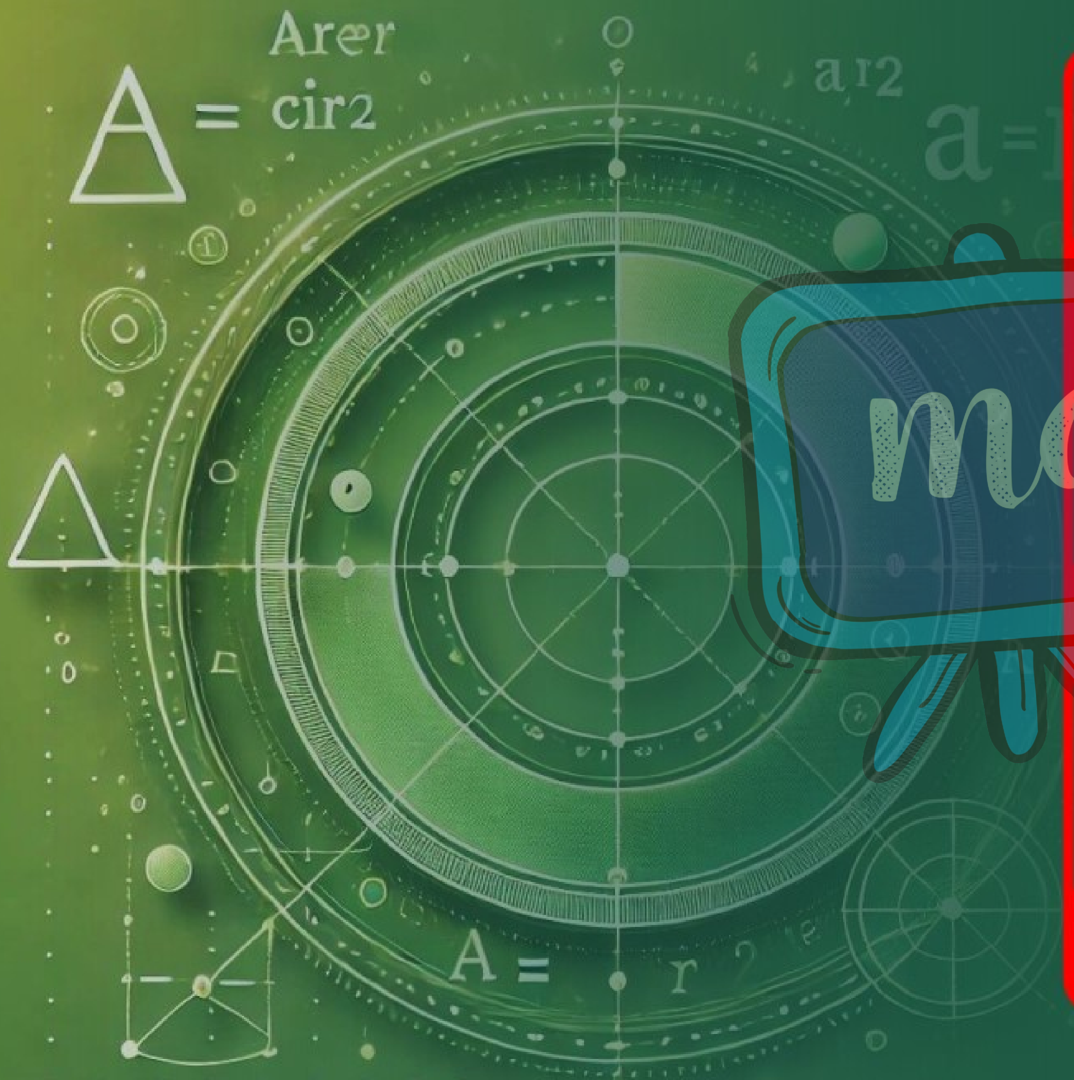

CLASS – 10

MATHEMATICS

CH – 10 : CIRCLES

CBSE Board

Most Important Questions – 1

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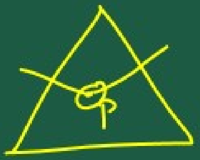
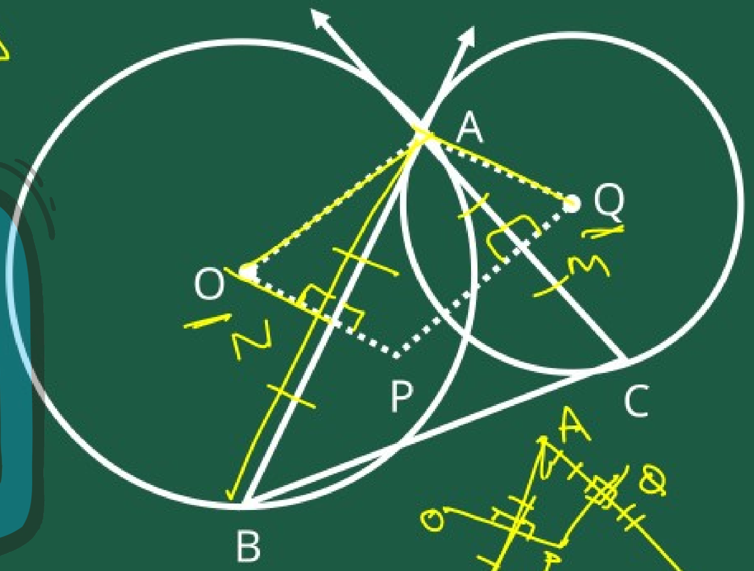
Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER

1. Let A be one point of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C, respectively. Let the point P be located so that AOPQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC.



Soln.



$QA \perp AB$

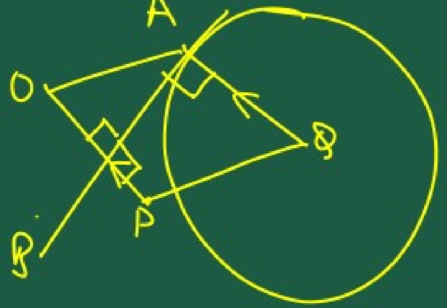
$OP \perp AB$

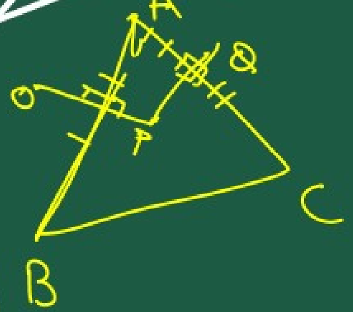
$OA \parallel PQ$

$OA \perp AC$

$PQ \perp AC$

$QA \parallel OP$





$ON \perp AB$

$AN = NB$

$QM \perp AC$

$AM = MC$

in ΔABC OP and QP are perpendicular bisector at P, hence it is proved that P is a circumcentre of ΔABC.

