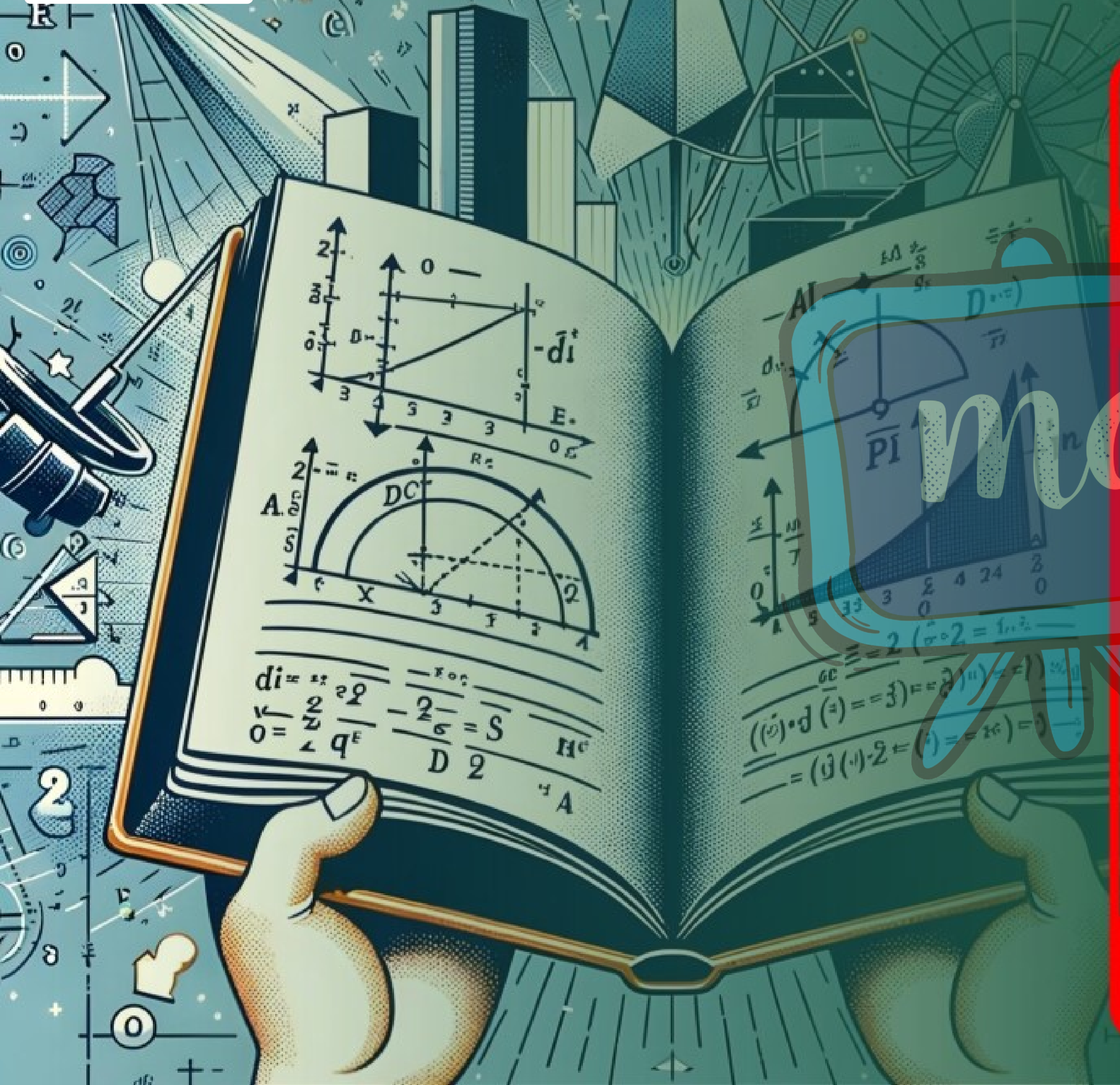


CLASS – 10

MATHEMATICS

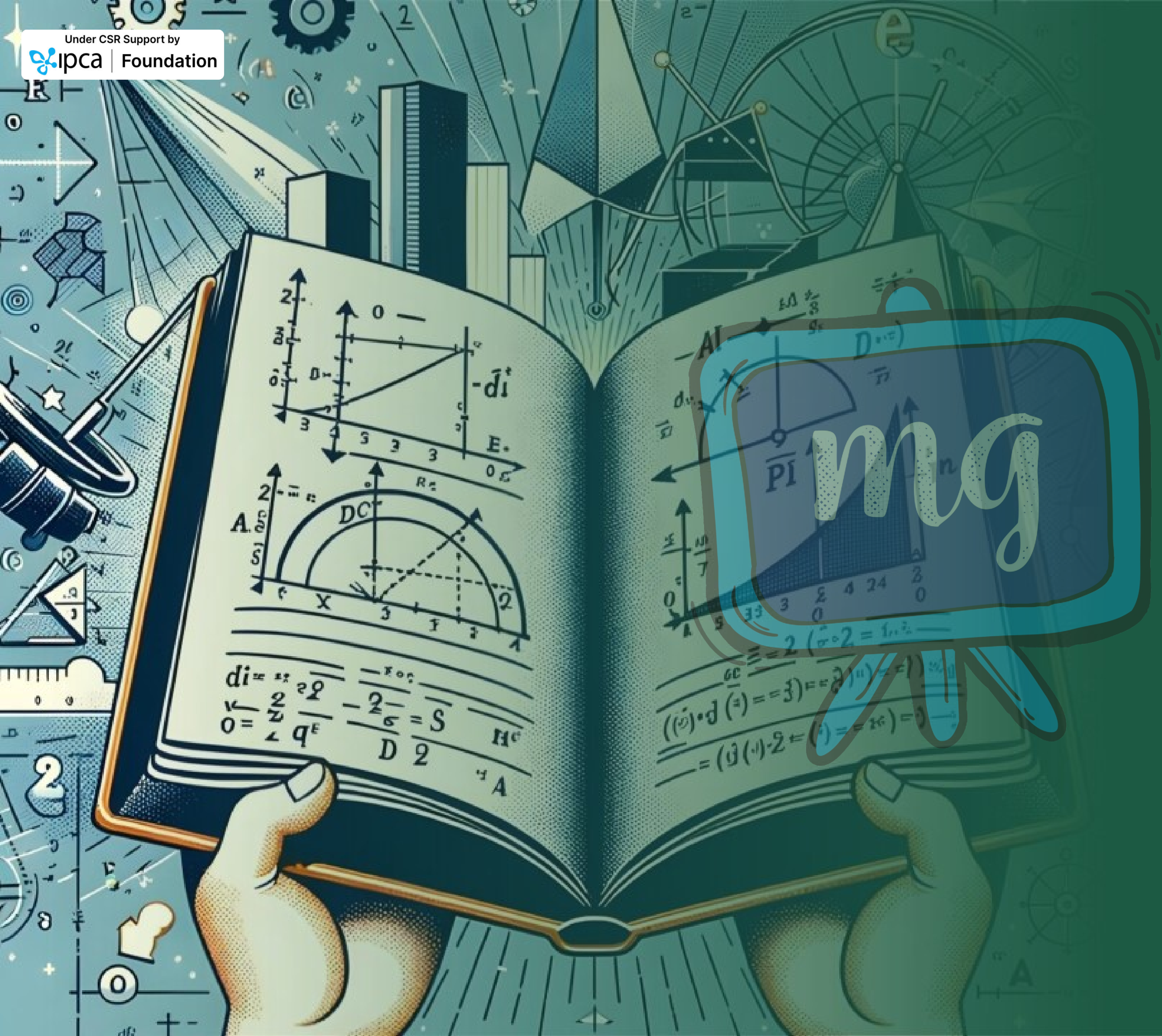
Chapter – 8

Introduction to Trigonometry

Part – 6

Exercise – 8.3

Shubham Tiwari



EXERCISE 8.3

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1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A

