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CLASS – 9

MATHEMATICS

Chapter – 10

Heron's Formula

Part – 1

Area of a Triangle – by Heron's formula

Vikas Saini

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OVERVIEW

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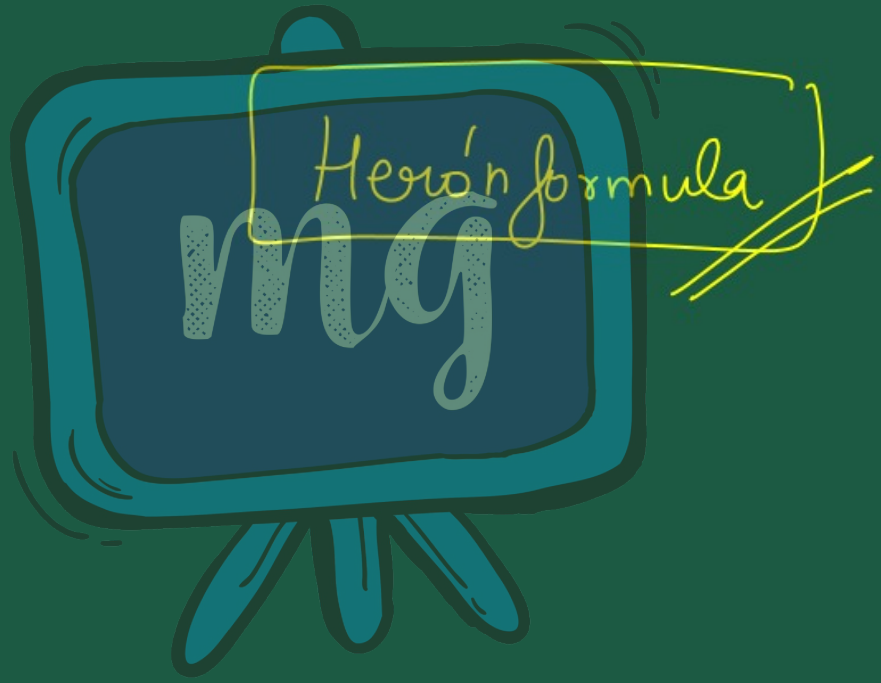

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3 vertex
3 Sides
3 Angle

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PERIMETER

- It is a 1 dimensions space of a 2 dimension figure that means the length of the distance around the 2 dimension shape.
- Perimeter of any polygon is sum of its all sides.

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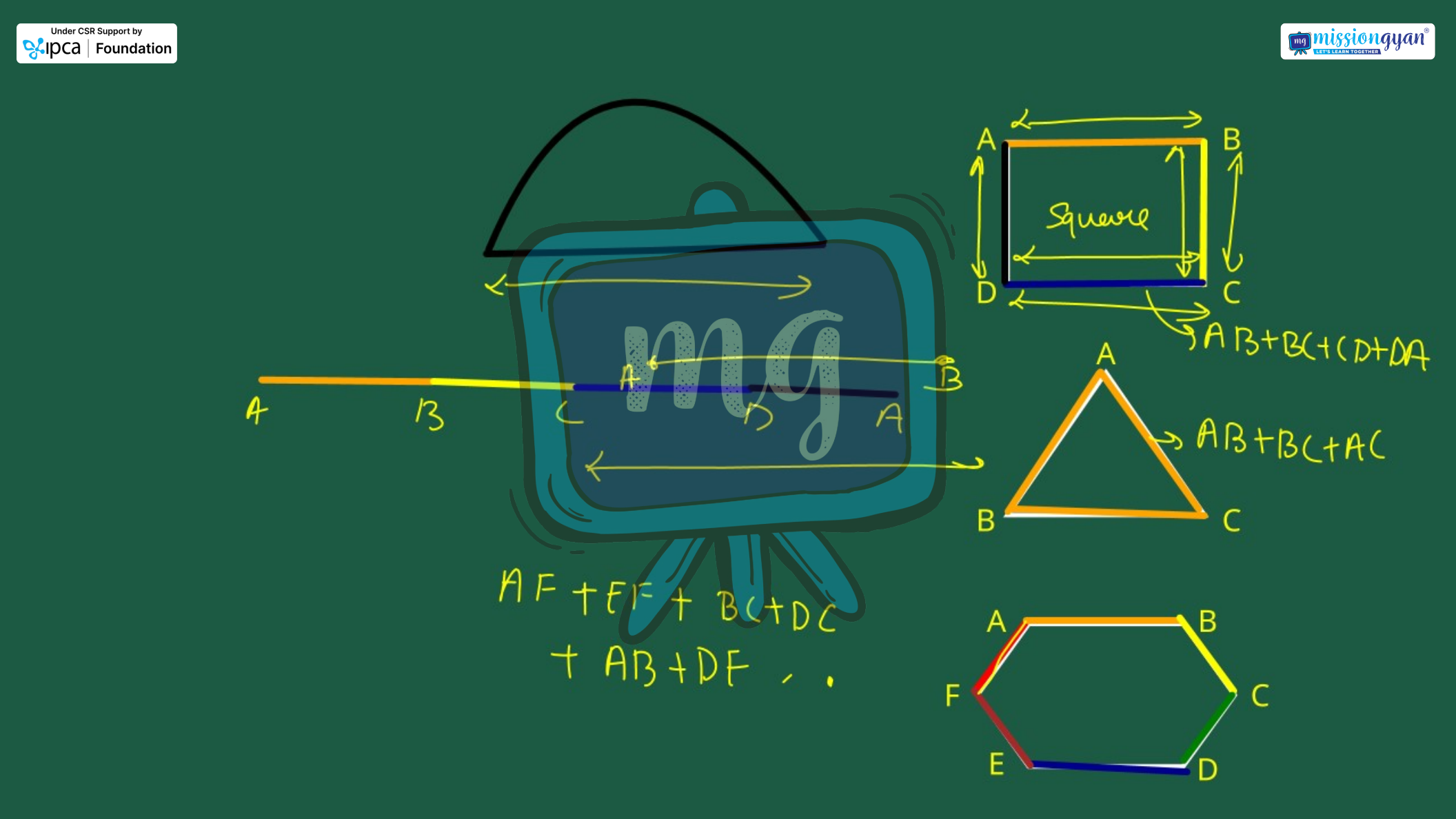