2. AB is a diameter of a circle. P is a point on the semi-circle APB. AH and BK are perpendiculars from A and B respectively to the tangent at P. Prove that AH + BK = AB.

in ΔMKB & ΔMHA

$\angle M = \angle M$ {common}

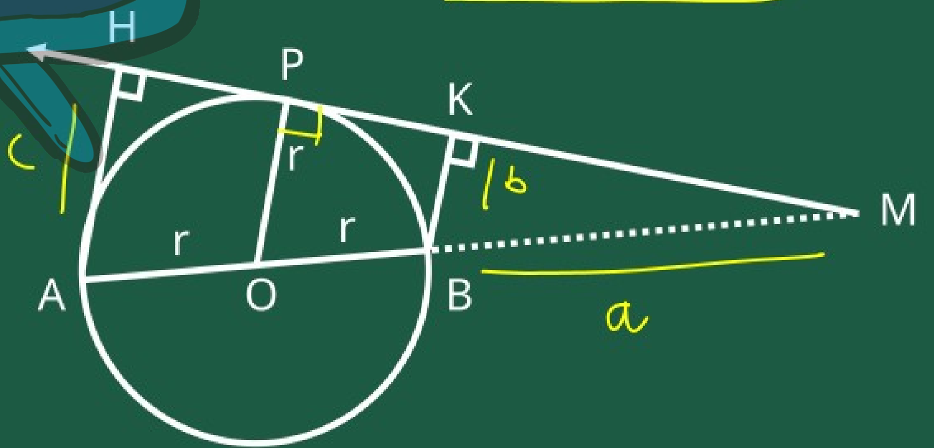
$\angle MKB = \angle MHA = 90^\circ$

AA similarity Rule

$\Delta MKB \sim \Delta MHA$

$$\frac{MB}{MA} = \frac{BK}{AH}$$

$$\frac{a}{a+2r} = \frac{b}{c}$$



$$ac = (a+2r)\,b$$

$$ac = ab + 2rb$$

$$ac - ab = 2rb$$

$$a(c-b) = 2rb$$

$$\boxed{a = \frac{2rb}{c-b}} \quad \text{—} \; 1$$

in ΔMKB & ΔMPO

$\angle M = \angle M$ {common}

$\angle MKB = \angle MPO = 90^\circ$

AA Similarity Rule

$\Delta MKB \sim \Delta MPO$

$$\frac{MB}{MO} = \frac{BK}{PO}$$

$$\frac{a}{a+r} = \frac{b}{r}$$

$$ar = ba + br$$

$$ar - ab = br$$

$$a[r-b] = br$$

$$a = \frac{rb}{r-b} \quad \text{—} \; 2$$

from eq (1) and eq (2)

