

# CLASS – 10

## MATHEMATICS

### Chapter – 11

#### AREAS RELATED TO CIRCLES

Part – 1

Areas of Sector and Segment of  
a Circle

Shubham Tiwari

mg

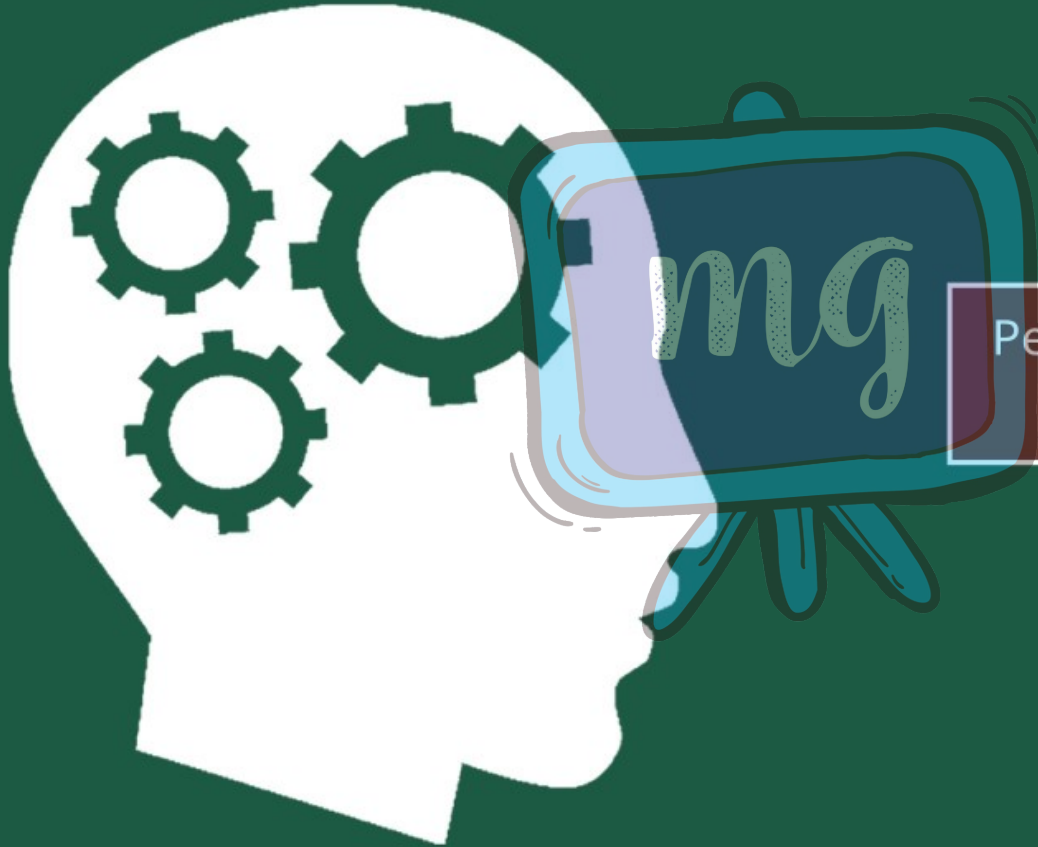
# OVERVIEW

## 1. Areas of Sector and Segment of a Circle

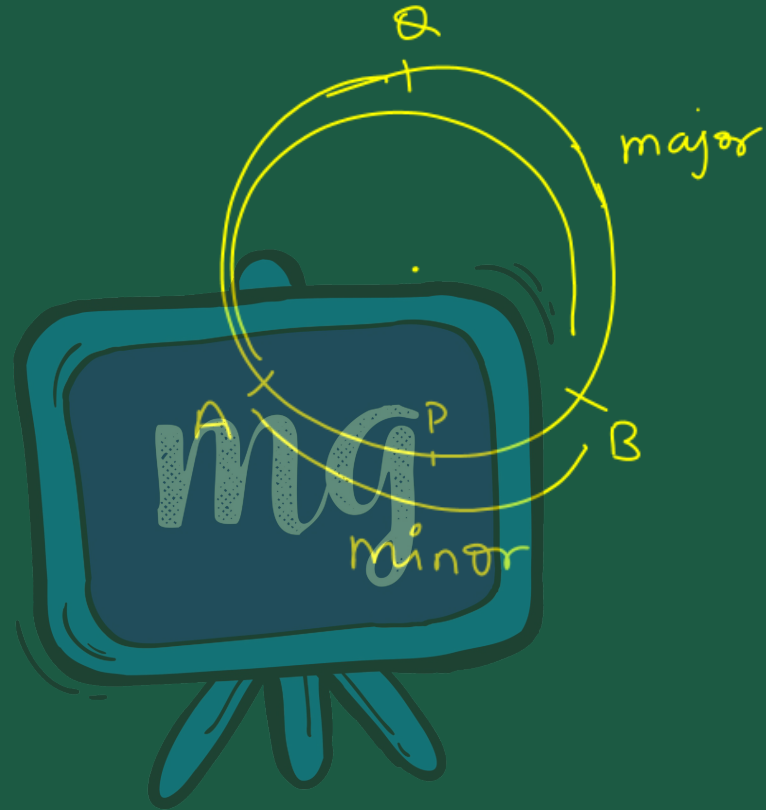
The logo for Missiongyan, featuring the letters 'mg' in a stylized, white, cursive font inside a blue rounded square with a white border. The logo is set against a background of a green and blue gradient with faint circular patterns.

