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CLASS – 10

MATHEMATICS

Chapter – 10

CIRCLES

Part – 5

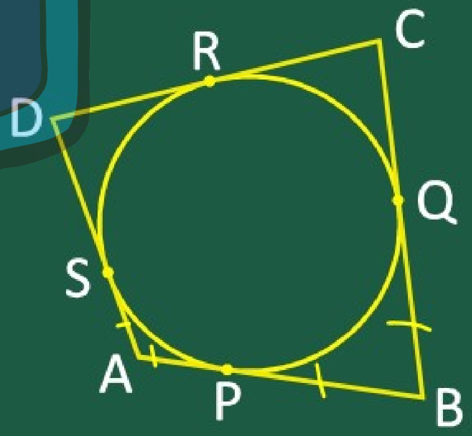
EXERCISE 10.2 (8–13)

Shubham Tiwari

EXERCISE 10.2

8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig.). Prove that

$$AB + CD = AD + BC$$



$AP = AS$ (1)

$PB = BQ$ (2)

$CR = QC$ (3)

$DR = DS$ (4)

{Tangents from an external point.}

By adding eq (1), (2), (3), (4)

$AP = AS$ — (1)

$PB = BQ$ — (2)

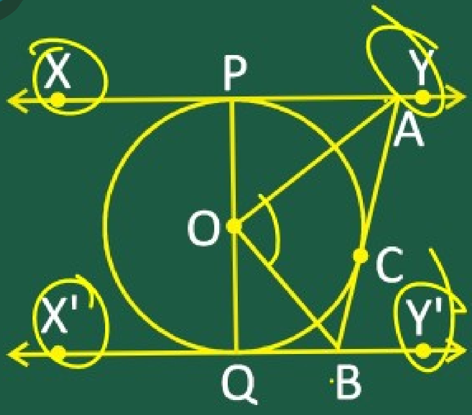
$CR = QC$ — (3)

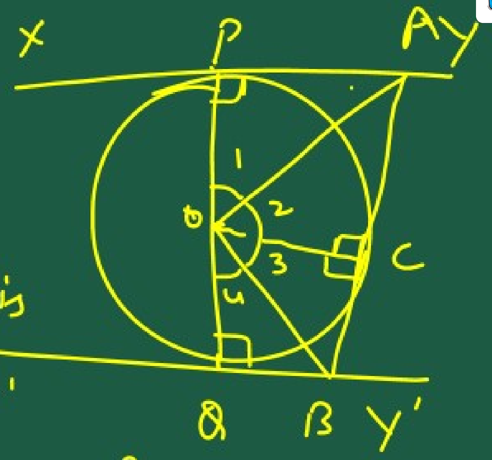
$DR = DS$ — (4)

$AP + PB + CR + DR = AS + BQ + QC + DS$

$$\boxed{AB + CD = AD + BC}$$

9. In Fig , XY and XY' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90°$.





in Δ OPA & ΔOCA.

$\angle OPA = \angle OCA = 90°$ {Radius is Perpendicular to the tangent}

OA = OA {common}

OP = OC {radius}

RHS Congruency Rule $\Delta OPA \cong \Delta OCA$.

By CPCT $\angle 1 = \angle 2$

Similarly in ΔOQB & ΔOCB

$\angle 3 = \angle 4$

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$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$

$\underbrace{\angle 2 + \angle 2} + \angle 3 + \angle 3 = 180^\circ$

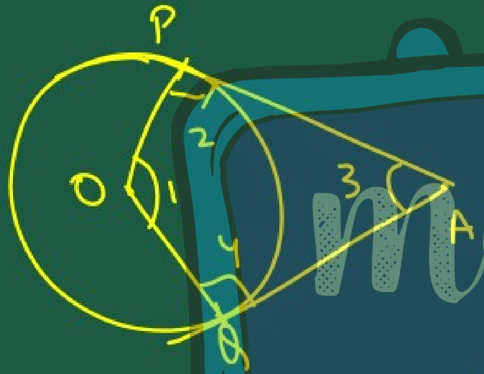
$2\angle 2 + 2\angle 3 = 180$

$2[\angle 2 + \angle 3] = 180^\circ$

$\boxed{\angle 2 + \angle 3 = 90^\circ}$

Hence $\angle AOB = 90^\circ$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.



$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$

$\angle 1 + 90^\circ + \angle 3 + 90^\circ$

$\angle 1 + \angle 3 + 180^\circ = 360$

$\boxed{\angle 1 + \angle 3} = 360^\circ - 180^\circ = 180^\circ$

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Soln.