$$\frac{2\cancel{rb}}{c-b} = \frac{\cancel{rb}}{r-b}$$

$$2(r-b) = c-b$$

$$2r - 2b = c - b$$

$$2r = 2b + c - b$$

$$2r = b + c$$

$$\boxed{AB = BK + AH}$$

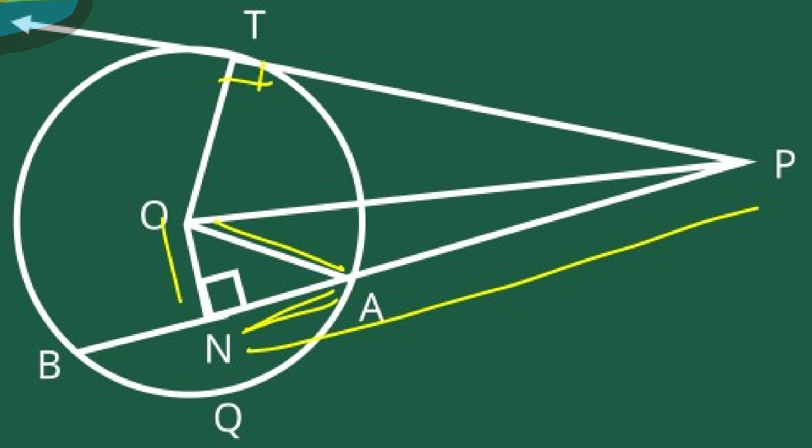
3. From an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that

(i) $PA.\,PB = PN^2 - AN^2$

$$PA \cdot PB = (PN - AN)(PN + BN)$$

$$= (PN - AN)(PN + AN) \quad \{BN = AN\}$$

$$= PN^2 - AN^2$$



(ii) $PN^2 - AN^2 = OP^2 - OT^2$

$PN^2 - AN^2 =$

in ΔONP, $\angle N = 90^\circ$

$OP^2 = ON^2 + PN^2$

$PN^2 - AN^2 = OP^2 - ON^2 - AN^2$

$= OP^2 - (ON^2 + AN^2)$

$= OP^2 - OA^2$ {frm ΔONA}

$= OP^2 - OT^2$ {$OA = OT$}

(iii) PA. PB = PT^2

$$\left\{\begin{aligned} PA \cdot PB &= PN^2 - AN^2 \quad \text{--- (1)} \\ OP^2 - OT^2 &= PN^2 - AN^2 \quad \text{--- (2)} \end{aligned}\right.$$

from eq (1) and eq (2)

