

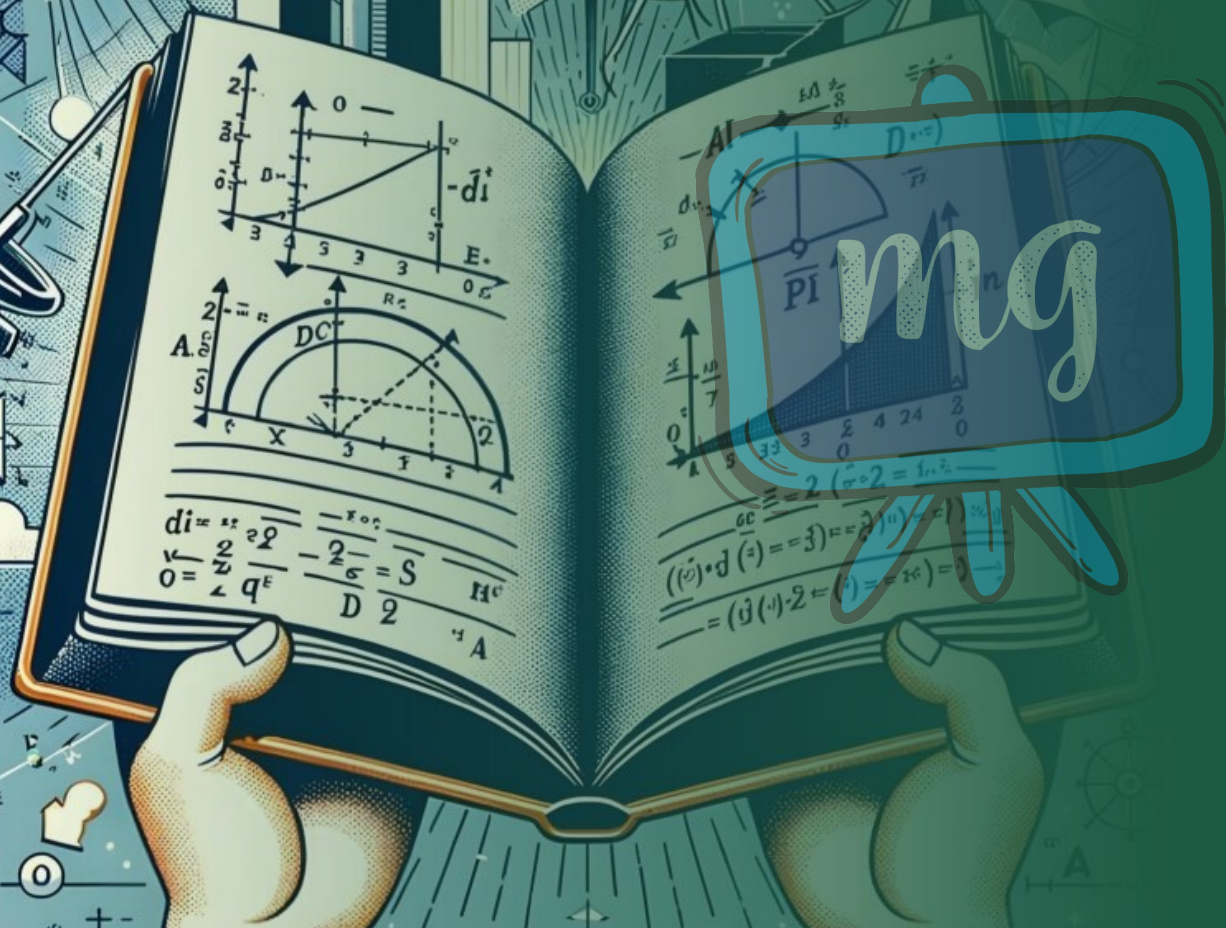
# CLASS – 10 MATHEMATICS

## Chapter – 8

### Introduction to Trigonometry

Part – 2

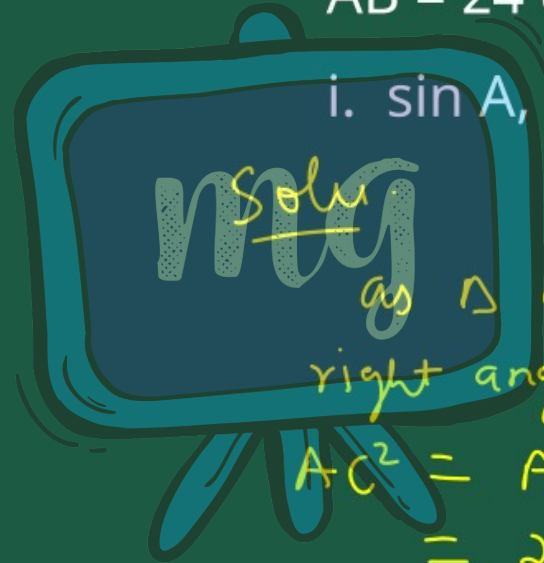
EXERCISE 8.1



# EXERCISE 8.1

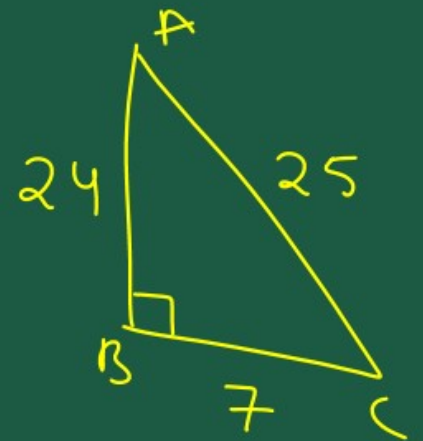
1. In  $\triangle ABC$ , right-angled at B,  
AB = 24 cm, BC = 7 cm. Determine :