$AP = AS$ — (1)

$PB = BQ$ — (2)

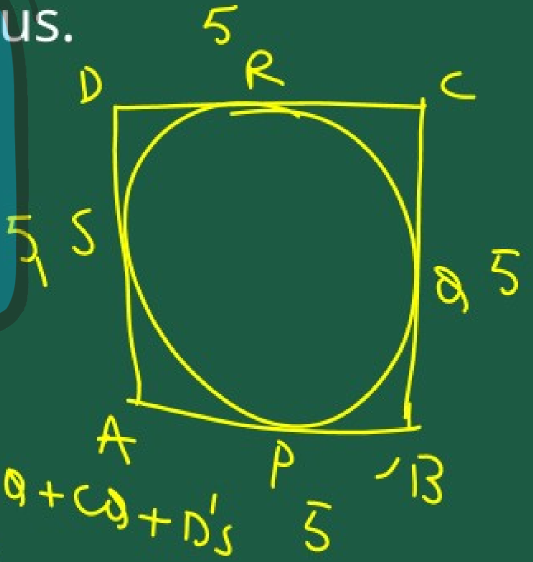
$CR = CQ$ — (3)

$DR = DS$ — (4)

eq (1) + eq (2) + eq (3) + eq (4)

$\underbrace{AP + PB} + \underbrace{CR + DR} = AS + BQ + CQ + DS$

$AB + CD = AD + BC$



$AB + CD = AD + BC$

$AB + AB = BC + BC$ {opposite sides of parallelogram}

$\cancel{2}AB = \cancel{2}BC$

$\boxed{AB = BC}$

Hence this parallelogram has adjacent sides equal, so this becomes a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig.). Find the sides AB and AC.

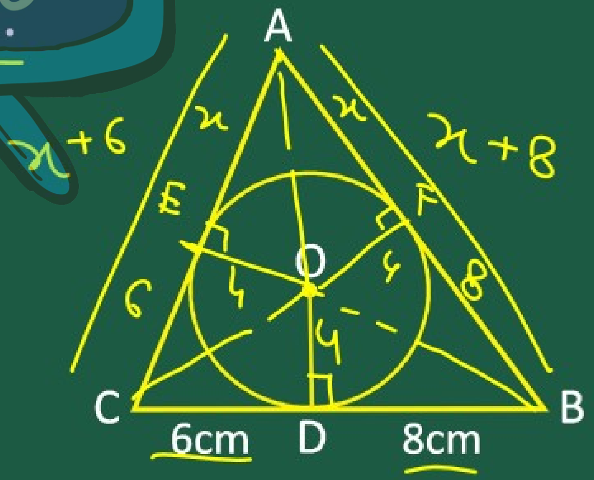
in ΔOAC

$Ar = \frac{1}{2} B \times h$

$Ar = \frac{1}{2} AC \times OE$

$= \frac{1}{2} (x+6)\, 4$

$Ar. = 2(x+6)\ cm^2$



in $\triangle OAB$

$Ar = \frac{1}{2} B \times h$

$= \frac{1}{2} AB \times OF$

$= \frac{1}{2} (x+8)\, 4$

$Ar = 2(x+8)\, cm^2$

in $\triangle OBC$

$Ar = \frac{1}{2} B \times h$

$= \frac{1}{\cancel{2}} (14)\, \cancel{4}^{2}$

$= 14 \times 2$

$Ar = 28\ cm^2$

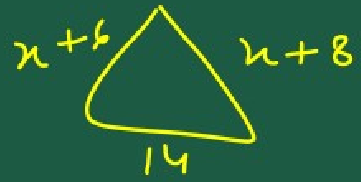
$Ar\ \triangle ABC = 28 + 2(x+8) + 2(x+6)$

$= 28 + 2x + 16 + 2x + 12$

$= 4x + 56$

$= 4(x+14)$

Ar of $\triangle ABC$ by Heron's formula.



$$s = \frac{a+b+c}{2}$$

$$= \frac{x+6+x+8+14}{2}$$

$$s = \frac{2x+28}{2} = x+14$$

$$Ar = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(x+14)\,(x+14-14)\,[(x+14)-(x+6)]\,[x+14-(x+8)]}$$

$$= \sqrt{(x+14)(x)\,(8)\,(6)}$$

$$Ar. = \sqrt{48\,x\,(x+14)}$$

$$\text{Area } \Delta = \sqrt{48x(x+14)}$$

$$4(x+14) = \sqrt{48x(x+14)}$$

$$\cancel{4^2}(x+14)^{\cancel{2}} = \cancel{48}x\,\cancel{(x+14)}$$

$$x+14 = 3x$$

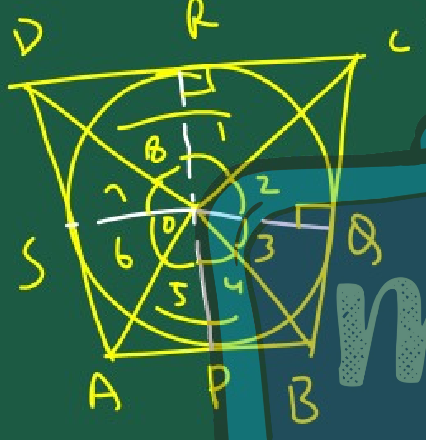
$$14 = 2x$$

$$\boxed{7 = x}$$

Hence the sides are $x+6 = 7+6 = 13$ cm

$x+8 = 7+8 = 15$ cm.

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



in ΔORC & ΔOQC

$OR = OQ = r$

$\angle ORC = \angle OQC = 90°$

$OC = OC$ (common)

RHS $\Delta ORC \cong \Delta OQC$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360$ {complete Angle}

$\angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360$

$2\angle 1 + 2\angle 4 + 2\angle 5 + 2\angle 8 = 360$

$\cancel{2}(\angle 1 + \angle 4 + \angle 5 + \angle 8) = \cancel{360}$

$= 180$

Hence $\angle AOB + \angle COD = 180^\circ$

Similarly $\angle BOC + \angle AOD = 180^\circ$