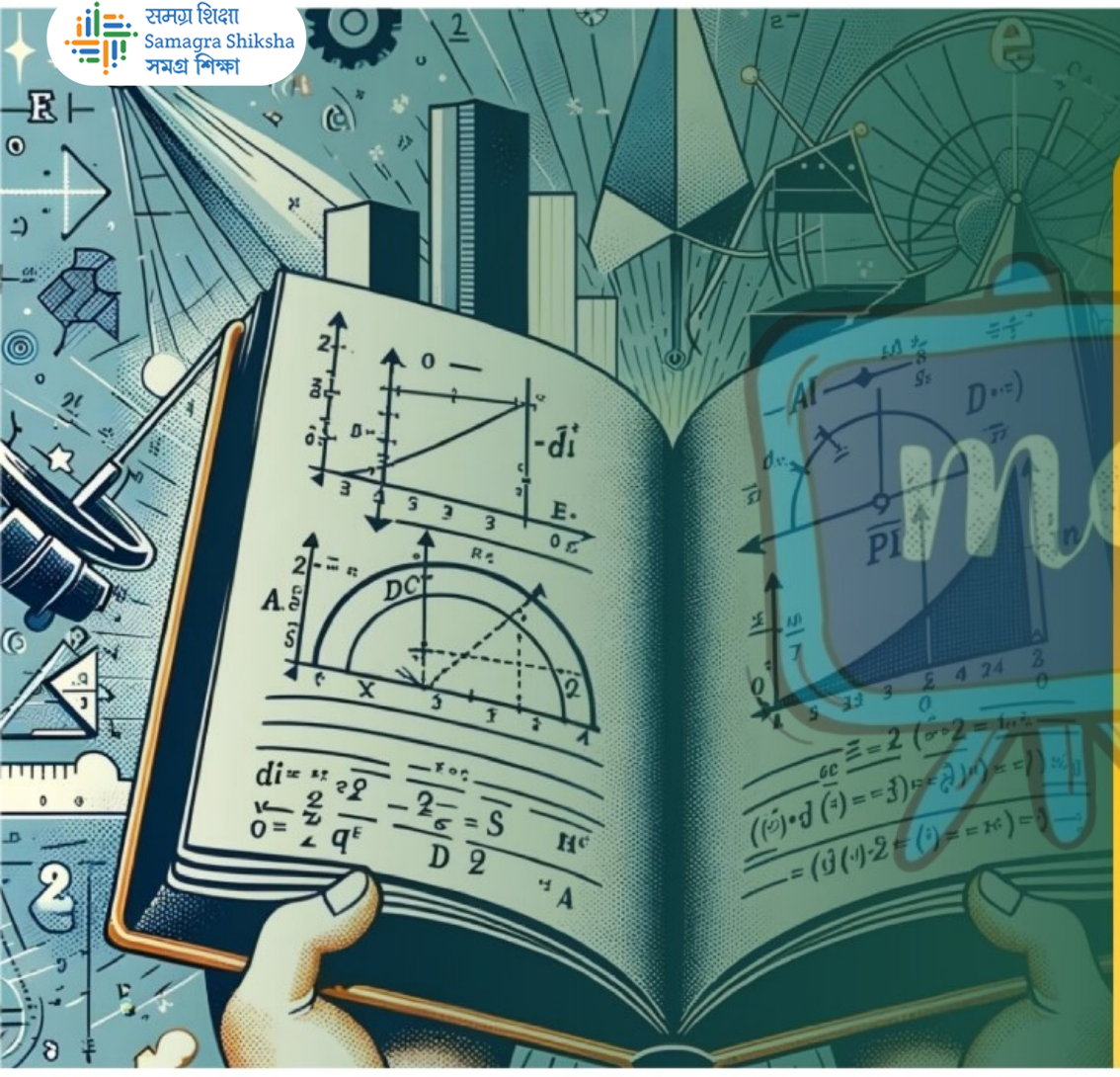
समग्र शिक्षा
Samagra Shiksha
সমগ্ৰ শিক্ষা

