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CLASS – 10

MATHEMATICS

Chapter – 10

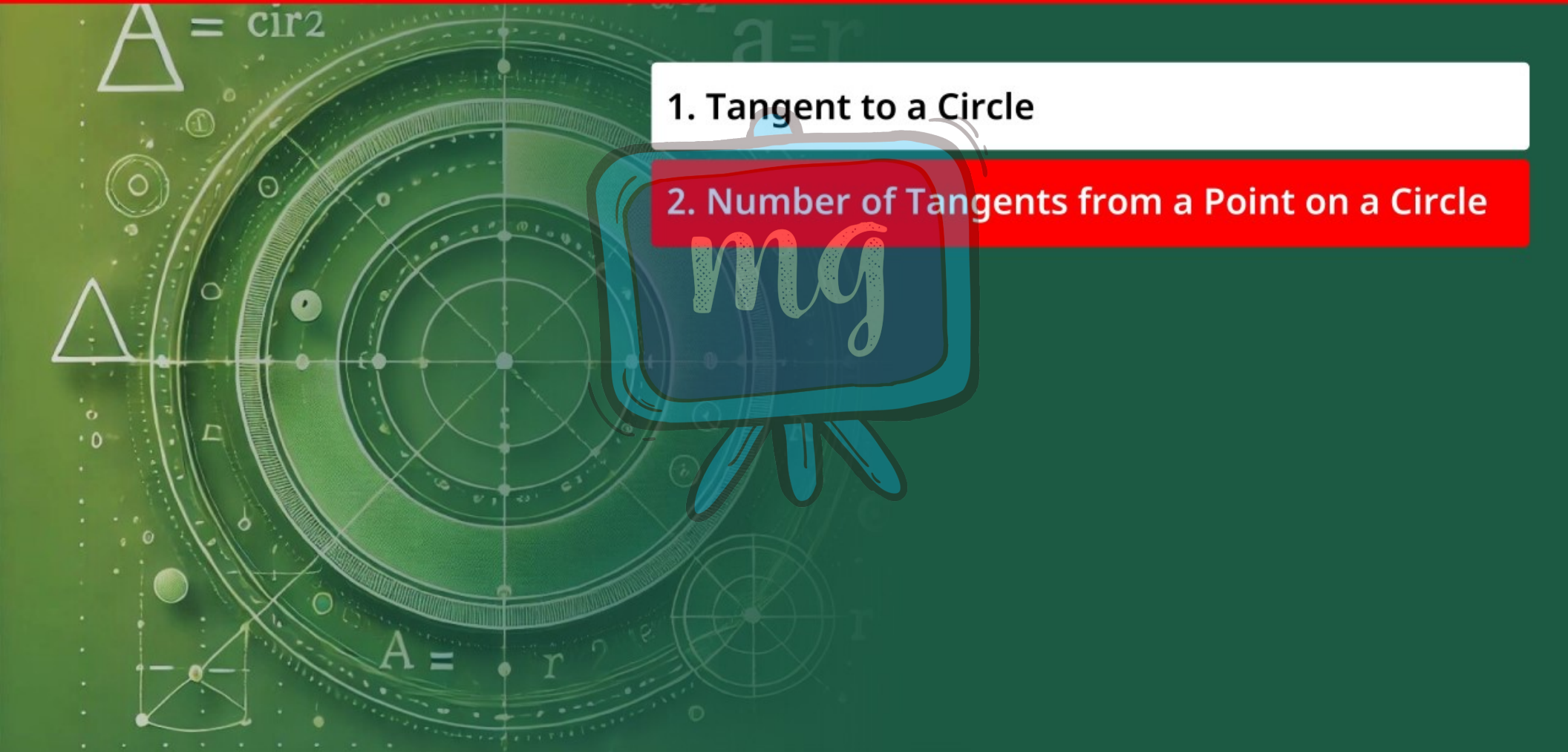
CIRCLES

Part – 3

Number of Tangents from a Point on a Circle

Shubham Tiwari

OVERVIEW



1. Tangent to a Circle
2. Number of Tangents from a Point on a Circle