mg

# COMPETENCY BASED LEARNING

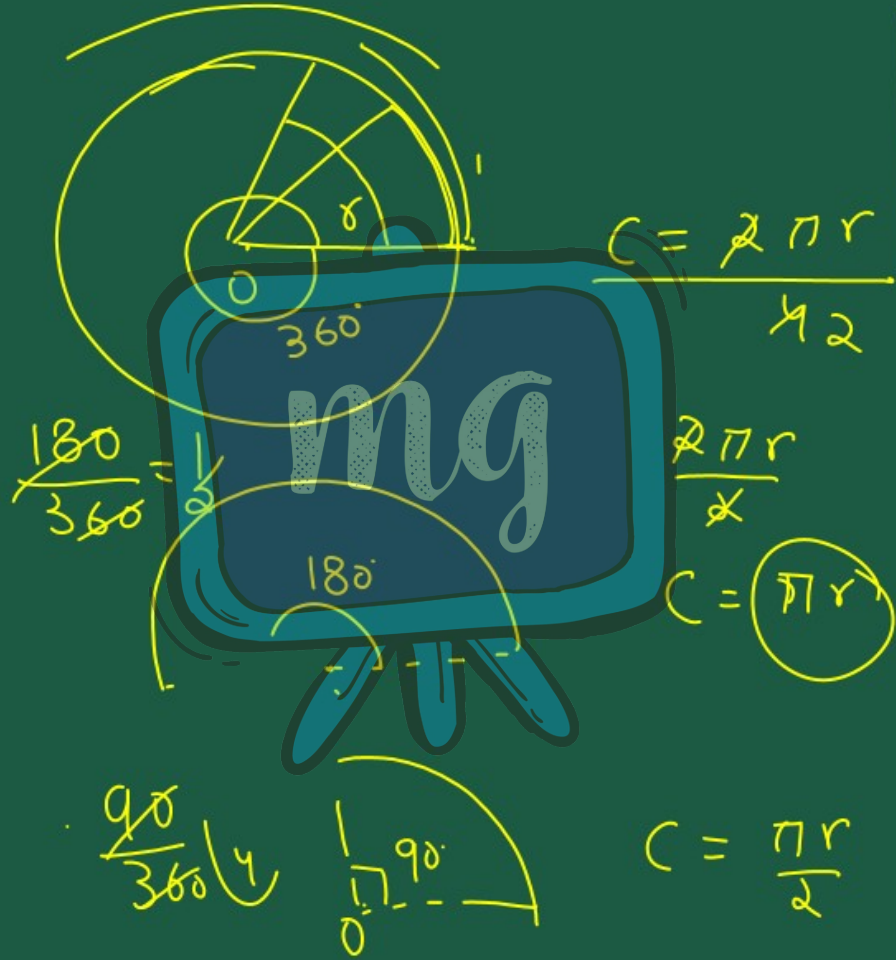


Perimeter and Area of a Sector  
and Segment of a circle



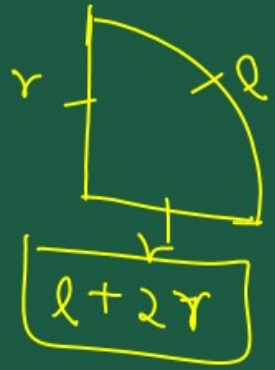
$\widehat{APB}$

$\widehat{AQB}$



$$\left[ \frac{\theta}{360} = \frac{l}{2\pi r} \right]$$

$$\frac{\theta}{360} \times 2\pi r = l$$



The diagram shows a circle with a central character 'mg'. Three sectors are drawn from the center, each with a different angle. The angles are labeled as  $\theta$ ,  $60^\circ$ , and  $45^\circ$ . Each sector is associated with a formula for its arc length  $l$ .

For the sector with angle  $\theta$ :

$$\frac{\theta}{360} \times 2\pi r = l$$

For the sector with angle  $60^\circ$ :

$$\frac{60}{360} \times 2\pi r = l$$
$$3 = \frac{\pi r}{3} = l$$

For the sector with angle  $45^\circ$ :

$$\frac{45}{360} \times 2\pi r = l$$
$$\frac{1}{8} \times 2\pi r$$
$$4 = \frac{\pi r}{4} = l$$

## ARC

A piece of a circle between any two different points is called an **Arc** of that circle.

Between any two points on a circle, there are two arcs. One is smaller than the other. The smaller arc is known as Minor arc and the larger is called Major Arc.

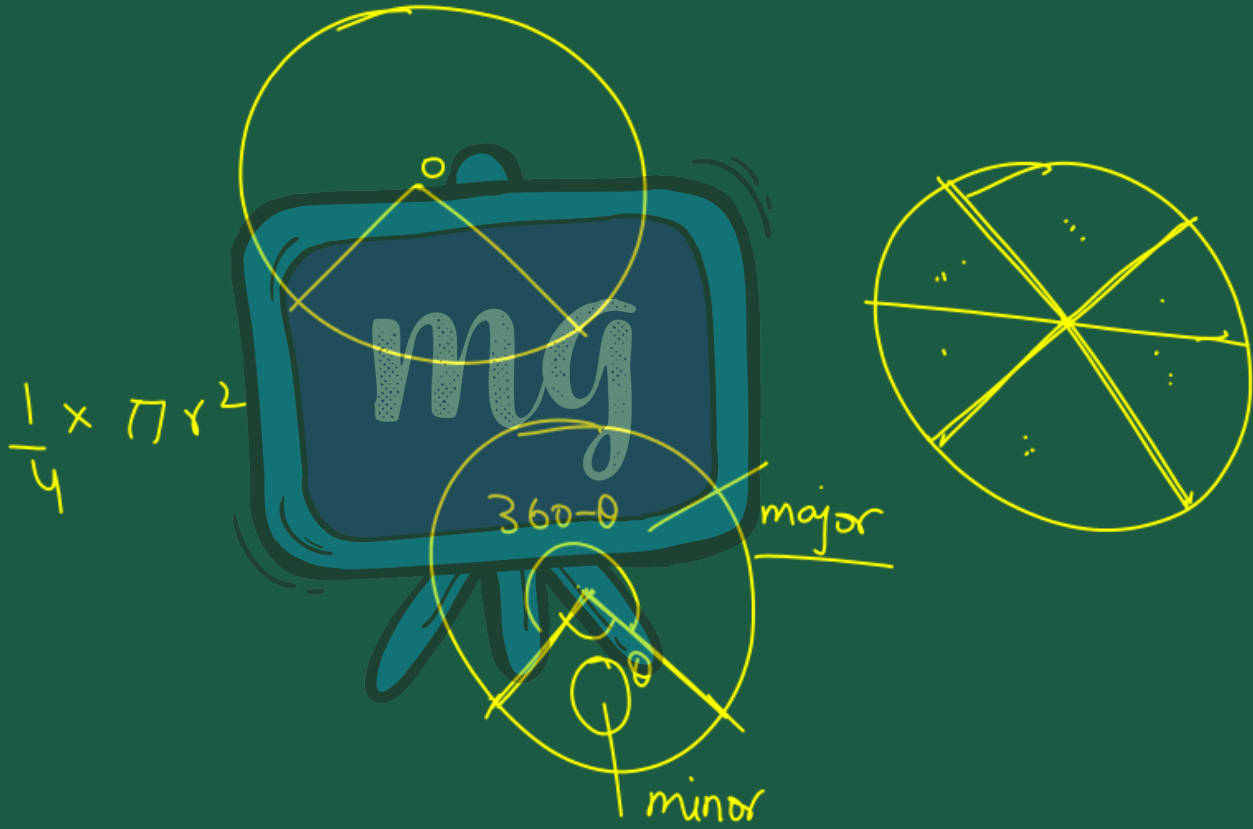
In the figure, AB (solid) represents a minor arc and AB (dotted) represents a major arc.





When both the arcs, major and minor, are equal then each is called Semicircle.