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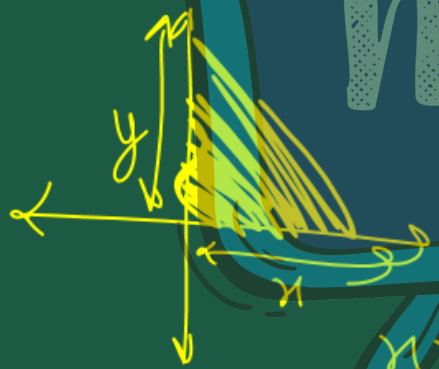
AREA

☛ The area of any closed figure is the surface bounded by its perimeter. Its unit area is the square of the unit length.





$m \times m = m^2$

$km \times km = km^2$

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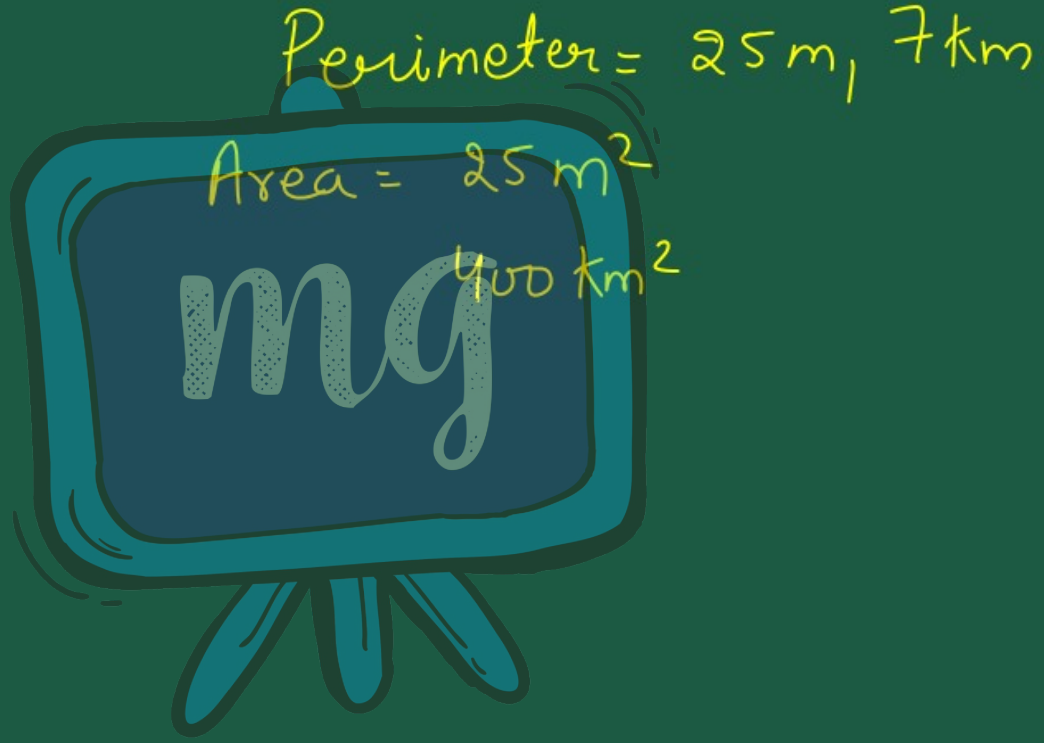

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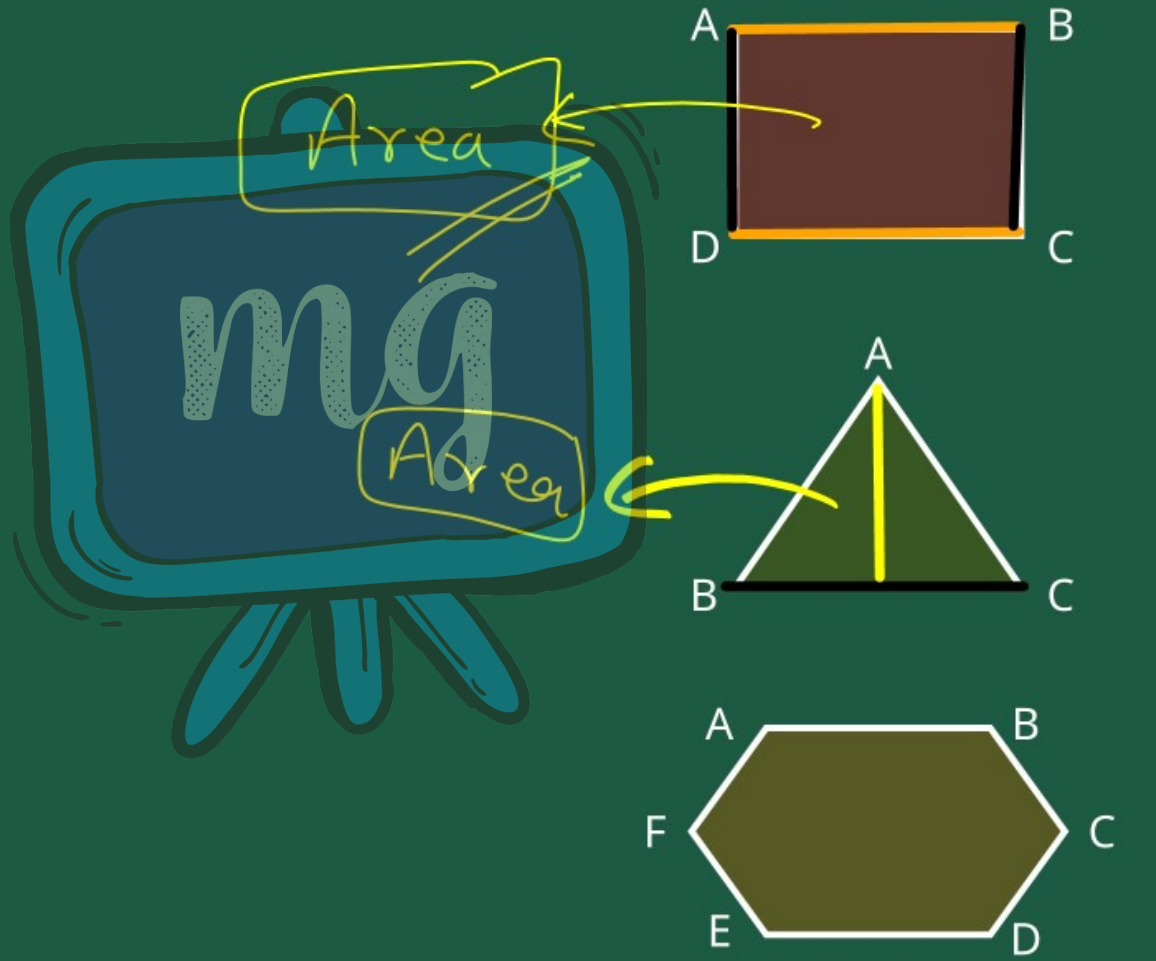