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COMPETENCY BASED LEARNING

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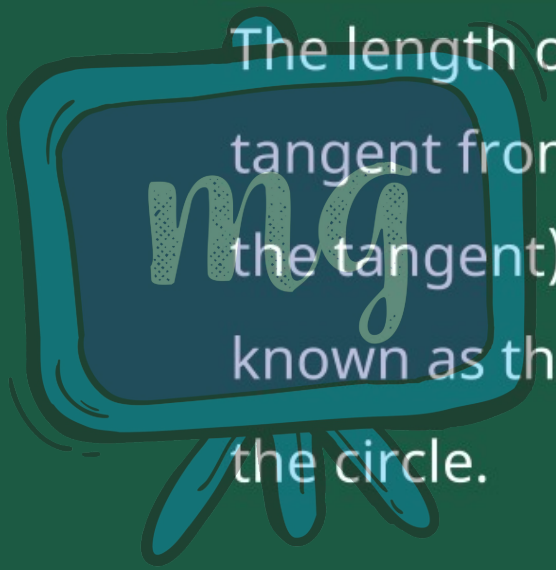
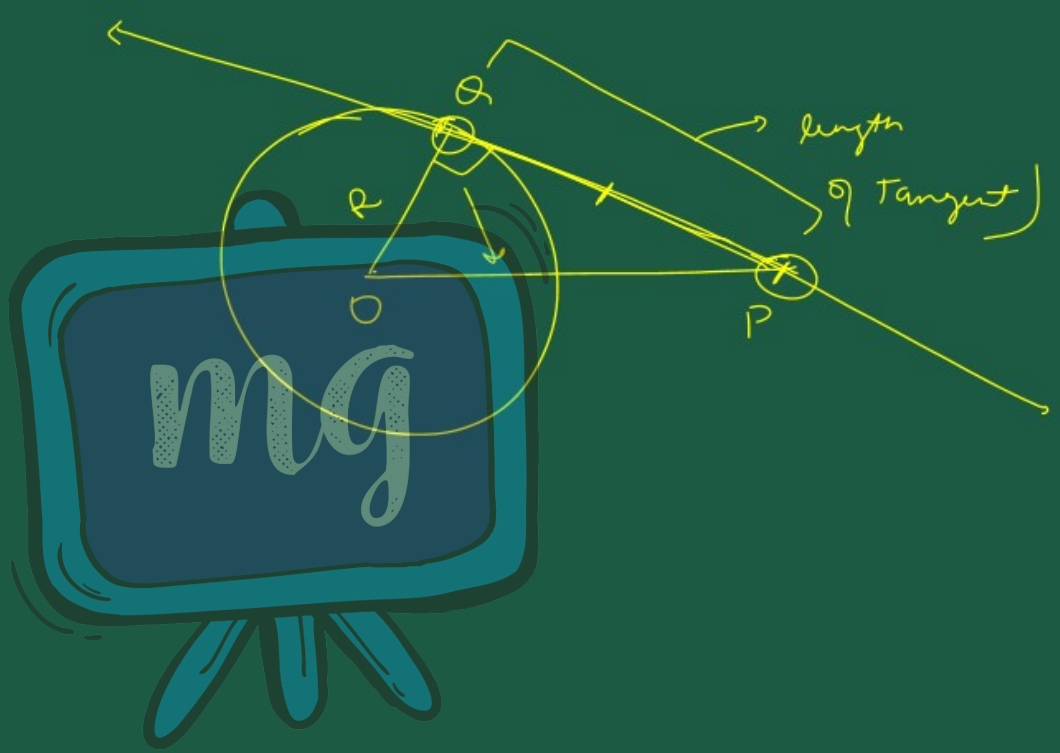
Length of a Tangent from an external point to a circle