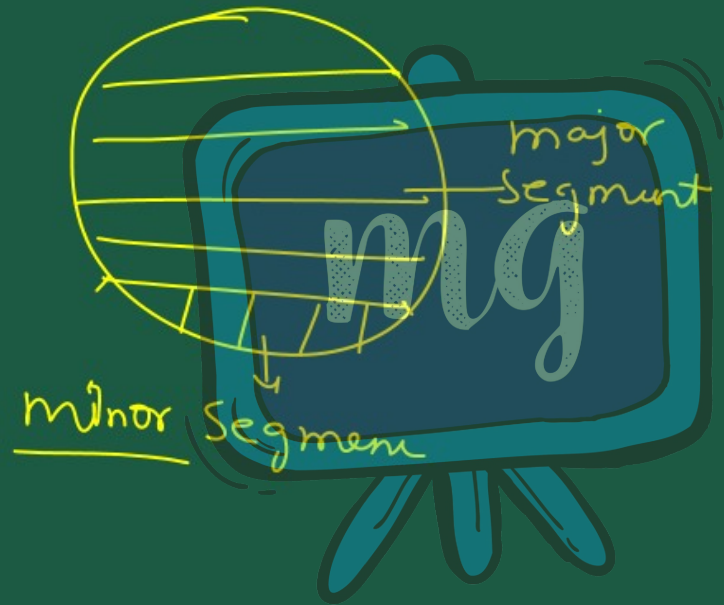


$$Area = \frac{\theta}{360} \times \pi r^2$$

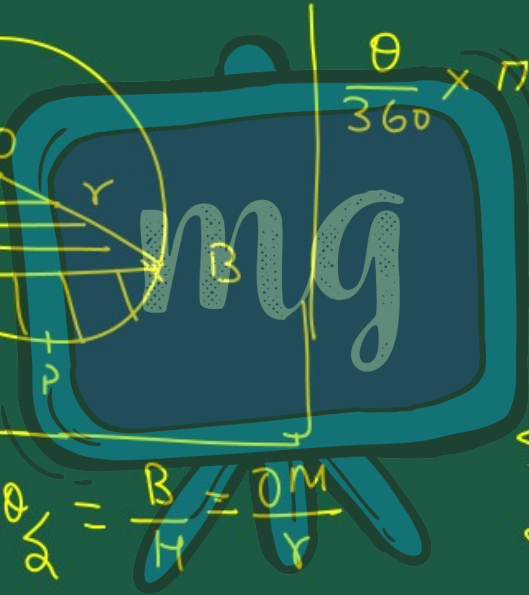
$$\frac{1}{2} \times \pi r^2$$


$$= \frac{1}{4} \times \pi r^2$$

# Segment

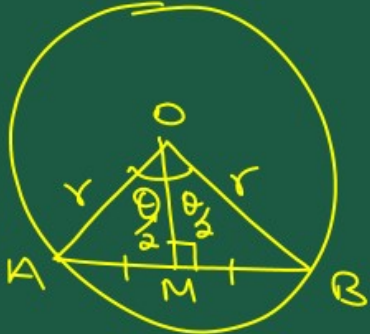


Area of Segment = Area of sector - Area of  $\Delta$





$\frac{\theta}{360} \times \pi r^2 -$



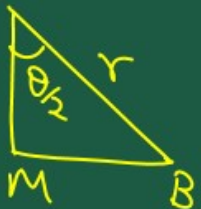
$$\cos \theta_2 = \frac{B}{H} = \frac{OM}{r}$$

$$r \cos \theta_2 = OM$$

$$\sin \theta = \frac{P}{H}$$

$$\sin \theta_2 = \frac{MB}{r}$$

$$r \sin \theta_2 = MB$$



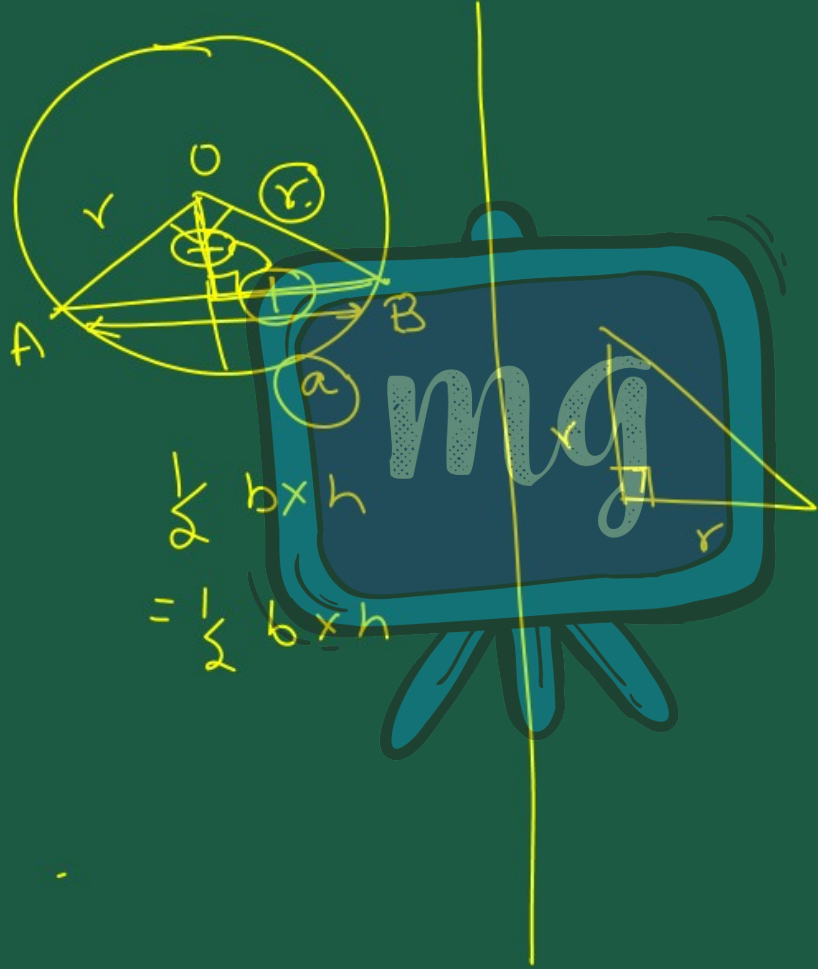
$$Ar = \frac{1}{2} B \times h$$

$$= \frac{1}{2} r \sin \theta_2 \times r \cos \theta_2$$

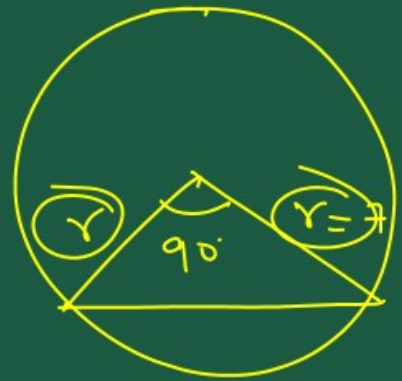
$$Ar = \frac{1}{2} r^2 \sin \theta_2 \cdot \cos \theta_2$$

$$Ar \text{ of } \Delta OAB = 2 \times \frac{1}{2} r^2 \sin \theta_2 \cos \theta_2$$