Soln:

$1 + \cot^2 A = \operatorname{cosec}^2 A$

$\sqrt{1 + \cot^2 A} = \operatorname{cosec} A$

$\sin A = \dfrac{1}{\operatorname{cosec} A}$

$\sin A = \dfrac{1}{\sqrt{1 + \cot^2 A}}$

$$\sec A = \sqrt{1 + \tan^2 A}$$

$$= \sqrt{1 + \frac{1}{\cot^2 A}}$$

$$= \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}}$$

$$= \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

$$\boxed{\tan A = \frac{1}{\cot A}}$$

2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Soln:

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sqrt{\sec^2 A}}$$

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

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3. Choose the correct option. Justify your choice.

i. $9\sec^2 A - 9\tan^2 A =$

A. 1

B. 9

C. 8

D. 0

$9(\sec^2 A - \tan^2 A)$

$9(1)$

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ii. $(1+\tan\theta+\sec\theta)(1+\cot\theta-\csc\theta)=$

A. 0

B. 1

C. 2

D. -1

$1+\tan\theta+\sec\theta$

$= 1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}$

$= \frac{(\cos\theta+\sin\theta)+(1)}{\cos\theta}$

$1+\cot\theta-\csc\theta$

$1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}$

$\frac{(\sin\theta+\cos\theta)-(1)}{\sin\theta}$

$\Rightarrow \frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\times\cos\theta}$

$\cancel{\sin^2\theta+\cos^2\theta}+2\sin\theta\cdot\cos\theta\cancel{-1}$

$\frac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$

iii. (sec A + tan A) (1 – sin A) =

A. sec A

B. sin A

C. cosec A

D. cos A

$$\frac{1}{\cos A} + \frac{\sin A}{\cos \theta}$$

$$\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$= \frac{1-\sin^2 A}{\cos A}$$

$$= \frac{\cos^{\cancel{2}} A}{\cancel{\cos A}} = \cos A$$

iv. $\dfrac{1+\tan^2 A}{1+\cot^2 A} =$

A. $\sec^2 A$

B. -1

C. $\cot^2 A$

D. $\tan^2 A$

$$\frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1/\cos^2 A}{1/\sin^2 A}$$

$$\frac{\sin^2 A}{1} \times \frac{1}{\cos^2 A}$$

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most Imp 4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

i. $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Soln. Solving LHS

$$(\operatorname{cosec}\theta - \cot\theta)^2$$

$$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$\left[\frac{1-\cos\theta}{\sin\theta}\right]^2$$

$$\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos^2\theta)}$$

$$\frac{\cancel{(1-\cos\theta)}(1-\cos\theta)}{\cancel{(1-\cos\theta)}(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

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ii. $$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$$

Solu:

$$\frac{\cos A(1-\sin A)}{(1+\sin A)(1-\sin A)}+\frac{1+\sin A}{\cos A}$$

$$\frac{\cos A(1-\sin A)}{(1-\sin^2 A)}+\frac{(1+\sin A)}{\cos A}$$

$$\frac{\cancel{\cos A}(1-\sin A)}{\cos^2 A}+\frac{(1+\sin A)}{\cos A}$$

$$\frac{1}{\cos A}\left[1-\cancel{\sin A}+1+\cancel{\sin A}\right]$$

$$=\frac{2}{\cos A}=2\sec A$$

iii. $$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$$

Solu.