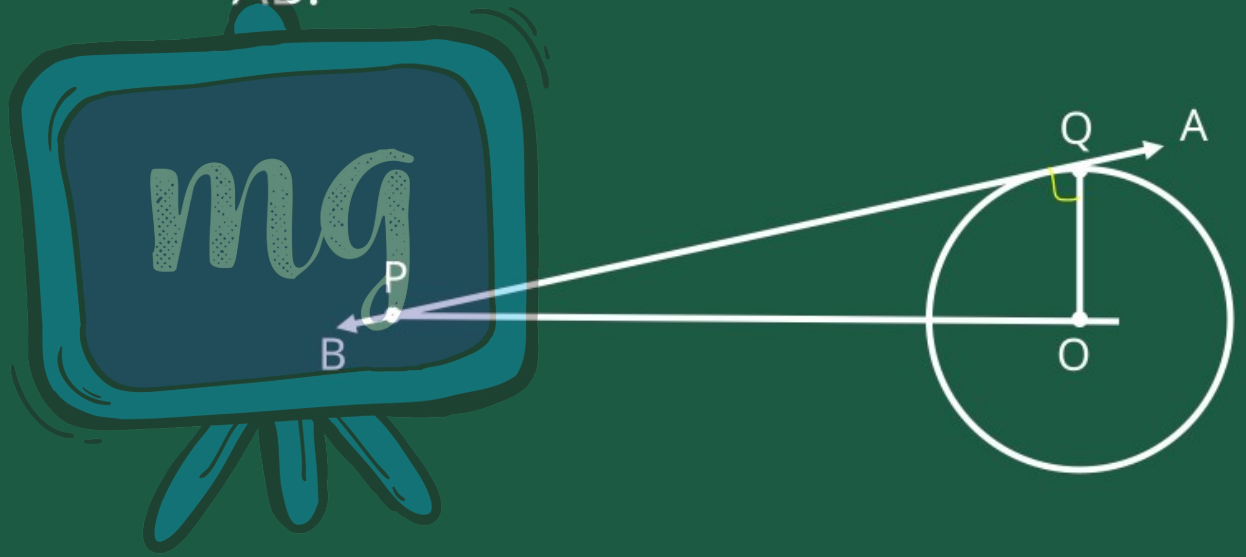
LENGTH OF A TANGENT



The length of the segment of a tangent from an external point (on the tangent) to the point of contact is known as the length of the tangent to the circle.



Here, PQ is the length of the tangent AB.

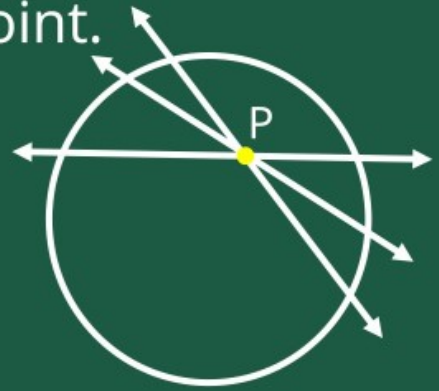


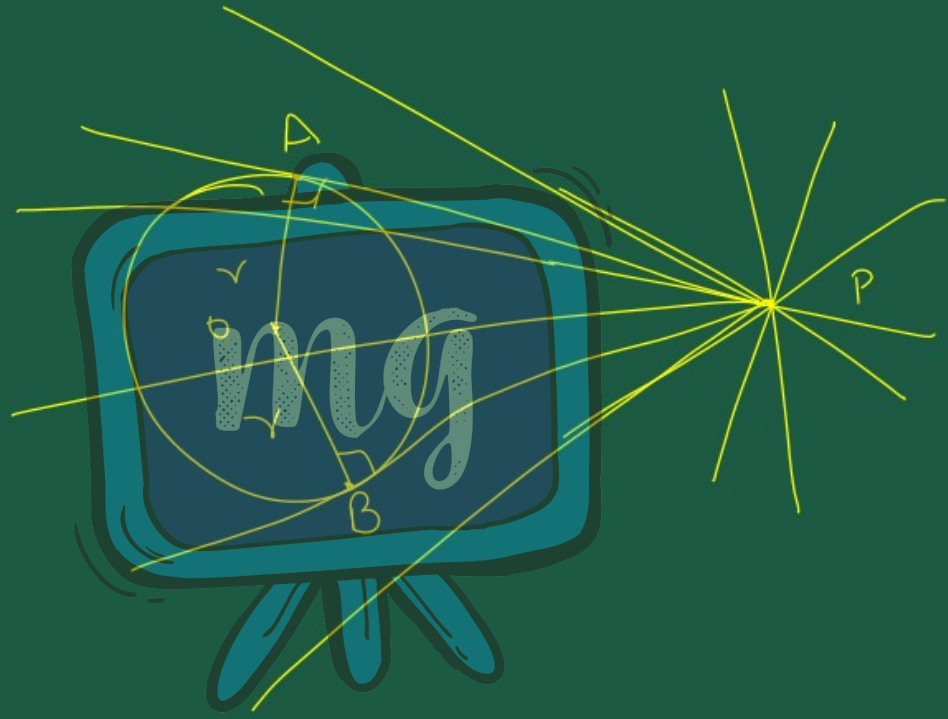
NUMBER OF TANGENTS FROM A POINT ON A CIRCLE

Case : 1

When the point lies inside the circle :

There are no tangents to the circle, passing through the point.