$$\underline{Ar \text{ of } \Delta OAB = r^2 \sin \theta_2 \cos \theta_2}$$



$$\frac{1}{2} b \times h$$
$$= \frac{1}{2} b \times h$$



$$\frac{1}{2} b \times h$$
$$\frac{1}{2} r \times r = \frac{1}{2} r^2$$

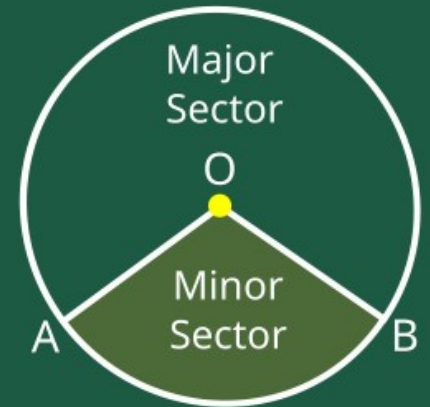
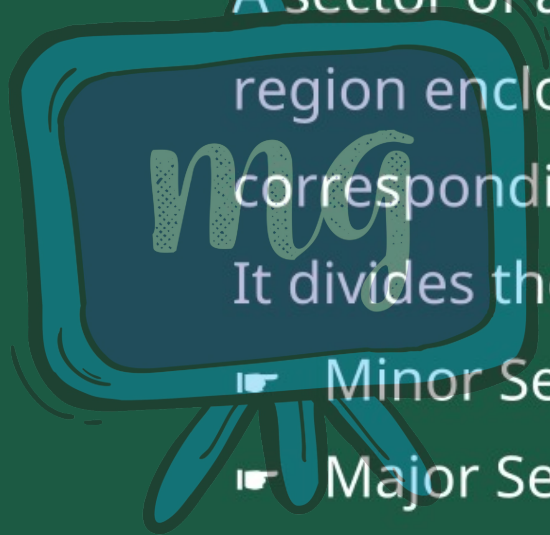
# SECTOR

A sector of a circle is the circular region enclosed by two radii and the corresponding arc.

It divides the circle in two parts –

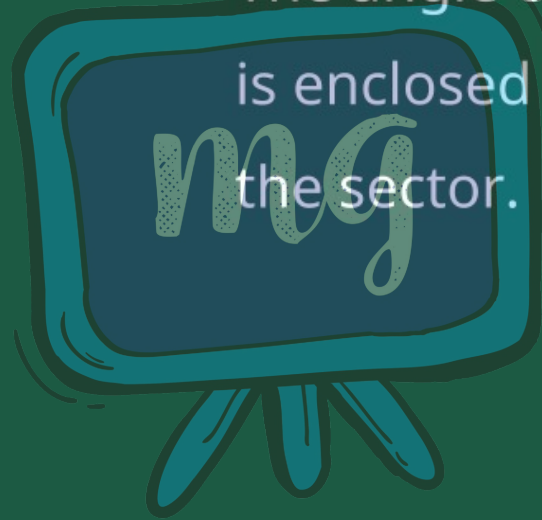
▣ Minor Sector

▣ Major Sector



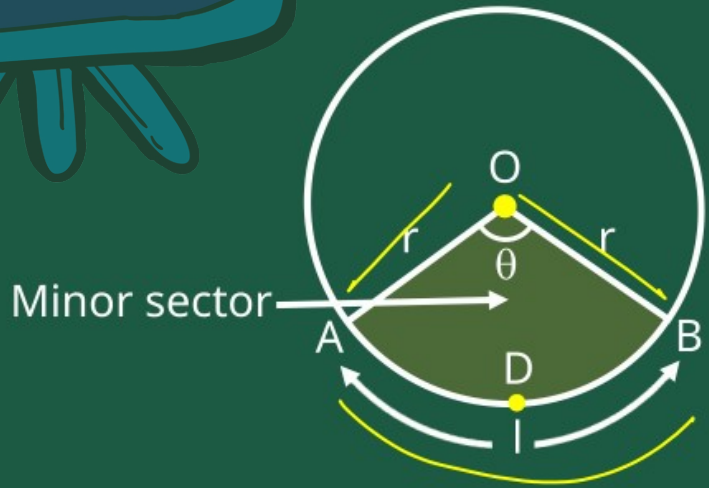
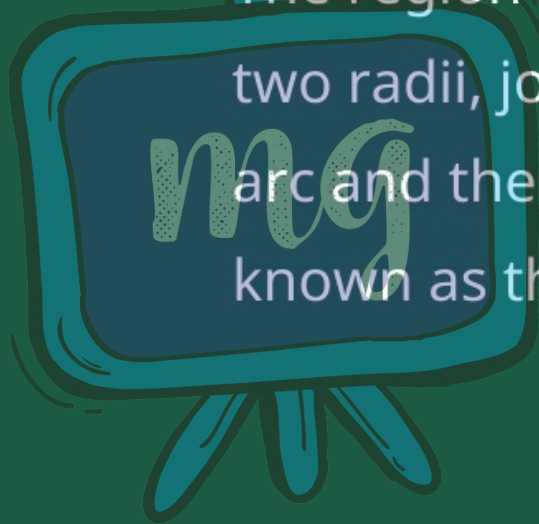
## ANGLE OF A SECTOR

The angle of a sector is the angle that is enclosed between the two radii of the sector.



# MINOR SECTOR

The region between a minor arc and two radii, joining the endpoints of the arc and the centre, of a circle is known as the Minor Sector of the circle.



# MINOR SECTOR

Length of Arc

$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

Perimeter

$$2r + \left( \frac{\theta}{360^\circ} \times 2\pi r \right)$$

Or

$$2r + l$$

Area

$$\frac{\theta}{360^\circ} \times \pi r^2$$

Or

$$\frac{1}{2}lr$$

$$\frac{\theta}{360} \times \pi r^2$$

$\frac{1}{2} \times \frac{\theta}{360} \times \pi r \times r$

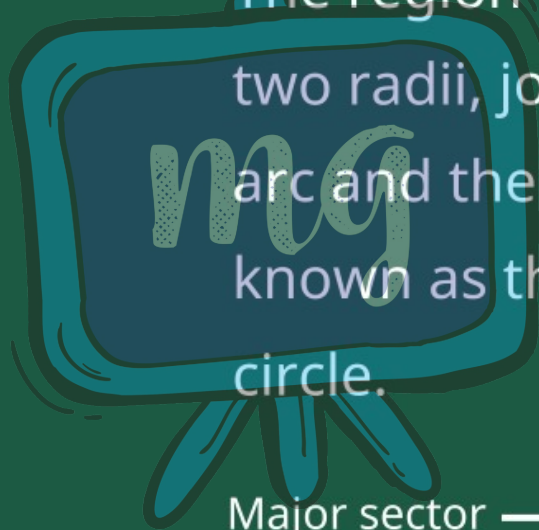
$\frac{1}{2} \times \frac{\theta}{360} \times 2\pi r \times r$