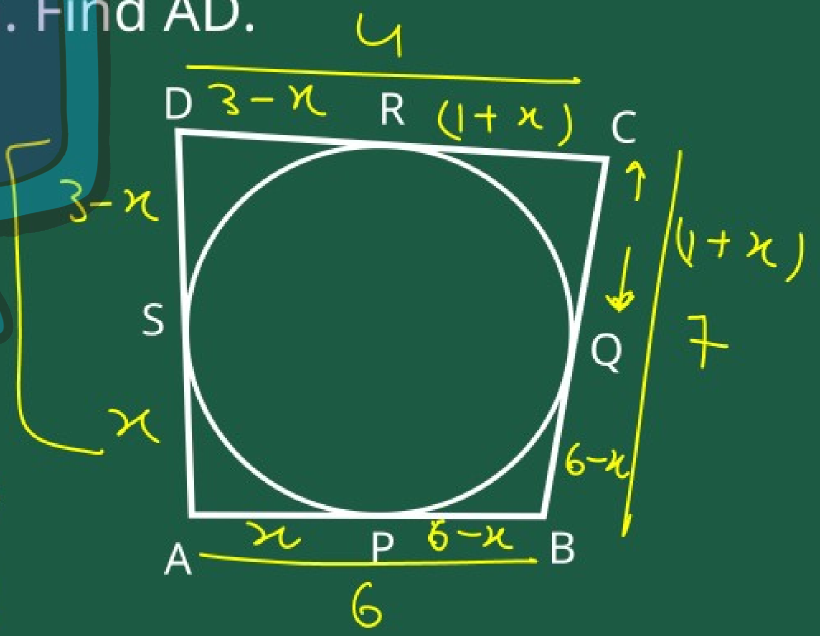
$$PA \cdot PB = OP^2 - OT^2$$

$$\boxed{PA \cdot PB = PT^2} \; \{\text{from } \Delta\, OPT\}$$

4. In Fig. a circle touches all the four sides of a quadrilateral ABCD with AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.