Case : 2

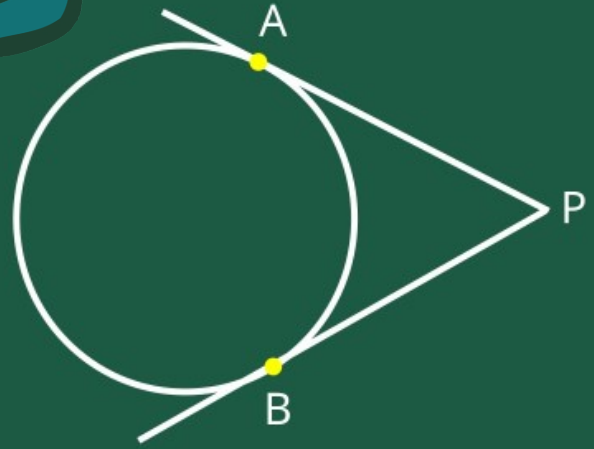
When the point lies on the circle :

There is one and only one tangent to the circle, passing through the point.

Case : 3

When the point lies outside the circle :

There are exactly two tangents (PA and PB) to the circle, passing through the point (P).



THEOREM 10.2

The lengths of tangents drawn from an external point to a circle are equal.

To prove: PA = PB

In $\triangle OAP$ & $\triangle OBP$

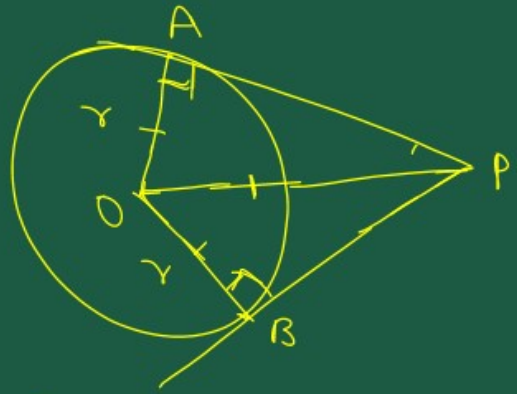
S – OA = OB = r

H – OP = OP [common]

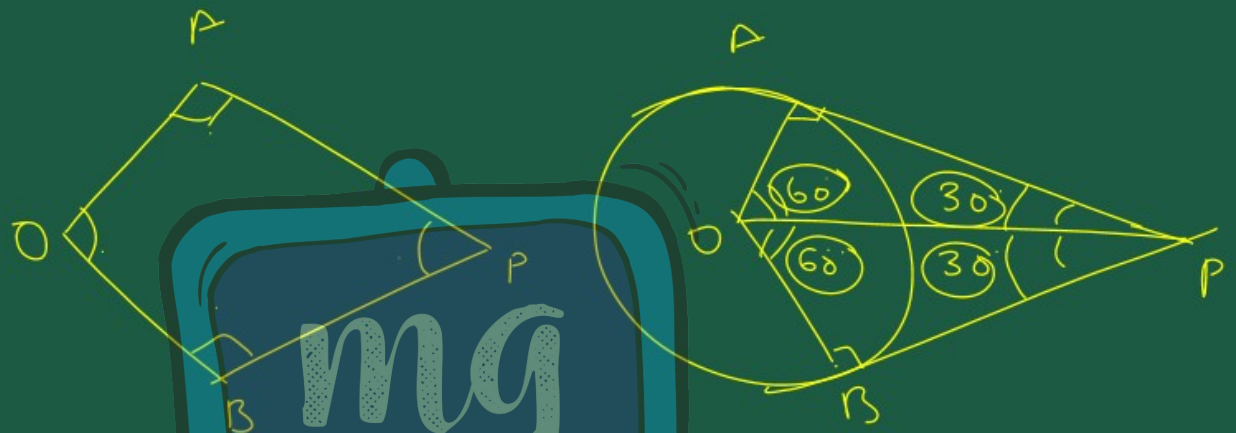
R – $\angle OAP = \angle OBP = 90^\circ$ [radius is ⊥ Tangent]

By RHS congruency Rule

$\triangle OAP \cong \triangle OBP$



AP = BP [C.P.C.T.]