i.  $\sin A$ ,  $\cos A$



as  $\triangle ABC$  is  
right angle at B

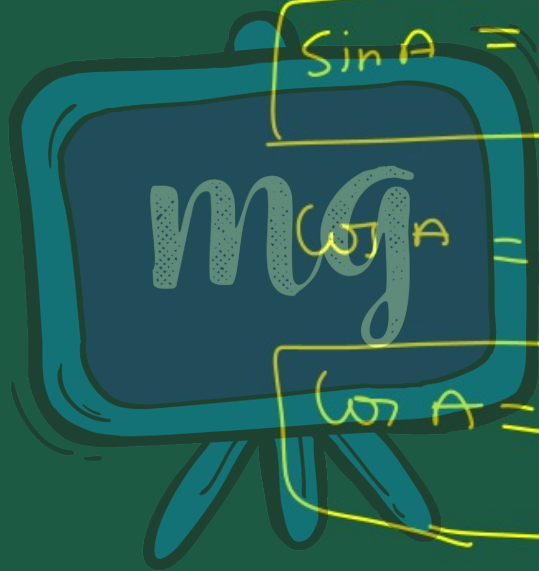
$$\begin{aligned}AC^2 &= AB^2 + BC^2 \\ &= 24^2 + 7^2 \\ &= 576 + 49 \\ AC^2 &= 625\end{aligned}$$



$$AC = 25$$

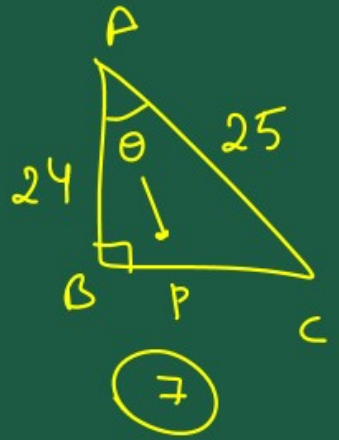
$$\sin A = \frac{P}{H} = \frac{BC}{AC}$$