Under CSR Support by
ipca | Foundation



দশম শ্ৰেণীৰ বাবে

গণিত

অধ্যায় - ৮

ত্ৰিকোণমিতিৰ পৰিচয়

অংশ – ১৩

অনুশীলনী-৮.৪

অন্তৰা বেনার্জী

समग्र शिक्षा
Samagra Shiksha
সমগ্ৰ শিক্ষা

Under CSR Support by
ipca | Foundation

(ix) $(\text{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

[ইংগিত : বাওঁপক্ষ আৰু সোঁপক্ষ বেলেগে সৰল কৰা।]

$$\text{LHS.} = (\text{Cosec}A - \text{Sin}A)(\text{Sec}A - \text{Cos}A)$$

$$= \left(\frac{1}{\text{Sin}A} - \text{Sin}A\right)\left(\frac{1}{\text{Cos}A} - \text{Cos}A\right)$$

$$= \left(\frac{1 - \text{Sin}^2A}{\text{Sin}A}\right)\left(\frac{1 - \text{Cos}^2A}{\text{Cos}A}\right)$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= \frac{(1-\sin^2 A)(1-\cos^2 A)}{\sin A \cdot \cos A}$$

$$= \frac{\cos^2 A \cdot \sin^2 A}{\sin A \cdot \cos A}$$

$$= \sin A \cdot \cos A$$

$$\text{RHS.} = \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}}$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= \frac{\sin A \cdot \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\sin A \cdot \cos A}{1}$$

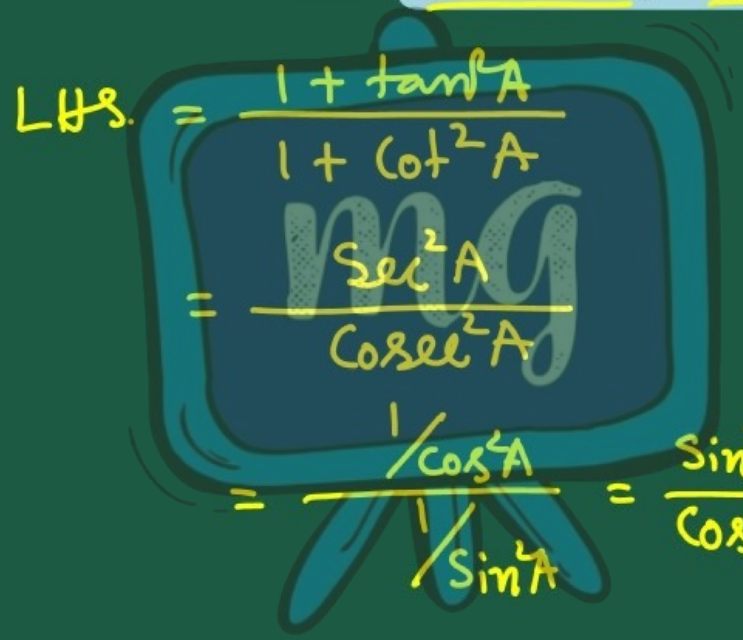
$$= \sin A \cdot \cos A$$

LHS = RHS.

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$



$$\text{LHS} = \frac{1+\tan^2 A}{1+\cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1/\cos^2 A}{1/\sin^2 A} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$\text{RHS.} = \left(\frac{1-\tan A}{1-\cot A}\right)^2$$

$$= \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\frac{\cos A-\sin A}{\cos A}}{\frac{\sin A-\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\cos A-\sin A}{\cos A} \times \frac{\sin A}{\sin A-\cos A}\right)^2$$

$$= \left\{\frac{-(\sin A-\cos A)\sin A}{(\sin A-\cos A)\cdot\cos A}\right\}^2$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= (-\tan A)^2$$

$$= \tan^2 A .$$



समग्र शिक्षा
Samagra Shiksha
সমগ্ৰ শিক্ষা

Under CSR Support by
ipca | Foundation

6. প্ৰমাণ কৰা —

(i) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

$$\text{LHS.} = \tan^4\theta + \tan^2\theta$$
$$= \tan^2\theta(\tan^2\theta + 1)$$
$$= \tan^2\theta \times \sec^2\theta$$
$$= (\sec^2\theta - 1)\cdot\sec^2\theta$$
$$= \sec^4\theta - \sec^2\theta$$
$$= \text{RHS.}$$