$4-(1+x)$
$4-1-x$
$3-x$

$7-(6-x)$
$7-6+x$
$1+x$



$AD = AS + SD$
$= \cancel{x} + 3 - \cancel{x}$

$\boxed{AD = 3 \text{ cm}}$

5. In Fig. there are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

$12^2+5^2 = OP^2$

$144+25 = OP^2$

$169 = OP^2$

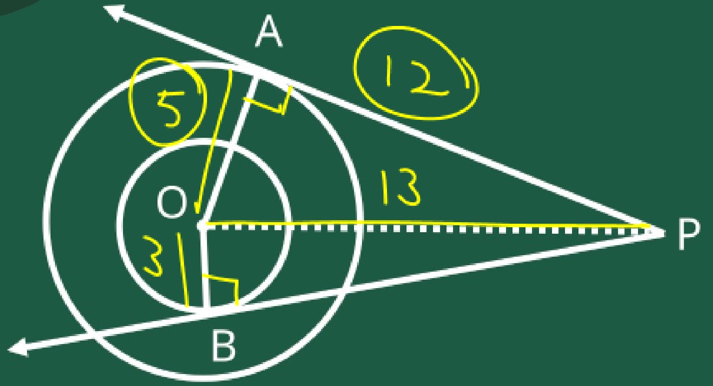
$\boxed{13 = OP}$

in ΔOPB, $\angle OBP = 90^\circ$

$OP^2 - OB^2 = BP^2$

$(13)^2-(3)^2 = BP^2$

$169-9 = BP^2$



$$160 = BP^2$$

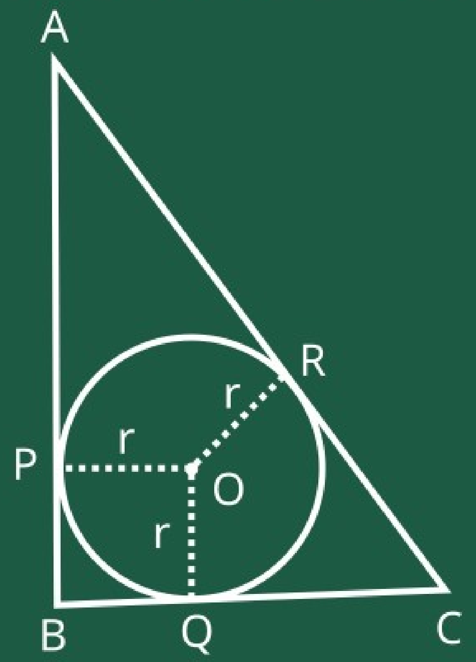
$$\sqrt{160} = BP^2$$

$$\sqrt{40 \times 4} = BP$$

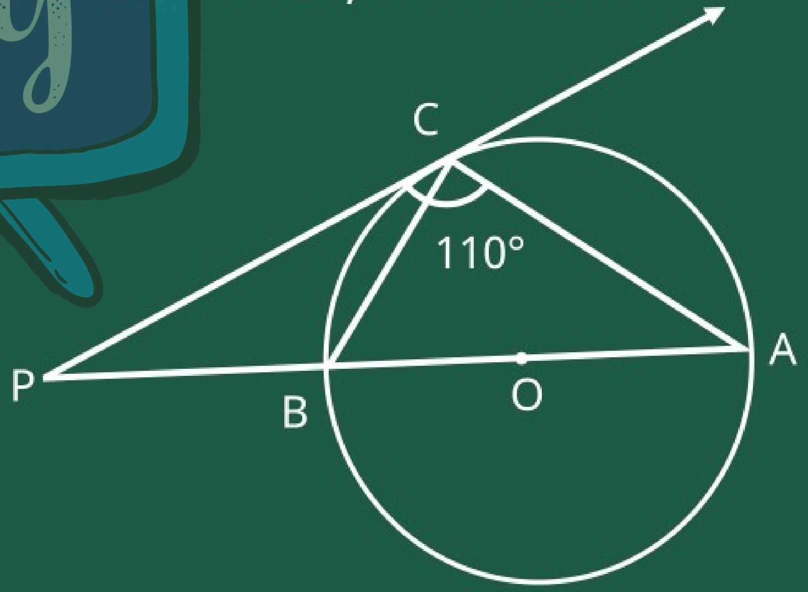
$$\sqrt{4 \times 10 \times 4} = BP$$

$$\boxed{4\sqrt{10}\ \text{cm} = BP}$$

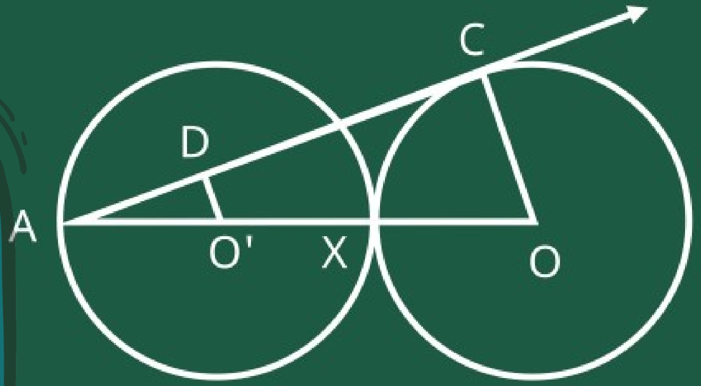
6. In Fig. ABC is a right triangle right-angled at B such that BC=6 cm and AB = 8 cm. Find the radius of its incircle.



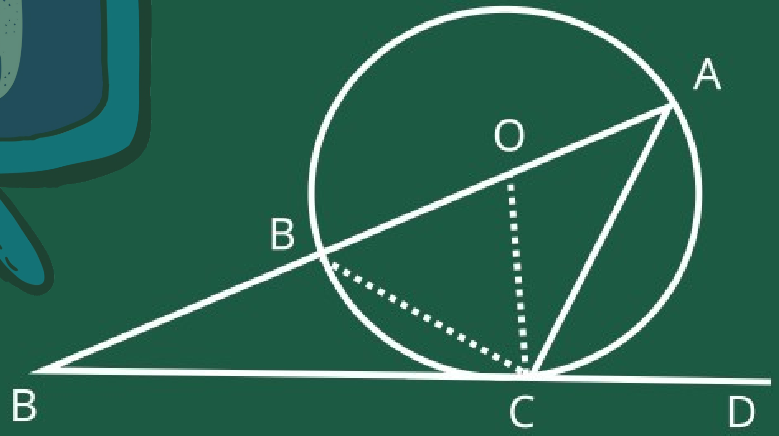
7. In Fig. the tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110°$, find $\angle CBA$.



8. In Fig. equal circles with centres O and O' touch each other at X. OO' produced to meet a circle with centre O', at A. AC is a tangent to the circle whose centre is O. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$


C
D
A
O'
X
O

9. In Fig. O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90°$.



10. From a point P two tangents PA and PB are drawn to a circle with centre at O. If $OP = 2r$, show that ΔPAB is equilateral.

in $\triangle OAB$

By BPT $\{QP \parallel OA\}$

$$\frac{\cancel{BQ}}{\cancel{QO}} = \frac{BP}{PA} \quad \{BQ = QO = 75\}$$

$$1 = \frac{BP}{PA}$$

$$BP = PA$$

Hence $PA = \frac{1}{2} AB = \frac{1}{2}\, 75\sqrt{3}$

$$PA = \frac{75\sqrt{3}}{2}$$

in ΔOAB & ΔBQP

$\angle B = \angle B$ (Common)

$\angle BAO = \angle BPQ = 90^\circ$ { $QP \parallel OA$ }

By AA similary Rule

$\Delta BQP \sim \Delta BOA$

$$\boxed{\frac{BQ}{BO} = \frac{QP}{OA}} = \frac{BP}{BA}$$

$$\frac{BQ}{BO} = \frac{QP}{OA}$$

$$\frac{\cancel{75}}{\cancel{150}} = \frac{QP}{75}$$

$$\frac{1}{2} = \frac{QP}{75}$$

$$\boxed{\frac{75}{2} = QP = 37.5}$$