$$\sin A = \frac{7}{25}$$



$$\cos A = \frac{B}{H} = \frac{AB}{AC}$$

$$\cos A = \frac{24}{25}$$



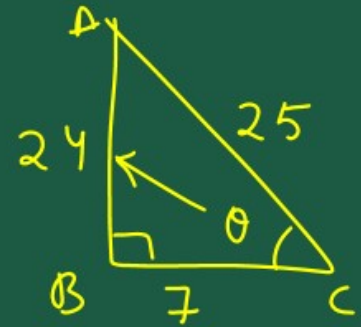
## ii. sin C, cos C

$$\sin C = \frac{P}{H} = \frac{AB}{AC}$$

$$\sin C = \frac{24}{25}$$

$$\cos C = \frac{B}{H} = \frac{BC}{AC}$$

$$\cos C = \frac{7}{25}$$



2. In Fig, find tan P - cot R.

Solu.  
In  $\Delta PQR$

$$PR^2 = PQ^2 + QR^2$$

$$(13)^2 = (12)^2 + QR^2$$

$$13^2 - 12^2 = QR^2$$

$$(13-12)(13+12) = QR^2$$

$$1 \times 25$$

$$25 = QR^2$$

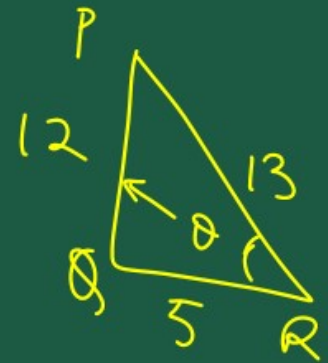
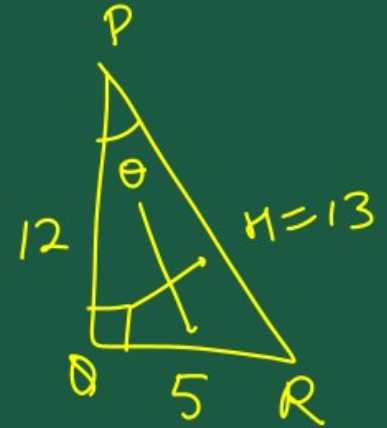
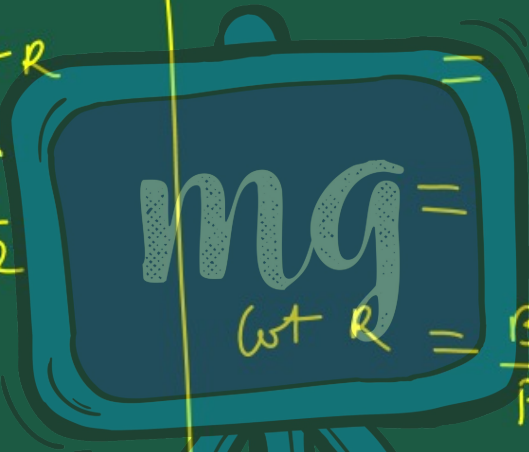


$$QR = 5$$

hence

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

$\tan P = \frac{P}{B} = \frac{QR}{PQ} = \frac{5}{12}$   
 $\cot R = \frac{B}{P} = \frac{QR}{PQ} = \frac{5}{12}$



3. If  $\sin A = \frac{3}{4}$ , calculate  $\cos A$  and  $\tan A$

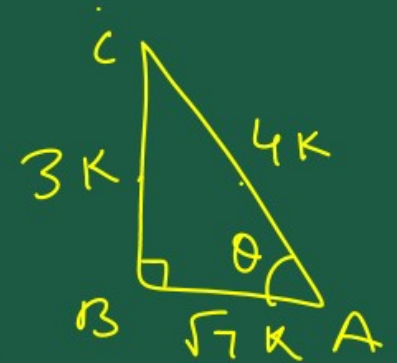
Soln.

$\sin A = \frac{3}{4}$

$\frac{p}{H} = \frac{3}{4}$

$p = 3K$

$H = 4K$



in  $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

$$(4K)^2 = AB^2 + (3K)^2$$

$$16K^2 - 9K^2 = AB^2$$

$$16K^2 - 9K^2 = AB^2$$

$$7K^2 = AB^2$$

$$\sqrt{7} K = AB$$

$$\cos A = \frac{B}{H} = \frac{AB}{AC}$$

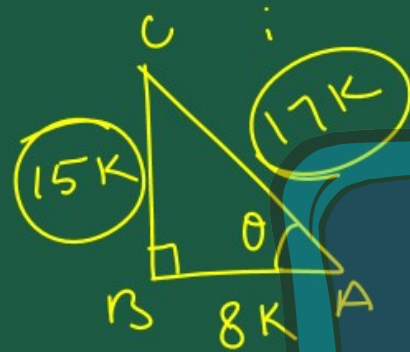
$$\cos A = \frac{\sqrt{7} K}{4K}$$
$$= \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{P}{B} = \frac{BC}{AB}$$

$$\tan A = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$$

$$\tan A = \frac{3\sqrt{7}}{7}$$

4. Given 15 cot A = 8, find sin A and sec A.



Solu.