$\angle O + \angle A + \angle P + \angle B = 360$

180

$\angle O + \angle P = 360 - 180$

$\angle O + \angle P = 180$

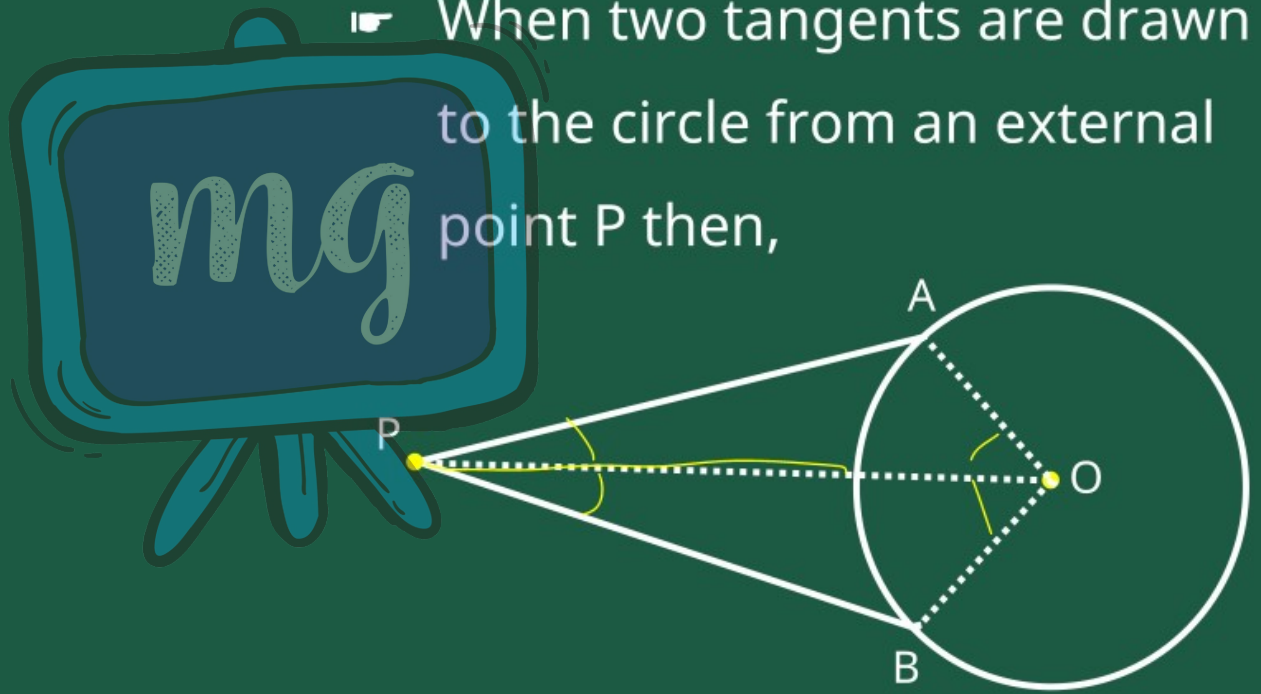
$\triangle OAP$

$\angle A = 90$

$\angle O + \angle P = 90$

Note

☛ When two tangents are drawn to the circle from an external point P then,

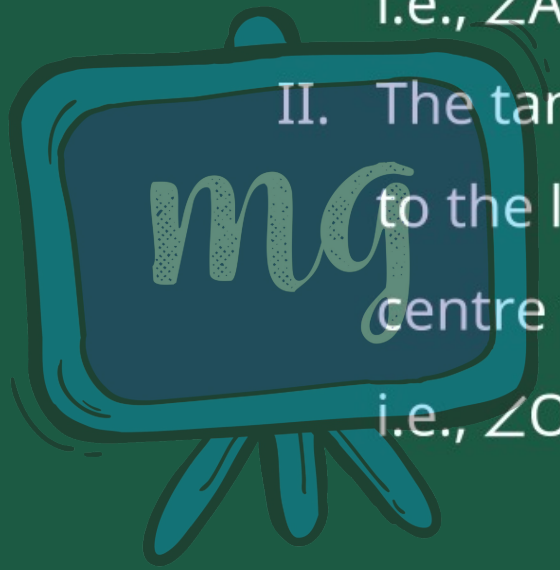


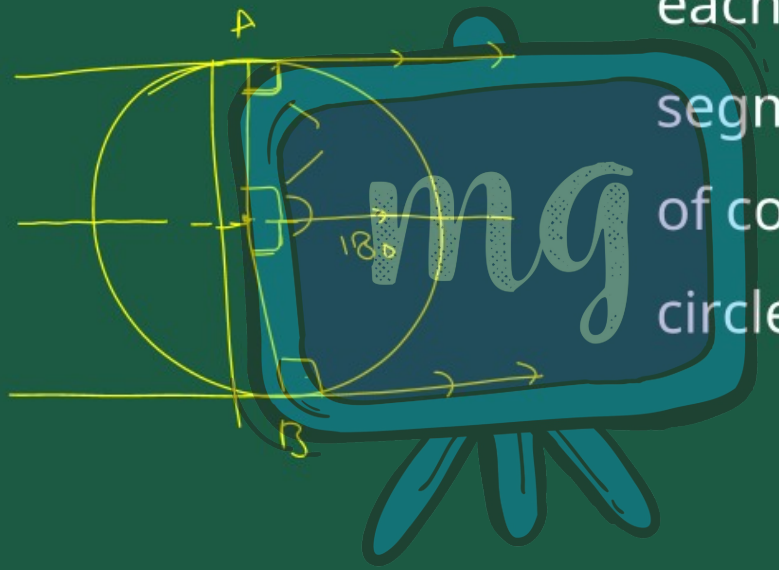
I. $\angle AOB$ is divided equally by OP

i.e., $\angle AOP = \angle BOP$

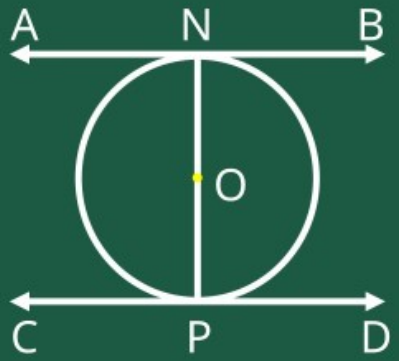
II. The tangents are equally inclined to the line segment joining the centre to the external point.

i.e., $\angle OPB = \angle OPA$



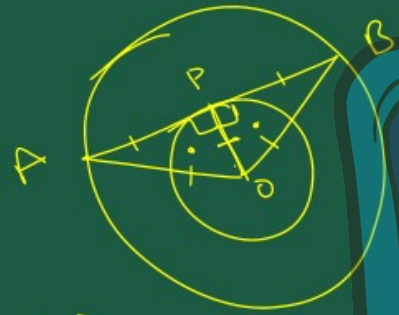


- If two tangents are parallel to each other, then the line segments joining their point of contact is a diameter of the circle.



Example : 1

Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.



In $\triangle OAP$ & $\triangle OBP$

$OA = OB = r$

$OP = OP$ (Common)

$\angle OPA = \angle OPB = 90^\circ$ {radius is perpendicular on the tangent}

By RHS Rule.

