases} 1, & \text{यदि } x > 0 \\ 0, & \text{यदि } x = 0 \\ -1, & \text{यदि } x < 0 \end{cases}$$

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$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

Sgn(x)



$$f(x) = \frac{|x|}{x}$$

$x > 0$

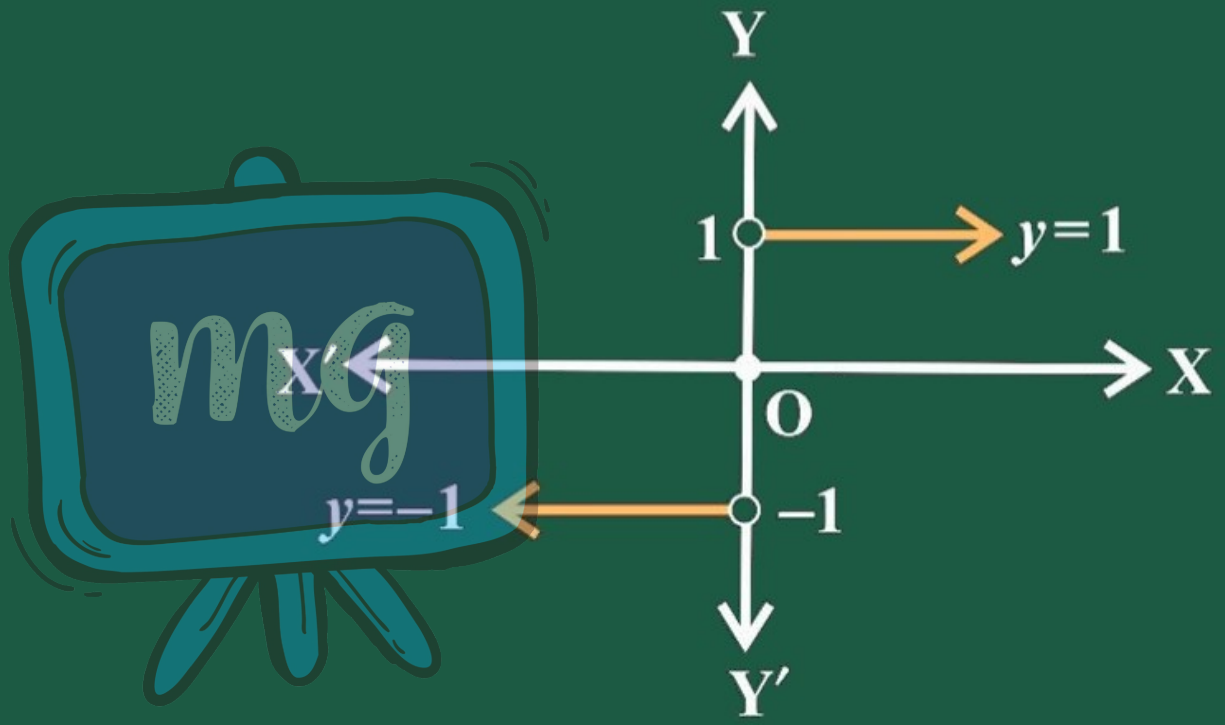
$$\frac{x}{x} = 1$$

$x < 0$

$$\frac{-x}{x} = -1$$

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$f(x) = \frac{|x|}{x}$ यदि $x \neq 0$ तथा 0 यदि $x = 0$

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महत्तम पूर्णांक फलन

$f(x) = [x]$

$n \le x < n + 1\ (n \in Z) \Rightarrow [x] = n$

x	[x]
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2

$[1, n+1)$

$x \in [1, 2)$

$[x] = 1$

$[1.2] = 1$

$[2.8] = 2$

$[2.988] = 2$

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$[2.4] = 2$

$[-0.2] = 0$ ✗

$[11.1]$

$= 11$

$[-11.1]$

$= -12$



$-0.2 \in [-1, 0)$

$[-0.2] = -1$

$[-2.5] =$

$-2.5 \in [-3, -2)$

$[-2.5) = -3$

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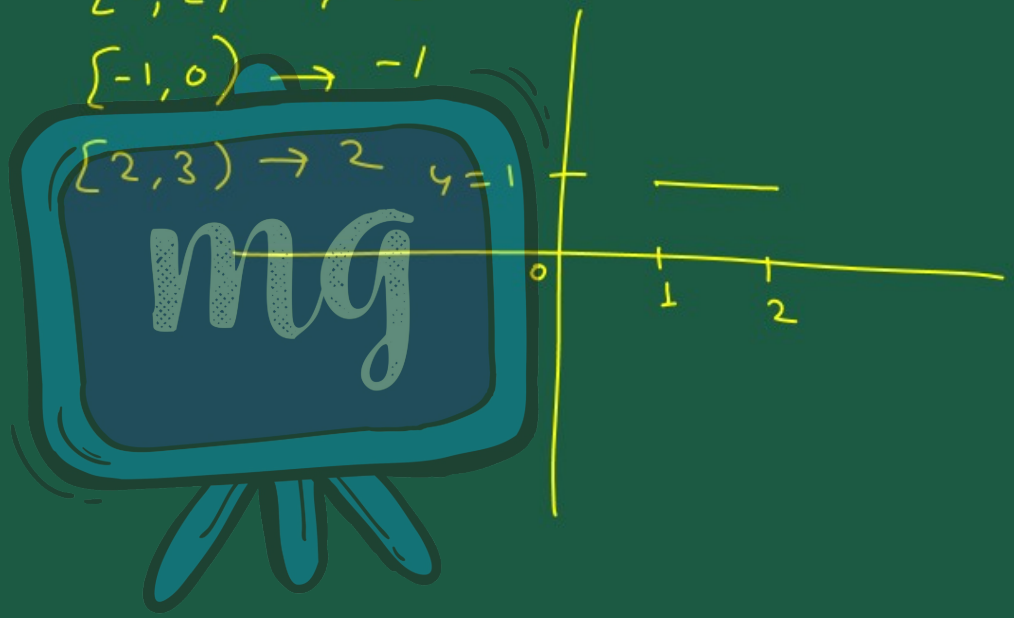
$[0,1) \rightarrow 0$