$$AC^2 = AB^2 + BC^2$$

$$= (8K)^2 + (15K)^2$$

$$= 64K^2 + 225K^2$$

$$AC^2 = 289K^2$$

$$15 \cot A = 8$$

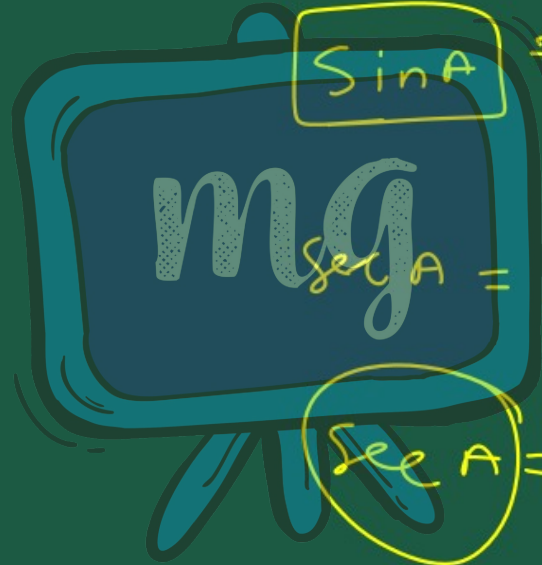
$$\cot A = \frac{8}{15}$$

$$\frac{B}{P} = \frac{8}{15}$$

$$\left. \begin{array}{l} B = 8K \\ P = 15K \end{array} \right\} \begin{array}{l} AC = 17K \end{array}$$

$$\sin A = \frac{P}{H} = \frac{BC}{AC}$$

$$\sin A = \frac{15K}{17K} = \frac{15}{17}$$



$$\sec A = \frac{H}{B} = \frac{AC}{AB}$$

$$\sec A = \frac{17K}{8K} = \frac{17}{8}$$

5. Given  $\sec \theta = \frac{13}{12}$ , calculate all other trigonometric ratios.

Solve:

$\sec \theta = \frac{13}{12}$

in  $\Delta ABC$

$$AC^2 = AB^2 + BC^2$$

$$(13K)^2 = AB^2 + (12K)^2$$

$$(13K)^2 - (12K)^2 = AB^2$$

$$169K^2 - 144K^2 = AB^2$$

$$\underline{25K^2 = AB^2}$$

$\frac{H}{B} = \frac{13}{12}$

$H = 13K$

$B = 12K$

$AB = P = 5K$



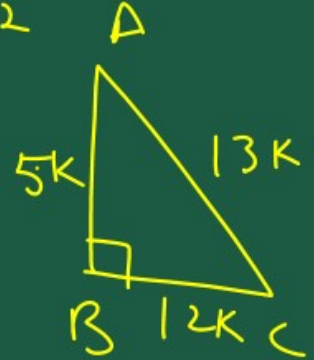
$$\sin A = \frac{P}{H} = \frac{AB}{AC} = \frac{5K}{13K} = \frac{5}{13}$$