$\triangle OAP \cong \triangle OBP$

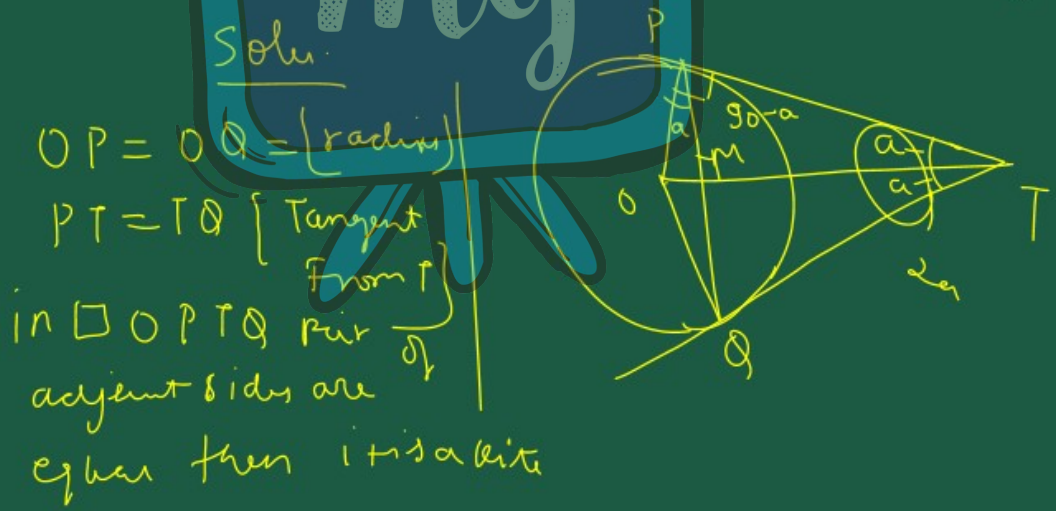
By CPCT

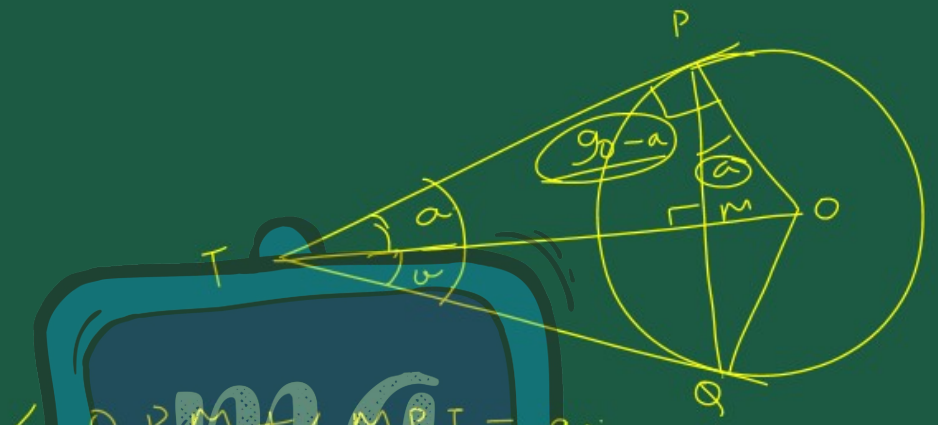
$AP = BP$

Hence we have proved that the chord is bisected at the point of Contact.

Example : 2

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ.





$\angle OPM + \angle MPT = 90^\circ$

$\angle MPT = 90 - \angle OPM$ — (1)

in $\triangle PTM$,

$\angle PTM + \angle TPM = 90^\circ$ { Angle sum property }

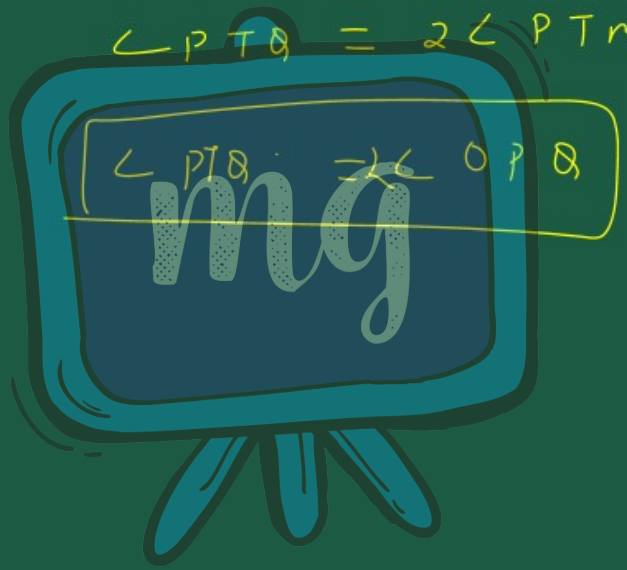
$\angle PTM + 90 - \angle OPM = 90$

$\angle PTM = \angle OPM$.

$\angle PTQ = \angle PTm + \angle mTQ$

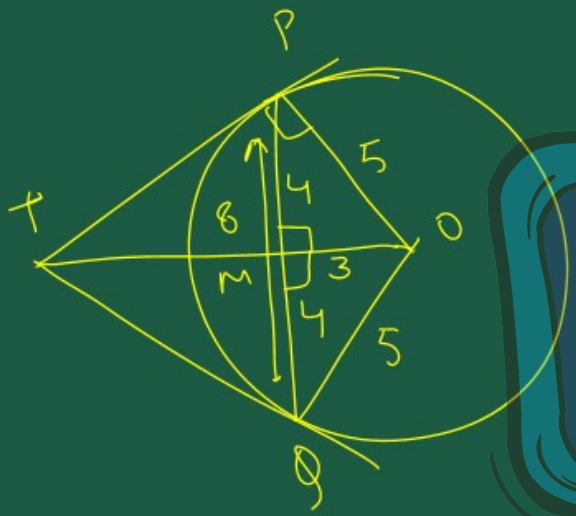
$\angle PTQ = 2\angle PTm \quad \{\angle PTm = \angle mTQ\}$

$\angle PTQ = \angle POQ$



Example : 3

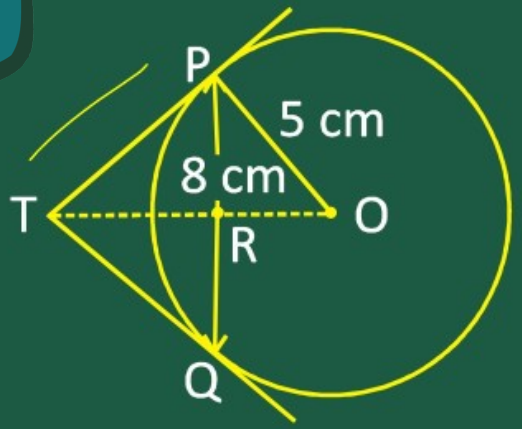
PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see Fig.). Find the length TP.