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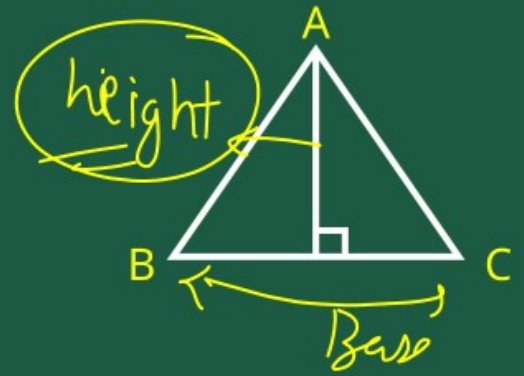
AREA OF A TRIANGLE

☛ If the height of a triangle is given, the general formula for finding its area is

$$\text{Area of a Triangle} = \frac{1}{2}\,(\text{Base} \times \text{Height})$$

Area of $\Delta = \frac{1}{2} \times$ Base $\times$ height



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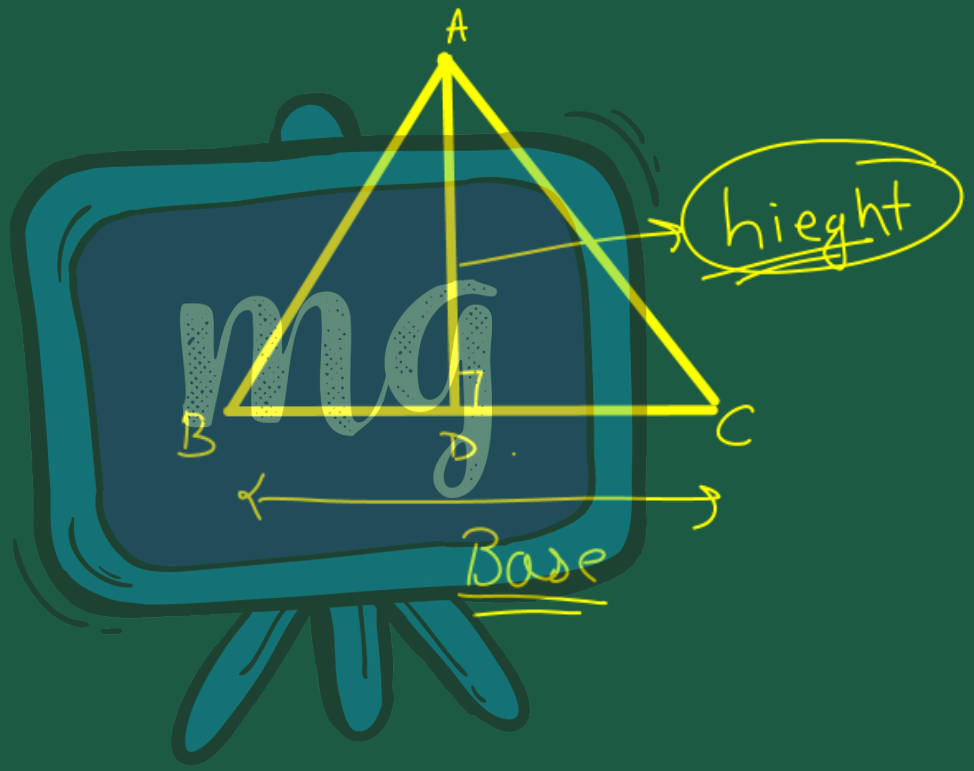

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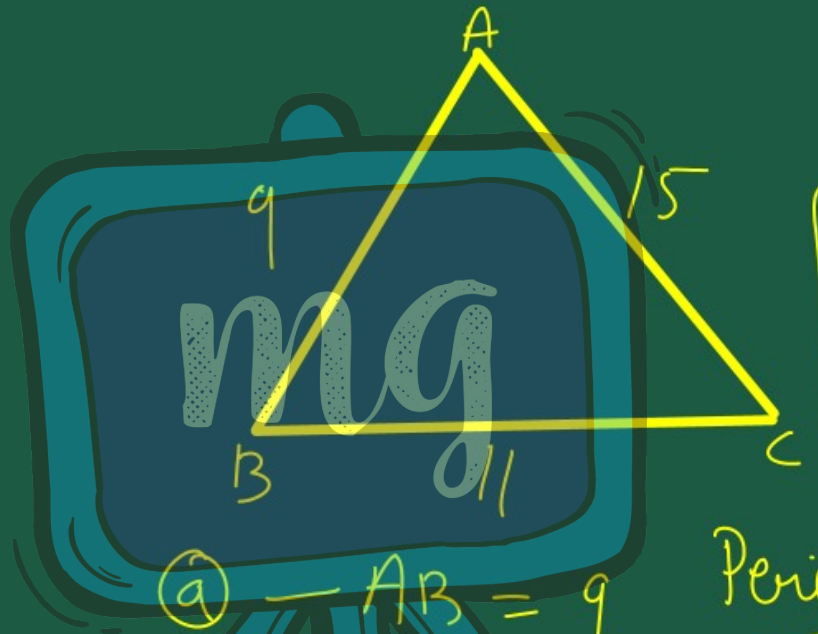