$$\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$\frac{(\tan\theta)^2}{(\tan\theta-1)}+\frac{1}{\tan\theta(1-\tan\theta)}$$

$$\frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$$

$$\frac{\tan^3\theta - (1)^3}{\tan\theta(\tan\theta - 1)}$$

$$\frac{\cancel{(\tan\theta - 1)}(\tan^2\theta + \tan\theta + 1)}{\tan\theta\,\cancel{(\tan\theta - 1)}}$$

$$\frac{\sec^2\theta + \tan\theta}{\tan\theta}$$

$$\frac{\sec\theta \times \sec\theta}{\tan\theta} + \frac{\cancel{\tan\theta}}{\cancel{\tan\theta}}$$

$$\boxed{\operatorname{cosec}\theta \times \sec\theta + 1}$$

$$\frac{\sec\theta \times \sec\theta}{\tan\theta}$$

$$\frac{\frac{1}{\cancel{\cos\theta}} \times \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cancel{\cos\theta}}}$$

$$\frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

iv. $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

Soln

Solving RHS

$$\frac{\sin^2 A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A}$$

$$\frac{\cancel{(1-\cos A)}(1+\cos A)}{\cancel{(1-\cos A)}}$$

$$= 1+\cos A = 1+\frac{1}{\sec A} = \frac{\sec A+1}{\sec A}$$

LHS = RHS Hence

v. $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$.

using the identity

$\operatorname{cosec}^2 A = 1 + \cot^2 A$.

Soln: $\dfrac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}}$

$= \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

$$= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\left[+1 - \cancel{(\operatorname{cosec} A + \cot A)}\right]}{\cancel{\cot A - \operatorname{cosec} A + 1}}$$

vi. $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Soln:

$$\sqrt{\frac{1+\sin A}{(1-\sin A)} \times \frac{(1+\sin A)}{(1+\sin A)}}$$

$$\sqrt{\frac{(1+\sin A)^2}{(1-\sin^2 A)}}$$

$$\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$\sqrt{\left(\frac{1+\sin A}{\cos A}\right)^2} = \frac{1+\sin A}{\cos A}$$

$$\frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\sec A + \tan A$$

vii. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$\Rightarrow \frac{\sin\theta\,(1 - \sin^2\theta - \sin^2\theta)}{\cos\theta\,(\cos^2\theta + \cos^2\theta - 1)}$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta}\left(\frac{\cancel{\cos^2\theta - \sin^2\theta}}{\cos^2\theta - (1 - \cos^2\theta)}\right) = \tan\theta$$

$$\cancel{\cos^2\theta - (\sin^2\theta)}$$

viii. $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$

Solu:

$\sin^2 A + \text{cosec}^2 A + 2\,\cancel{\sin A \times \text{cosec} A}$

$+ \cos^2 A + \sec^2 A + 2\,\cancel{\sec A \cos A}$

$1 + 2 + 2 + \sec^2 A + \text{cosec}^2 A$

$5 + 1 + \tan^2 A + 1 + \cot^2 A$

$7 + \tan^2 A + \cot^2 A$

ix. $(\text{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

$$\Rightarrow \left[\frac{1}{\sin A} - \sin A\right]\left[\frac{1}{\cos A} - \cos A\right]$$

$$\left[\frac{1 - \sin^2 A}{\sin A}\right]\left[\frac{1 - \cos^2 A}{\cos A}\right]$$

$$= \frac{\cos^2 A}{\cancel{\sin A}} \times \frac{\sin^2 A}{\cancel{\cos A}}$$

$$= \sin A \times \cos A$$

LHS = RHS

$$\frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$$\frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}}$$

$$\frac{1}{\dfrac{1}{\sin A \cdot \cos A}}$$

$$= \sin A \cos A$$

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x. $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Solu:

$$\frac{1+\tan^2 A}{1+\cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1/\cos^2 A}{1/\sin^2 A} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2$$

$$\left[\frac{1-\tan A}{1-\frac{1}{\tan A}}\right]^2$$

$$\left[\frac{(1-\tan A)\tan A}{\tan A - 1}\right]^2$$

$$\left[\frac{\cancel{(1-\tan A)}\tan A}{-\cancel{(1-\tan A)}}\right]^2$$

$$[-\tan A]^2 = \tan^2 A$$