$\frac{1}{2} \ell r$

$$\frac{\theta}{360} \times 2\pi r = \ell$$

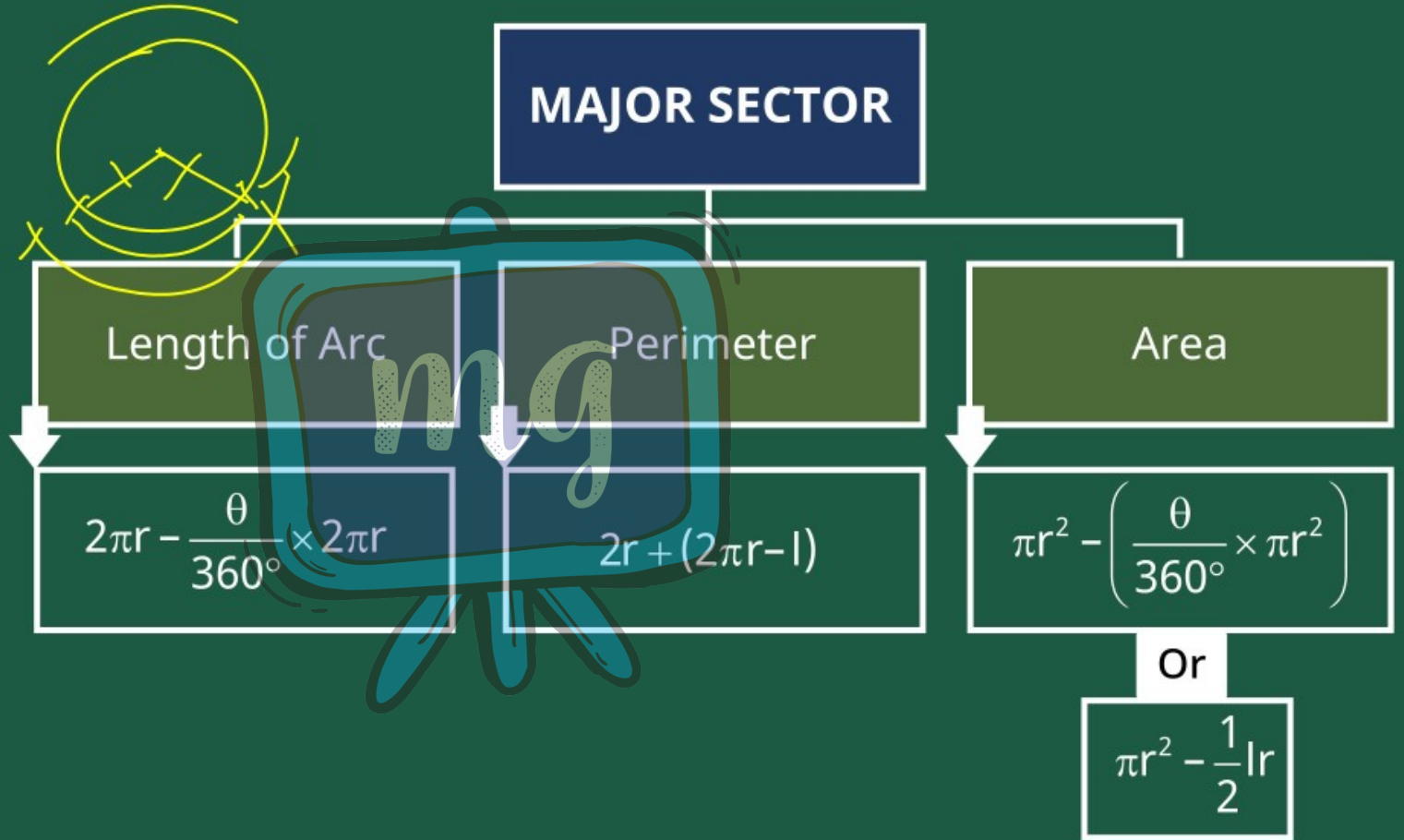
# MAJOR SECTOR

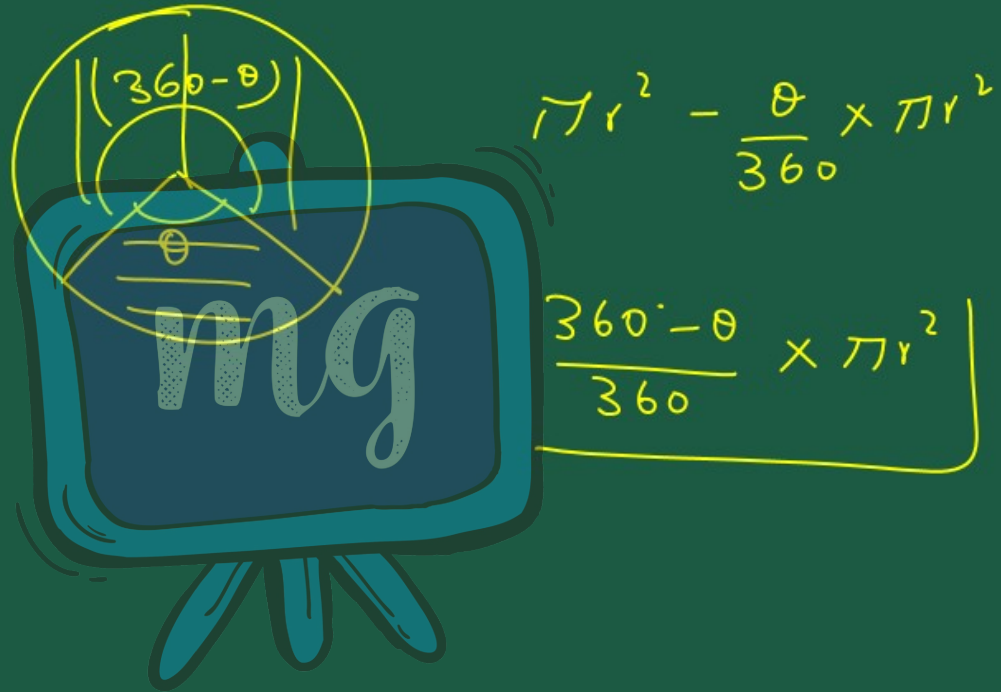
The region between a major arc and two radii, joining the endpoints of the arc and the centre, of a circle is known as the Major Sector of the circle.



Major sector







$$\pi r^2 - \frac{\theta}{360} \times \pi r^2$$

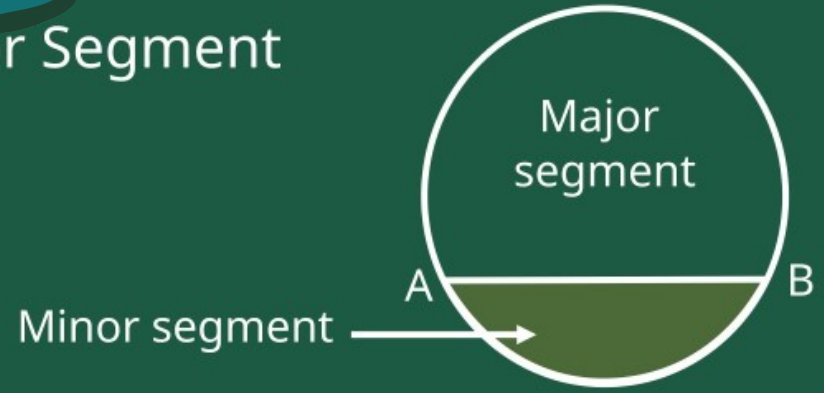
$$\frac{360 - \theta}{360} \times \pi r^2$$

# SEGMENT

In a circle, the region between a chord and an arc is called a **Segment** of the circle.