Heron's formula

(a) — AB = 9

(b) — BC = 11

(c) — AC = 15

Perimeter = AB + BC + AC

Semiperimeter (S) = $\frac{\text{Perimeter}}{2}$

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HERON'S FORMULA

Area of a Triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where S is semiperimeter of a triangle

which is $S = \frac{a+b+c}{2}$

Perimeter

S, a, b, c

Area of triangle = $\sqrt{S \times (S-a) \times (S-b) \times (S-c)}$

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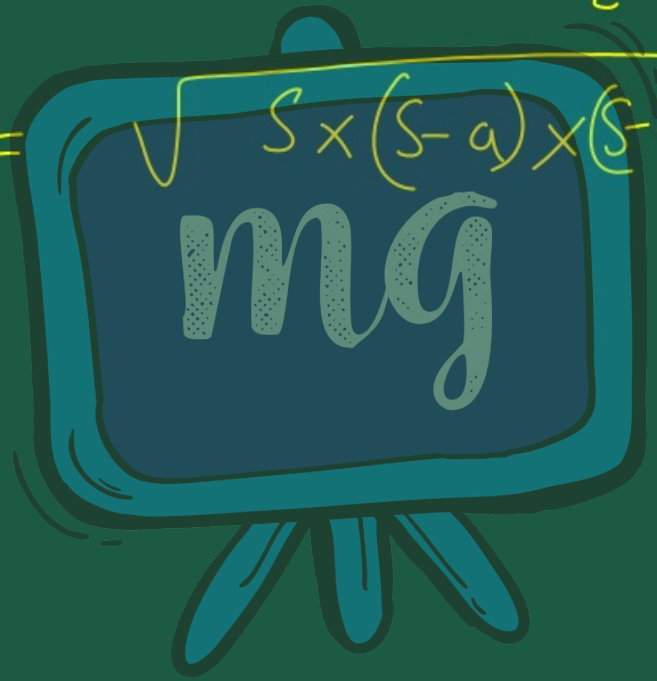
$$\begin{bmatrix} S \\ a \\ b \\ c \end{bmatrix}$$

$$\text{Area of } \Delta = \sqrt{S \times (S-a) \times (S-b) \times (S-c)}$$



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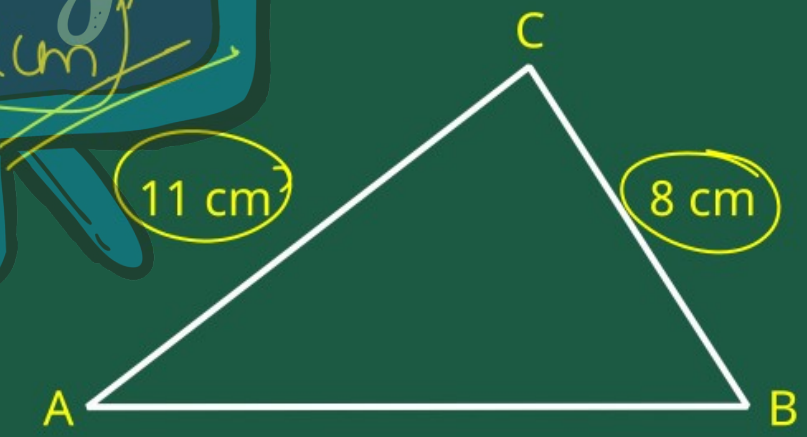
Example : 1

Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm.

Perimeter = 32 cm



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Sol^n :→ A.T.Q :→

S
a
b
c

In Δ. ABC,

$a = AC = 11\,cm$

$b = BC = 8\,cm$

and Perimeter of $\Delta ABC = 32\,cm$

Since we know that

Perimeter of Δs = Sum of its all sides

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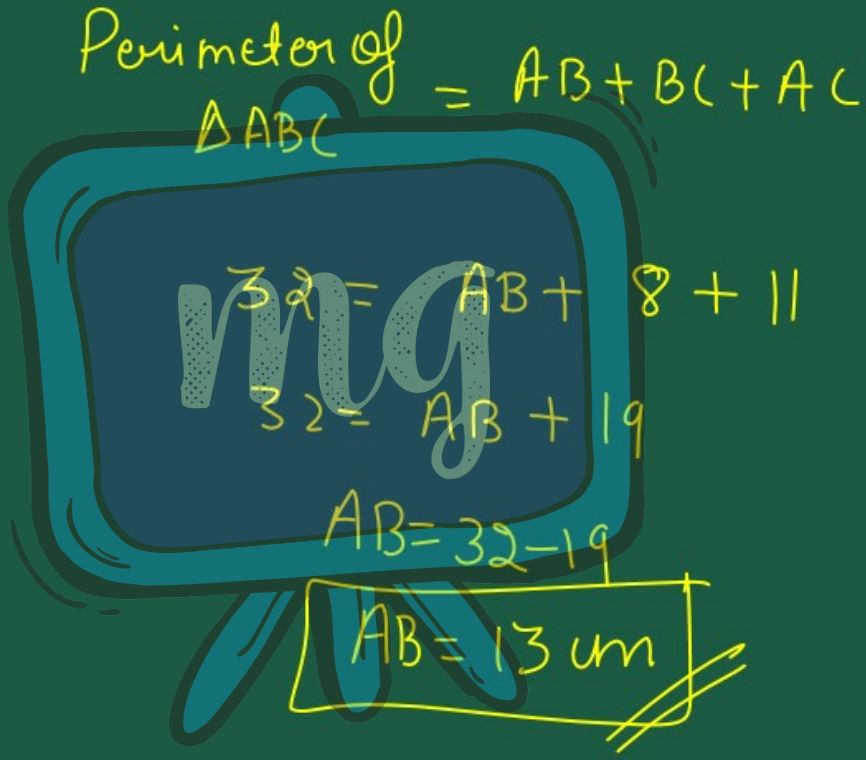