$[1,2) \rightarrow 1$

$[-1,0) \rightarrow -1$

$[2,3) \rightarrow 2$

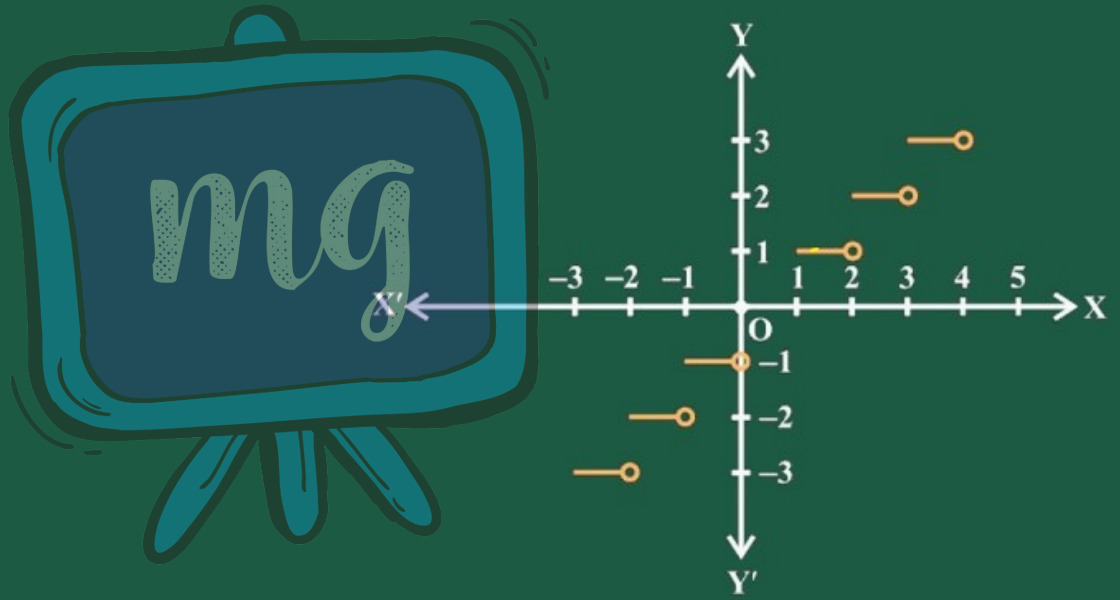
$[x]$



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$$f(x) = [x]$$



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$f(x) = x^2$

$g(x) = x^3$

$f(x) - g(x)$

$= x^2 - x^3$

$(f-g)(x)$

$= x^2 - x^3$

वास्तविक फलनों का बीजगणित (Algebra of real functions)

फलनों का बीजगणित फलन $f : X \to R$ तथा $g : X \to R$, के लिए हम निम्नलिखित परिभाषाएँ देते हैं।

1. $(f + g)(x) = f(x) + g(x), x \in X$
2. $(f - g)(x) = f(x) - g(x), x \in X$

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$f(x) = x$

$g(x) = 2x$

$= 2x \cdot x$

$f.g(x) = 2x^2$

3. $(f.g)(x) = f(x).g(x), x \in X$, k कोई अचर है।

4. $(kf)(x) = k(f(x)), x \in X$

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)}, x \in X, g(x) \neq 0$$

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सीखने के प्रतिफल (Learning Outcomes)

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1. फलन
2. फलनों के प्रकार
3. फलनों का बीजगणित

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मूल्यांकन (Assessment)

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1 यदि A = {1, 2, 3, 4} तथा B = {a, b, c, d} तो निम्न में से कौनसा A → B फलन है :

अ (1, a), (2, c), (3, b)

ब (1, a), (1, b), (2, c), (3, c), (4, c)

स (1, a), (2, c), (3, b), (4, d)

द (1, a), (2, b), (3, c), (3, d)

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मूल्यांकन (Assessment)

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2 निम्न सम्बन्धों में से कौनसा फलन है –

अ {(1, 4), (2, 6), (1, 5), (3, 9)}

ब {(1, 2), (2, 2), (3, 2), (4, 2)}

स {(3, 3), (2, 1), (1, 2), (2, 3)}

द {(3, 1), (3, 2), (3, 3), (3, 4)}

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मूल्यांकन (Assessment)

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3 | एक फलन $f(x) = x^2 + x$ द्वारा परिभाषित है। तो $f(2)$ होगा –

अ 8

ब 6

स 4

द 10

$f(x) = x^2 + x$

$x = 2$

$f(2) = (2)^2 + 2$

$= 4 + 2$

$= 6$