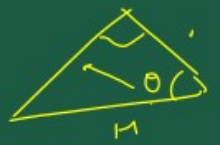


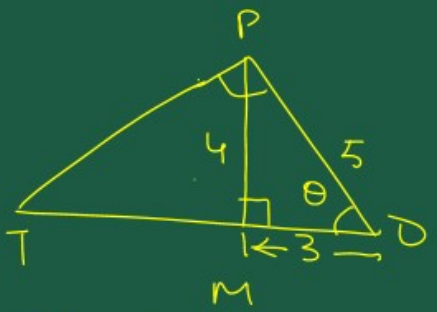
TP = TQ and OP = OQ

Hence OPTQ is a kite

$\angle OMQ = 90^\circ$







In $\triangle OPM$

$OP^2 = PM^2 + OM^2$

$5^2 = 4^2 + OM^2$

$5^2 - 4^2 = OM^2$

$3^2 = OM^2$

$3 = OM$

$\triangle PMO$

$\tan\theta = \frac{4}{3}$ — (1)

In $\triangle TPO$

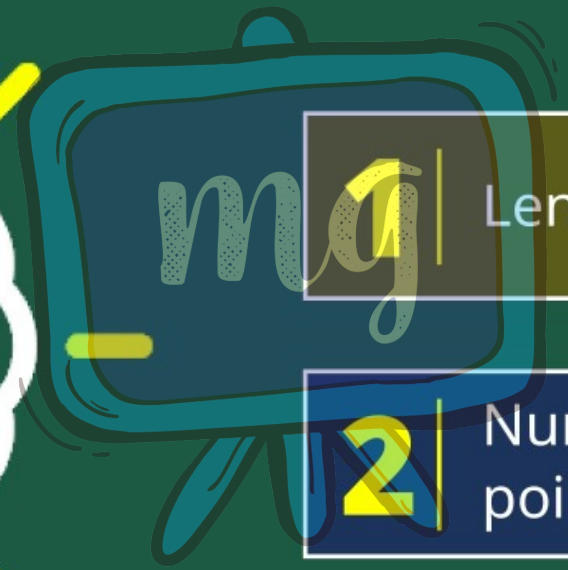
$\tan\theta = \frac{TP}{PO} = \frac{TP}{5}$

From (1)

$\frac{4}{3} = \frac{TP}{5}$

$\frac{20}{3}$ cm $= TP$

LEARNING OUTCOMES



1. Length of a tangent
2. Number of tangents from a point on a circle

ASSESSMENT

1 The length of the tangent from an external point A on a circle with centre O is :

A always greater than OA

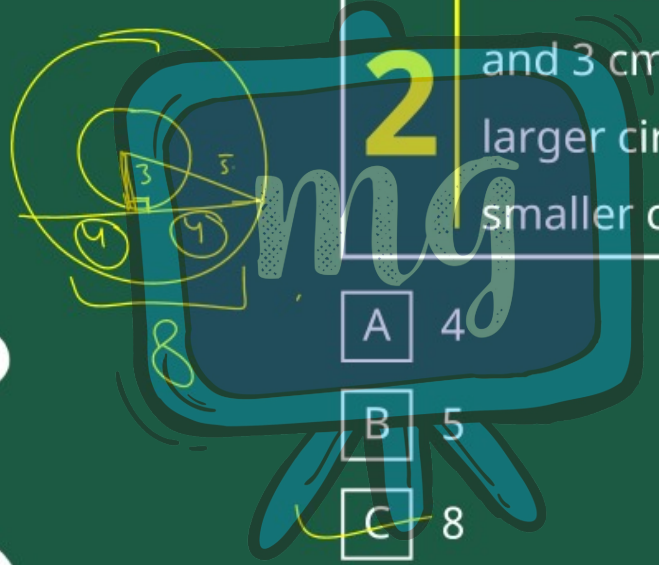
B equal to OA

C always less than OA

D cannot be estimated

ASSESSMENT

2 Two concentric circles are of radii 5 cm and 3 cm. Length of the chord of the larger circle, (in cm), which touches the smaller circle is



A 4

B 5

C 8

D 10