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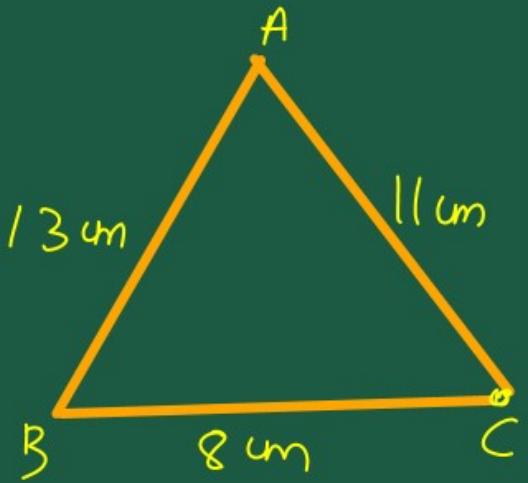
In ΔABC,

(a) — $AB = 13$ cm

(b) — $BC = 8$ cm

(c) — $AC = 11$ cm



Now Perimeter of $\Delta ABC = a+b+c$

$= 13+8+11$

$= 32$

Semi-perimeter $= \frac{P}{2} = \frac{a+b+c}{2} = \frac{32}{2} = 16$

$S = 16$ cm

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Now by using Heron's formula

$$\text{Area of } \Delta ABC = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$$

$$= \sqrt{16 \times (16-3) \times (16-8) \times 16-11}$$

$$= \sqrt{16 \times 3 \times 8 \times 5}$$

$$= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 5}$$

$$= \sqrt{(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2 \times 3 \times 5}$$

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$$= \sqrt{2^2 \times 2^2 \times 2^2 \times 2 \times 3 \times 5}$$

$$= \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2 \times 3 \times 5}$$

$$= 2 \times 2 \times 2 \times \sqrt{30}$$

$$= 8 \times \sqrt{30}$$

$$= 8\sqrt{30}\ cm^2$$

$$(a \times b)^m = a^m \times b^m$$

area of $\Delta ABC = 8\sqrt{30}\ cm^2$

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Example : 2

A triangular park ABC has sides 120m, 80m and 50m. A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of ₹20 per metre leaving a space 3m wide for a gate on one side.

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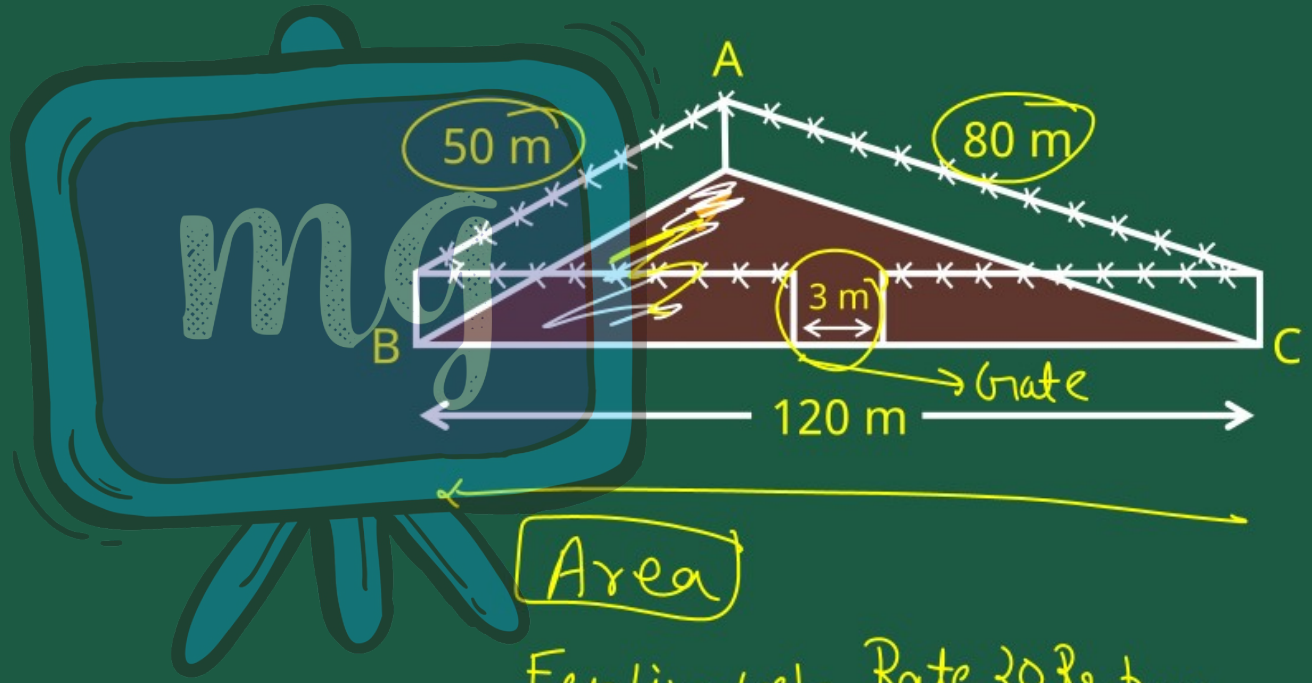