$1 + \tan^2\theta = \sec^2\theta$

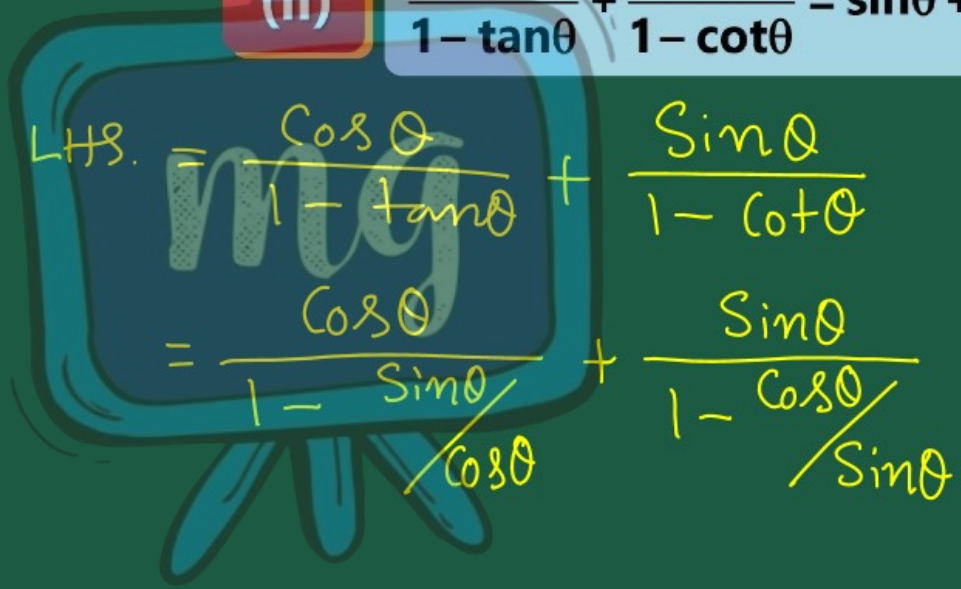
$\Rightarrow \tan^2\theta = \sec^2\theta - 1$

समग्र शिक्षा
Samagra Shiksha
সমগ্ৰ শিক্ষা

Under CSR Support by
ipca | Foundation

6. প্ৰমাণ কৰা —

(ii) $\dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$



$$\text{LHS} = \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$$

$$= \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}} + \frac{\sin\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}}$$

$$= \frac{\cos\theta \cdot \cos\theta}{\cos\theta - \sin\theta} + \frac{\sin\theta \cdot \sin\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{-(\cos\theta - \sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= \frac{\cos^2\theta - \sin^2\theta}{(\cos\theta - \sin\theta)}$$

$a^2 - b^2 = (a+b)(a-b)$

$$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)}$$

$$= \cos\theta + \sin\theta$$

$$= \text{RHS}.$$

समग्र शिक्षा
Samagra Shiksha
সমগ্ৰ শিক্ষা

Under CSR Support by
ipca | Foundation

6. প্ৰমাণ কৰা —

(iii) $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = \operatorname{cosec}\theta - \cot\theta$

$$\text{LHS.} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}}$$

$$= \sqrt{\frac{(\sec\theta - 1)(\sec\theta - 1)}{(\sec\theta + 1)(\sec\theta - 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\sec^2\theta - 1}}$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}}$$

$$= \frac{\sec\theta - 1}{\tan\theta}$$

$$= \frac{\sec\theta}{\tan\theta} - \frac{1}{\tan\theta}$$

$$= \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} - \cot\theta = \frac{1}{\sin\theta} - \cot\theta$$

$$= \operatorname{cosec}\theta - \cot\theta = \text{RHS}.$$

समग्र शिक्षा
Samagra Shiksha
সমগ্ৰ শিক্ষা

Under CSR Support by
ipca | Foundation

6. প্ৰমাণ কৰা —

(iv) $\cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$

$$\text{LHS.} = \cot\theta + \tan\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta \cdot \cos\theta + \sin\theta \cdot \sin\theta}{\sin\theta \cdot \cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cdot \cos\theta}$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= \frac{1}{\sin\theta \cdot \cos\theta}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

$$= \operatorname{cosec}\theta \cdot \sec\theta$$

$$= \text{RHS}.$$

समग्र शिक्षा
Samagra Shiksha
সমগ্ৰ শিক্ষা

Under CSR Support by
ipca | Foundation

6. প্ৰমাণ কৰা —

(v) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

$$\text{L.H.S.} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$

$$= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$$

$$= \quad \Big| \; (a+b)(a-b) = a^2-b^2$$

समग्र शिक्षा
Samagra Shiksha
समग्र शिक्षा

Under CSR Support by
ipca | Foundation

$$= 2 \times \frac{1}{\cos^2\theta}$$

$$= 2 \cdot \sec^2\theta$$

$$= \text{RHS}$$