$$\cos A = \frac{B}{H} = \frac{BC}{AC} = \frac{12K}{13K} = \frac{12}{13}$$

$$\tan A = \frac{P}{B} = \frac{AB}{BC} = \frac{5K}{12K} = \frac{5}{12}$$

$$\cot A = \frac{B}{P} = \frac{BC}{AB} = \frac{12}{5}$$

$$\operatorname{cosec} A = \frac{H}{P} = \frac{AC}{AB} = \frac{13}{5}$$



6. If  $\angle A$  and  $\angle B$  are acute angles such that  $\cos A = \cos B$ , then show that

$$\angle A = \angle B.$$

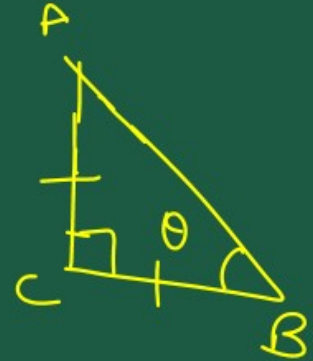
Soln

$$\cos A = \cos B$$

$$\frac{AC}{AB} = \frac{BC}{AB}$$

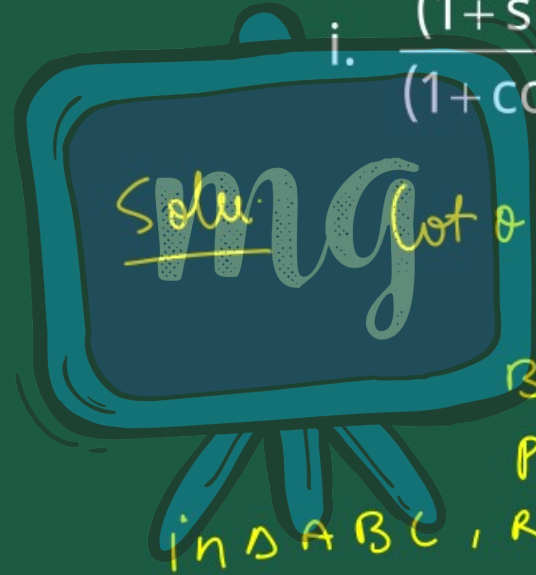
$$AC = BC$$

$\angle A = \angle B$  } angle opposite to  
equal sides are equal



7. If  $\cot \theta = \frac{7}{8}$ , evaluate :

i. 
$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$



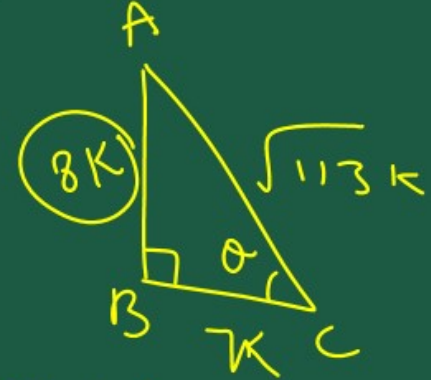
Soln:  $\cot \theta = \frac{7}{8} = \frac{B}{P}$

$$B = 7K$$

$$P = 8K$$

In  $\triangle ABC$ , Right angled at B

$$AC^2 = AB^2 + BC^2$$



$$AC^2 = AB^2 + BC^2$$

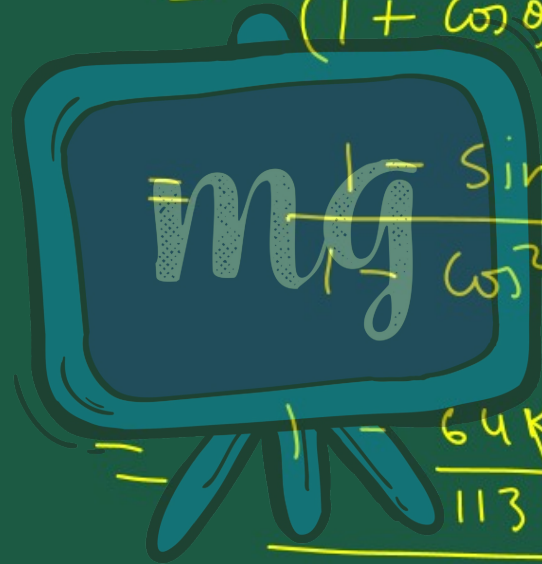
$$AC^2 = (8K)^2 + (7K)^2$$

$$= 64K^2 + 49K^2$$

$$AC^2 = 113K^2$$

$$AC = \sqrt{113} K$$

$$= \frac{(1 - \sin \theta)(1 + \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$




$$= \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{1 - \left[ \frac{8K}{\sqrt{113}K} \right]^2}{1 - \left[ \frac{7K}{\sqrt{113}K} \right]^2}$$