Area

Fencing cost Rate 20 Rs per metre

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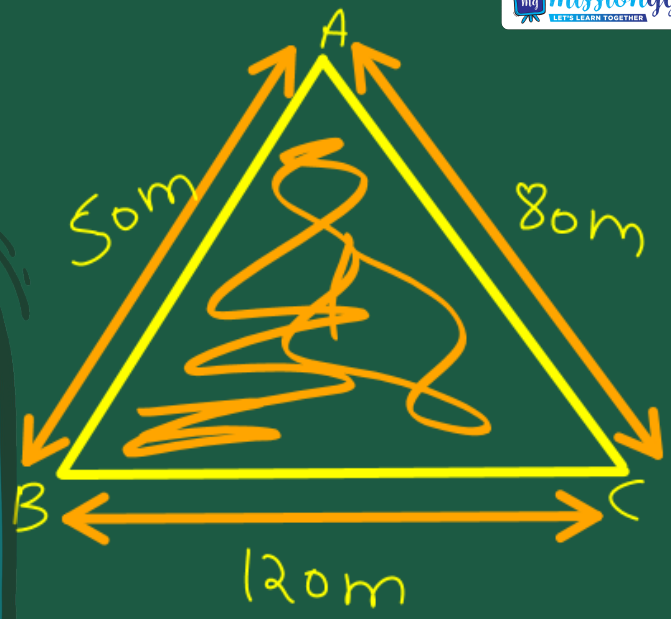
A.T.Q

ABC is triangular garden



So In $\triangle ABC$,

a — AB = 50m

b — BC = 120m

c — AC = 80m

Now, Perimeter of $\triangle ABC$ = AB + BC + AC

= 50 + 120 + 80

= 250m

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Sol

Semi perimeter of $\Delta ABC = \frac{\text{Perimeter}}{2} = \frac{250}{2} = 125\,m$

$S = 125\,m$

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Now by using heron's formula

$$\text{area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{125 \times (125-50) \times (125-120) \times (125-80)}$$

$$= \sqrt{125 \times 75 \times 5 \times 45}$$

$$= \sqrt{5\times5\times5 \times 5\times5\times3 \times 5 \times 5\times5\times3}$$

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$$= \sqrt{5\times5\times5\times5\times5\times3\times5\times5\times3\times3}$$

$$= \sqrt{5\times5\times5\times5\times5\times5\times3\times3\times5\times3}$$

$$= \sqrt{(5)^2\times(5)^2\times(5)^2\times(3)^2\times15}$$

$$= \sqrt{(5)^2}\times\sqrt{(5)^2}\times\sqrt{(5)^2}\times\sqrt{(3)^2}\times\sqrt{15}$$

$$= 5\times5\times5\times3\times\sqrt{15}$$

$$\left[\because (a\times b)^m = a^m\times b^m\right]$$

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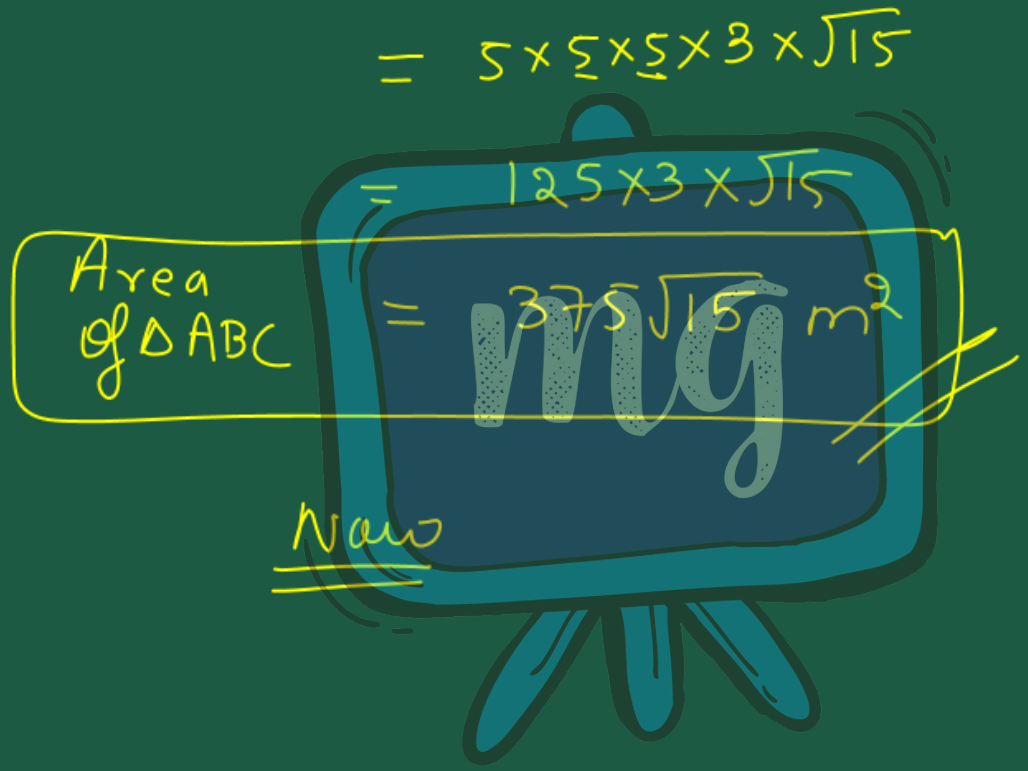

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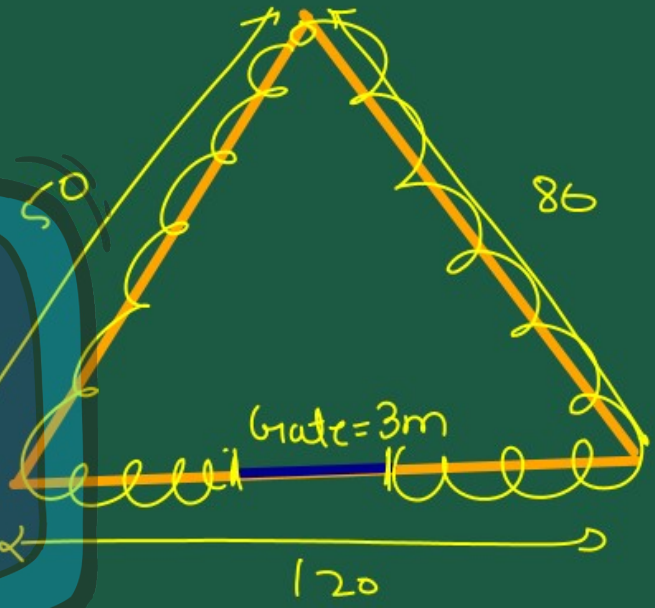

Length of Boundary where fence are going to plate = Perimeter of $\triangle ABC$ − length of Gate

$= 250 - 3 = 247\text{m}$.