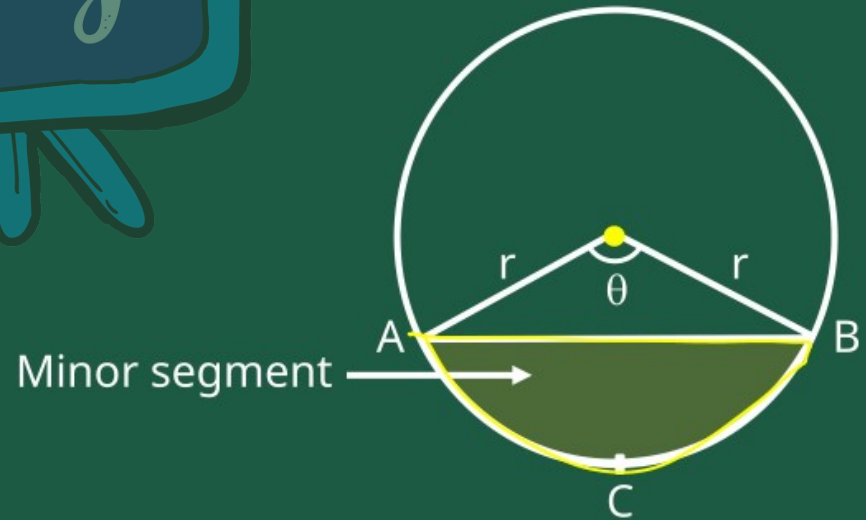
Minor Segment

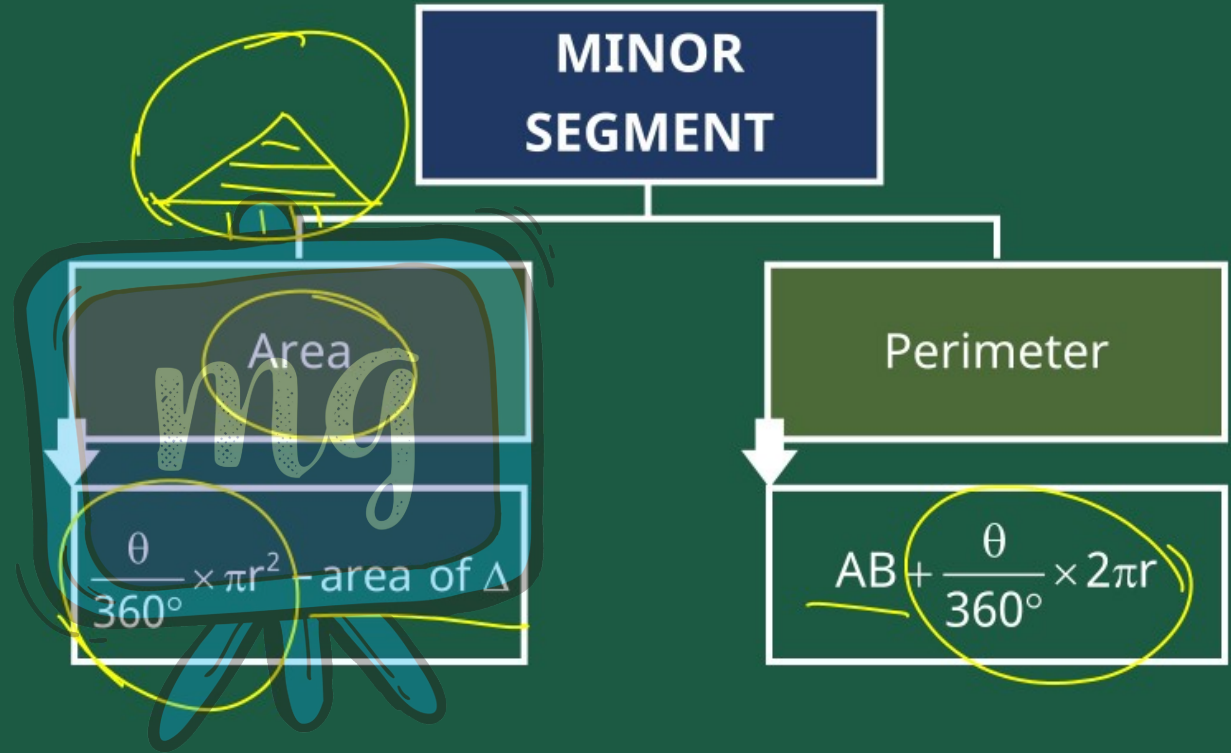
Major Segment



## MINOR SEGMENT

The region between a chord and a minor arc is called Minor Segment.  
Here ACB is minor segment.

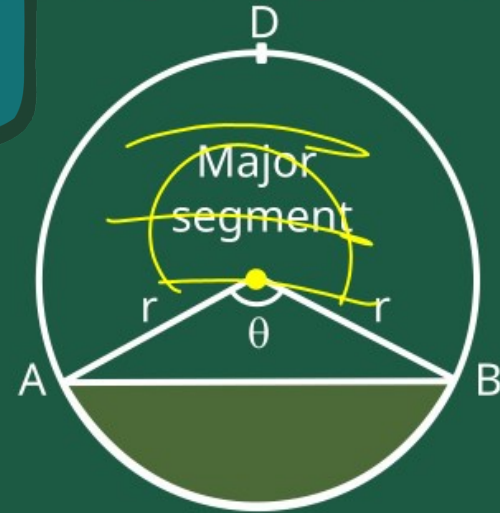


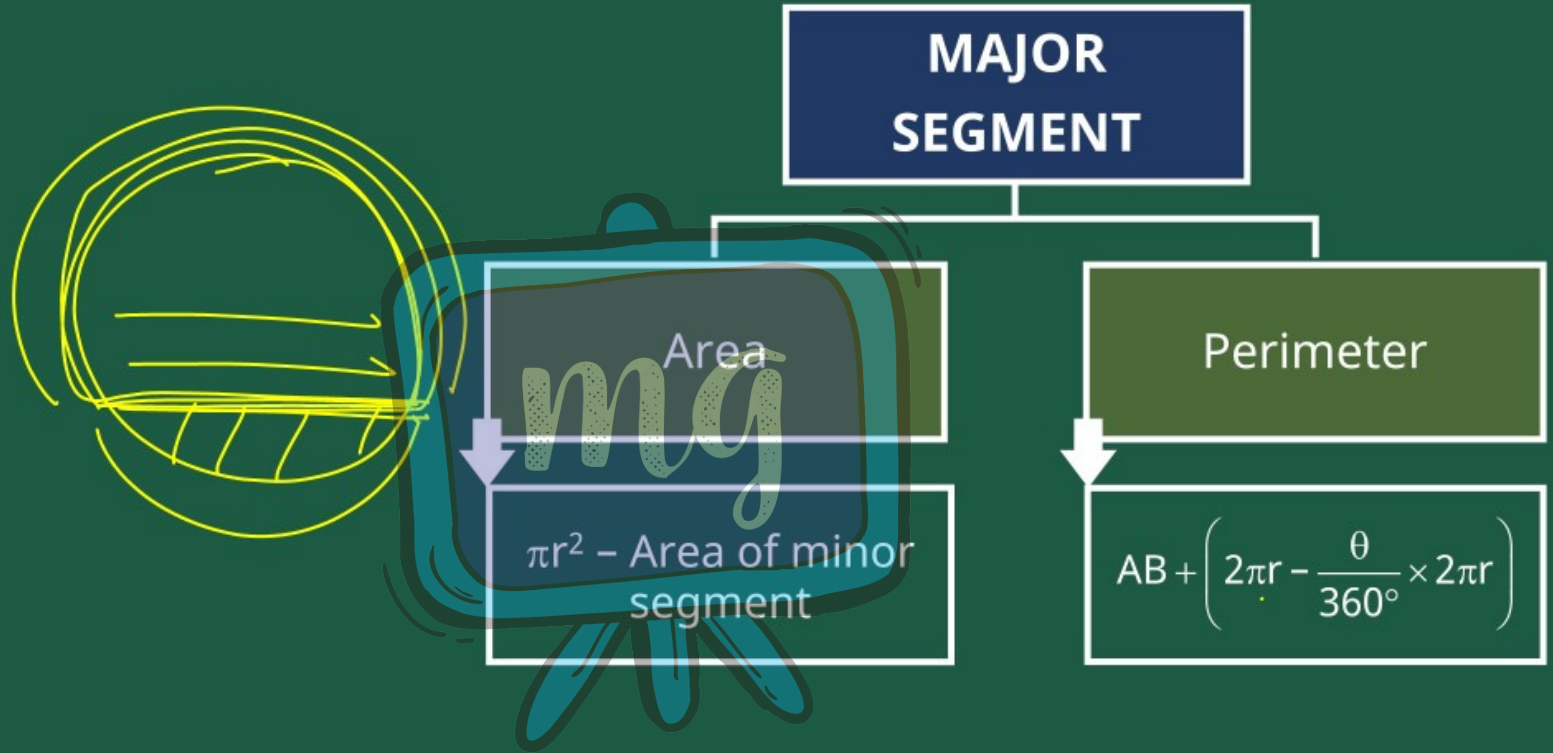


# MAJOR SEGMENT

The region between a chord and a major arc is called Major Segment.

Here ADB is Major Segment.

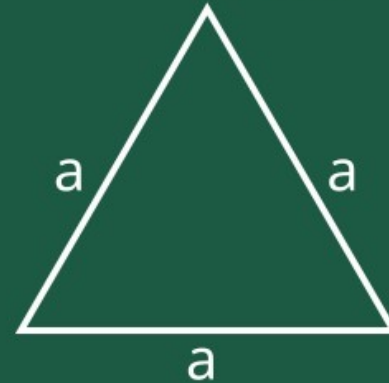




# AREA OF EQUILATERAL TRIANGLE

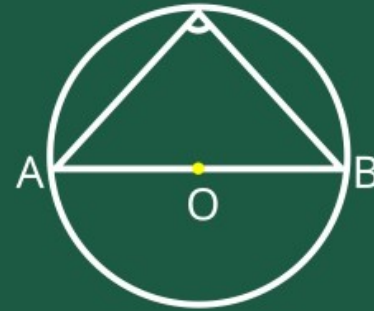
mg

$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$



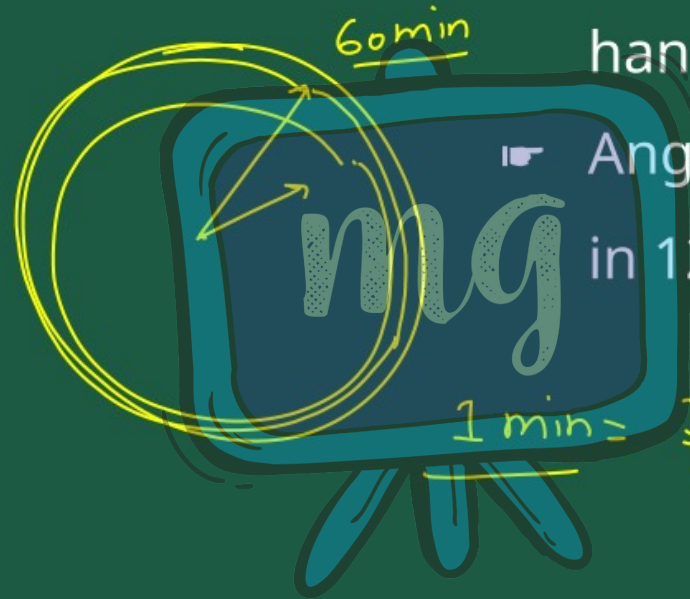


Angle subtended at the circle by a diameter is always a right angle.



▮ Angle described by minute hand in 60 minutes is  $360^\circ$ .

▮ Angle described by hour hand in 12 hours is  $360^\circ$ .

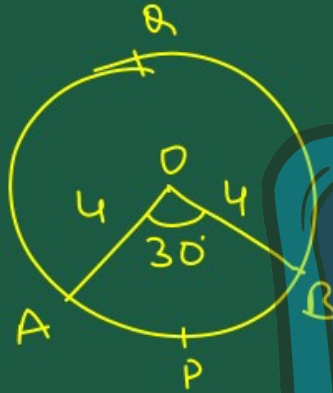


$$1 \text{ min} = \frac{360}{60} = 6^\circ$$

$$\begin{array}{l} 12 \text{ hr} = 360 \\ 1 \text{ hr} = \frac{360}{12} \\ \hline 1 \text{ hr} = 30 \end{array}$$

## Example : 1

Find the area of the sector of a circle with radius 4 cm and of angle  $30^\circ$ . Also, find the area of the corresponding major sector (Use  $\pi = 3.14$ ).



$$\begin{aligned} \text{Area of Sector OAPB} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{30}{360} \times \pi (4)^2 \end{aligned}$$

$$= \frac{30}{360} \times 3.14 \times 4 \times 4$$

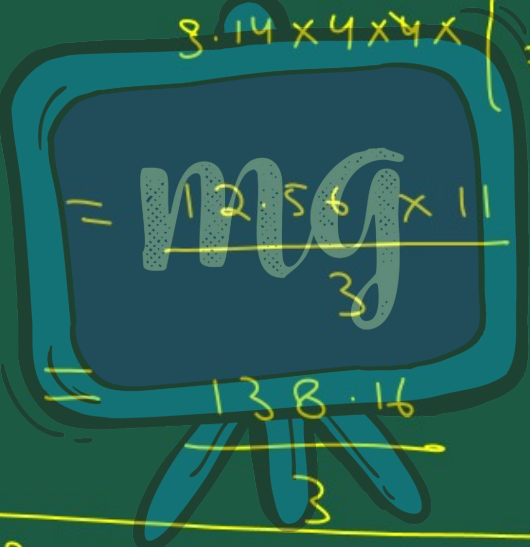


$$= \frac{3.14 \times 4}{3} = \frac{12.56}{3}$$

$$Ar = 4.19 \text{ cm}^2$$

$$\begin{aligned} \text{Area of } AOB &= \pi r^2 - \text{Area of } \triangle OAPB \\ &= \pi 4^2 - \frac{\theta}{360} \times \pi r^2 \\ &= \pi 4^2 \left[ 1 - \frac{30}{360} \right] \end{aligned}$$