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The Rate of fencing = 20 Rs per m



So, the cost for fencing of 247 m = 247 × 20

= 4940 Rs

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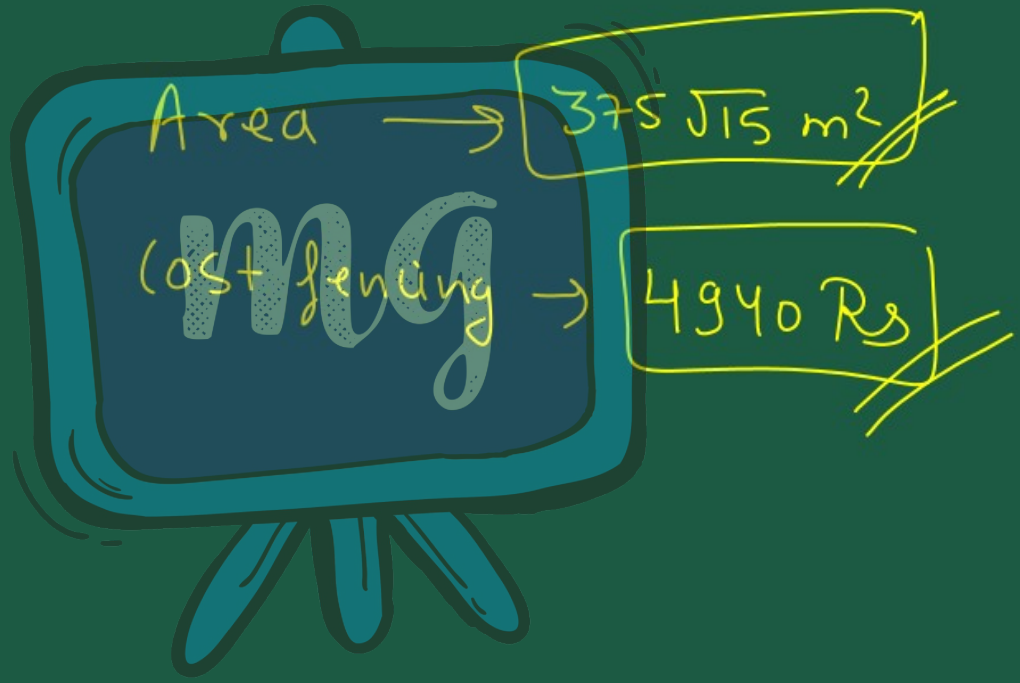

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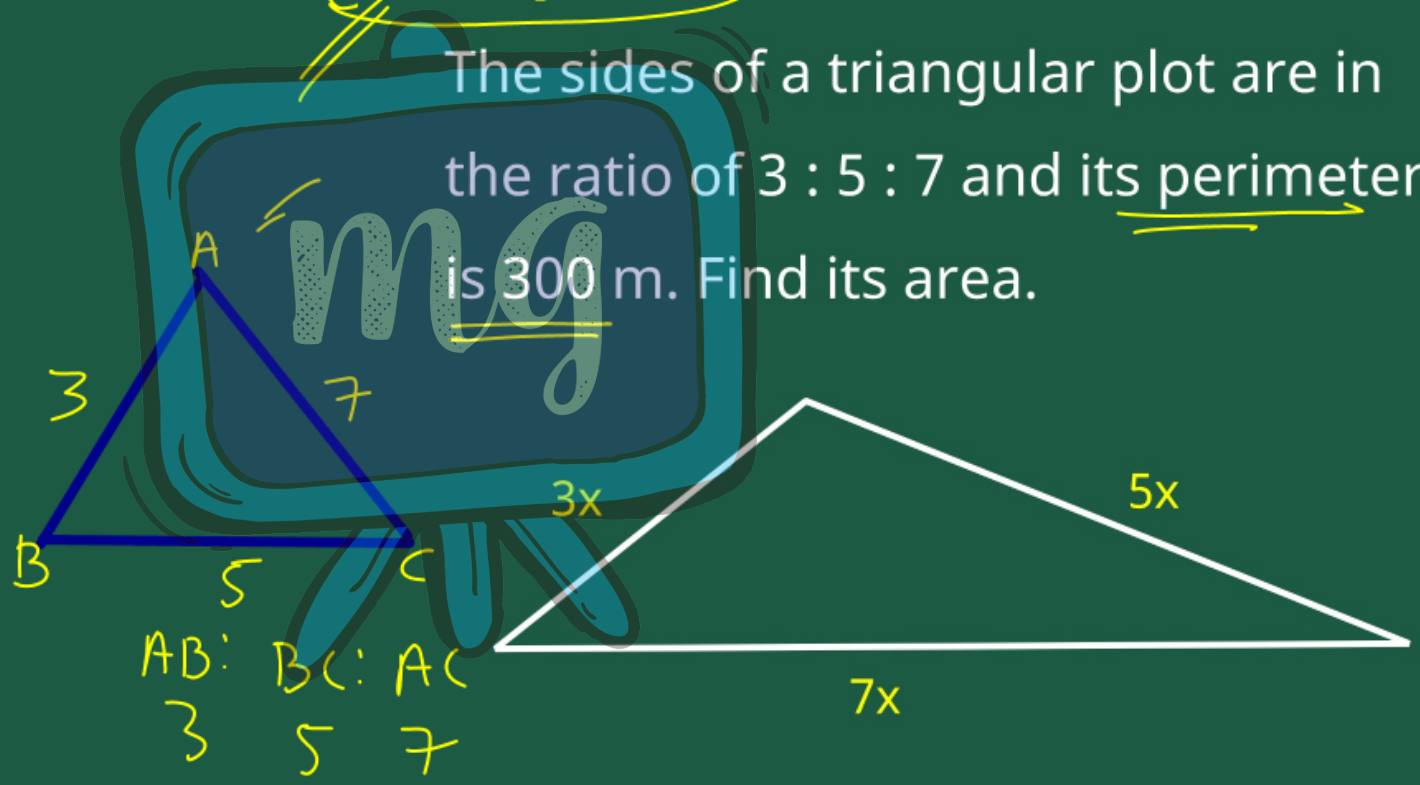
Example : 3