$$= 3.14 \times 4^2 \left[ 1 - \frac{1}{12} \right]$$

$$= 3.14 \times 4 \times 4 \times \left[ \frac{11}{12} \right]$$

$$= 12.56 \times 11$$
$$= 138.16$$

$$\begin{array}{r} 266 \\ 1256 \times 11 \\ \hline 13816 \end{array}$$

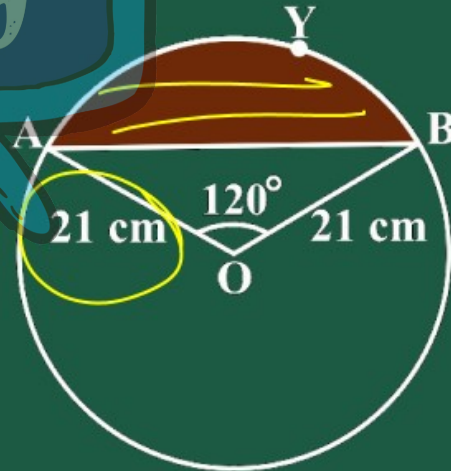
Are of major sector =  $46.05 \text{ cm}^2$

## Example : 2

Find the area of the segment AYB shown in Fig., if radius of the circle is 21 cm and  $\angle AOB = 120^\circ$ .

$$\begin{aligned} \text{Area of sector} &= \\ &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21 \\ &= \end{aligned}$$

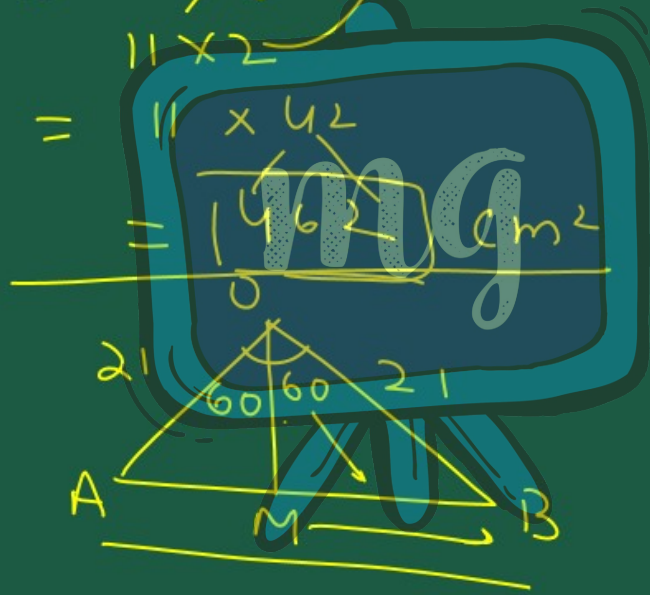
$$\pi = \frac{22}{7}$$



$$= \frac{22 \times 21}{11 \times 2}$$

$$= \frac{11 \times 42}{11 \times 2}$$

$$= \frac{462}{2} \text{ cm}^2$$



$$21$$

$$60 \quad 60 \quad 21$$

$$A \quad M \quad B$$

$$\frac{52 \times 11}{5 \times 7 \times 2}$$

$$\frac{76 \times 11}{7 \times 3 \times 6}$$

$$836$$

$$\sin 60 = \frac{P}{H} = \frac{MB}{21}$$

$$\frac{\sqrt{3}}{2} = \frac{MB}{21}$$

$$\frac{21\sqrt{3}}{2} = MB$$

$$AB = 2(MB)$$

$$= \frac{21\sqrt{3}}{2} \times 2$$

$$AB = 21\sqrt{3}$$

$$\cos 60 = \frac{OM}{21}$$

$$\frac{1}{2} = \frac{OM}{21}$$

$$\frac{21}{2} = OM$$

Ar of  $\Delta OAB$

$$= \frac{1}{2} B \times h$$

$$= \frac{1}{2} 21\sqrt{3} \times \frac{21}{2}$$

$$= \frac{441\sqrt{3}}{4} \text{ cm}^2$$

$$\text{Area of segment} = \text{Area of sector} - \text{Area of } \Delta$$

$$\begin{aligned}
 &= \frac{462}{4} - \frac{441\sqrt{3}}{4} \\
 &= 22 \times 21 - \frac{(21)^2 \sqrt{3}}{4} \\
 &= 21 \left[ 22 - \frac{21\sqrt{3}}{4} \right] \\
 &= 21 \left[ \frac{88 - 21\sqrt{3}}{4} \right] \\
 \text{Ar.} &= \frac{21}{4} [88 - 21\sqrt{3}] \text{ cm}^2
 \end{aligned}$$

# LEARNING OUTCOMES



1 | Area of a Sector of a Circle

2 | Area of a Segment of a Circle

# ASSESSMENT

**1** | Area of a sector of angle  $p$  (in degrees) of a circle with radius  $R$  is :

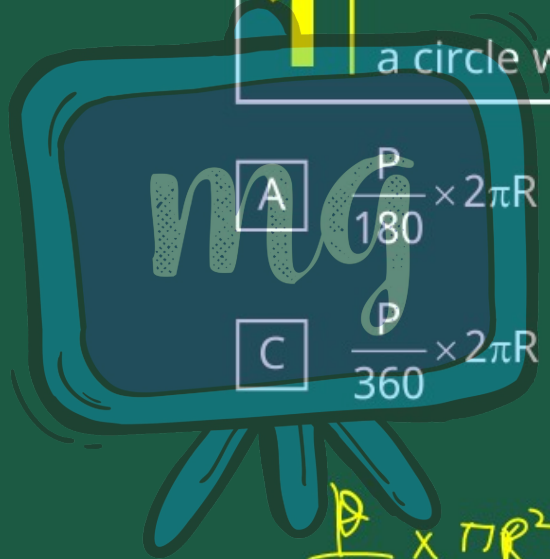
A  $\frac{p}{180} \times 2\pi R$

B  $\frac{p}{180} \times \pi R^2$

C  $\frac{p}{360} \times 2\pi R$

D  $\frac{p}{360} \times \pi R^2$

$\frac{p}{360} \times \pi R^2$



# ASSESSMENT

If the length of an arc of a segment in a circle of radius  $r$  is  $\pi$  cm and the angle of the segment is  $120^\circ$ , then radius of the circle is :

A

$\frac{2}{\pi}$  cm

B

2 cm

C

$\frac{2\pi}{3}$  cm

~~D~~

~~$\frac{3}{2}$  cm~~

$l = \pi$   
 $\theta = 120^\circ$

~~$l = \frac{120}{360} \times 2\pi r$~~

$\frac{3}{2} = r$

# ASSESSMENT

3

If the length of an arc of a segment of angle  $90^\circ$  in a circle is  $6\pi$  cm, what is the radius of the circle?

- A 6 cm
- B 12 cm
- C 18 cm
- D 24 cm



$$l = 6\pi$$

$$\theta = 90^\circ$$

$$l = \frac{\theta}{360} \times 2\pi r$$

$$3 \times 6\pi = \frac{90}{360} \times 2\pi r$$

$$12 = r$$