The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area.



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A·T·Q

ΔABC is triangular plot in which

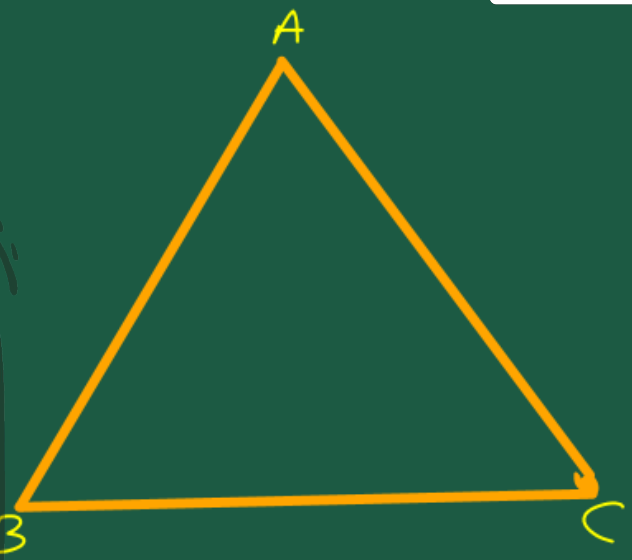
$AB : BC : AC = 3 : 5 : 7$

$AB = 3 \times x = 3x$

$BC = 5 \times x = 5x$

$AC = 7 \times x = 7x$ for x be any constant

& perimeter of $\Delta ABC = 300m$.



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Since we know that

perimeter of $\triangle ABC$ = Sum of all its Sides

$= AB + BC + AC = 300$

$\Rightarrow 3x + 5x + 7x = 300$

$\Rightarrow 15x = 300$

$x = \frac{300}{15} = 20$

$x = 20$

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So

(a) $AB = 3 \times n = 3 \times 20 = 60\ m$

(b) $BC = 5 \times n = 5 \times 20 = 100\ m$

(c) $AC = 7 \times n = 7 \times 20 = 140\ m.$

$\therefore$ Perimeter of $\triangle ABC = 300\ m$

So, Semiperimeter $= \frac{300}{2} = 150\ m$

$S = 150\ m$

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Now by using Heron's formula

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150 \times (150-60) \times (150-100) \times (150-140)}$

$= \sqrt{150 \times 90 \times 50 \times 10}$

$= \sqrt{3 \times 5 \times 10 \times 9 \times 10 \times 5 \times 10 \times 10}$

$= \sqrt{3 \times 5 \times 10 \times 3 \times 3 \times 10 \times 5 \times 10 \times 10}$

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$$= \sqrt{3\times5\times10\times3\times3\times10\times5\times10\times6}$$

$$= \sqrt{3\times3\times5\times5\times10\times10\times10\times10\times3}$$

$$= \sqrt{3^2\times(5)^2\times(10)^2\times(10)^2\times3}$$

$$= 3\times5\times10\times10\sqrt{3}$$

$$= 15\times100\times\sqrt{3}$$

Area of $\triangle ABC = 1500\sqrt{3}\ m^2$

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LEARNING OUTCOME

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Heron's formula

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ASSESSMENT

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1 By Heron's formula, the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ What is the S in this?

A Perimeter of a triangle

B Semiperimeter of a triangle

C Twice the perimeter of a triangle

D None of these

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ASSESSMENT

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2 The sides of a triangular lamina are 3m, 4m and 5m. Which of the following will be the area of the lamina?

A $6m^2$

B $12m^2$

C $15m^2$

D None of these

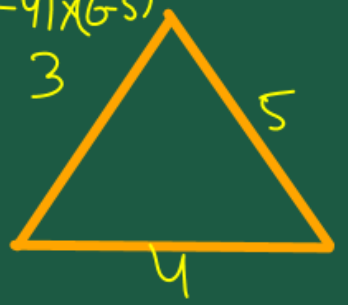
$= \sqrt{6 \times (6-3) \times (6-4) \times (6-5)}$

$= \sqrt{6 \times 3 \times 2 \times 1}$

$= \sqrt{6 \times 6}$

$= \sqrt{6^2}$

$= 6m^2$



Perimeter $= 3+4+5 = 12$

$S = \frac{P}{2} = \frac{12}{2} = 6$

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ASSESSMENT

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3 The sides of a triangle are in the ratio of 3 : 4 : 5 and its perimeter is 60 cm. Which of the following is the largest side of the triangle?

A 15 cm

B 25 cm

C 35 cm

D None of these

$a = 3 \times n = 3n = 3 \times 5 = 15$ cm

$b = 4 \times n = 4n = 4 \times 5 = 20$ cm

$c = 5 \times n = 5n = 5 \times 5 = 25$ cm

$a + b + c = 60$ cm

$3n + 5n + 4n = 60$

$12n = 60$

$n = \frac{60}{12} = 5$