$$= \frac{1 - \frac{64K}{113K}}{1 - \frac{49K}{113K}}$$

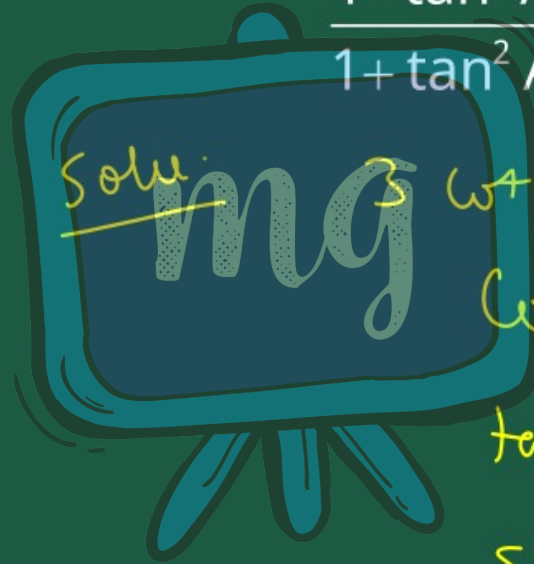
$$= \frac{113 - 64}{113 - 49} = \frac{49}{64}$$

ii.  $\cot^2 \theta$


$$\cot \theta = \frac{7}{8}$$
$$\cot^2 \theta = \left(\frac{7}{8}\right)^2$$
$$= \frac{49}{64}$$

8. If  $3 \cot A = 4$ , check whether

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A \text{ or not.}$$



Solu.

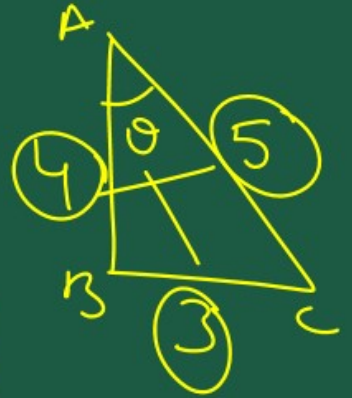
$$3 \cot A = 4$$

$$\cot A = \frac{4}{3}$$

$$\tan A = \frac{3}{4}$$

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$



$$\frac{1 - \tan^2 A}{1 + \tan^2 A}$$
$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$
$$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$
$$= \frac{7}{25}$$
$$\cos^2 A - \sin^2 A$$
$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25}$$
$$= \frac{16-9}{25}$$
$$= \frac{7}{25}$$

Hence L.H.S. = R.H.S.

9. In triangle ABC, right-angled at B,  
if  $\tan A = \frac{1}{\sqrt{3}}$ , find the value of:

i.  $\sin A \cos C + \cos A \sin C$

in  $\Delta ABC$

$$AC^2 = AB^2 + BC^2$$

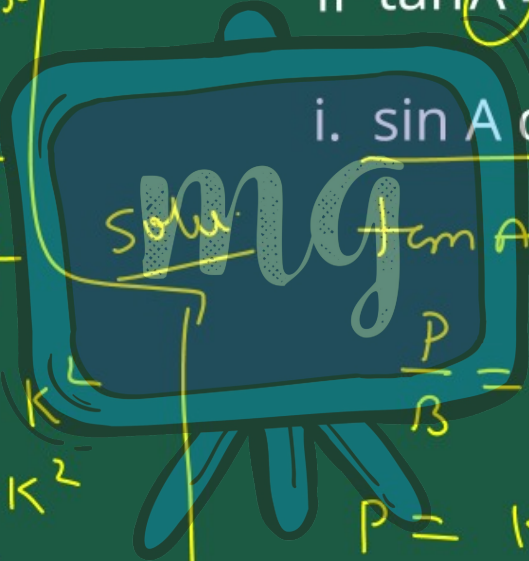
$$AC^2 = (\sqrt{3}K)^2 + K^2$$

$$= 3K^2 + K^2$$

$$AC^2 = 4K^2$$


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$$AC = 2K$$



Solu.

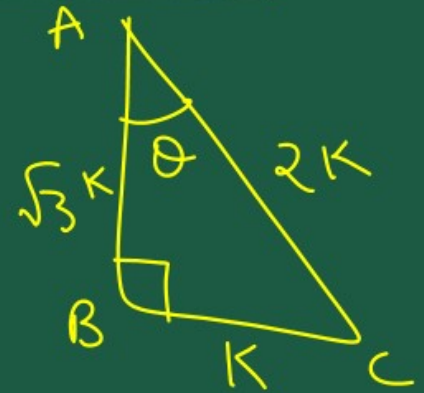
$$\tan A = \frac{1}{\sqrt{3}}$$

$$\frac{P}{B} = \frac{1}{\sqrt{3}}$$

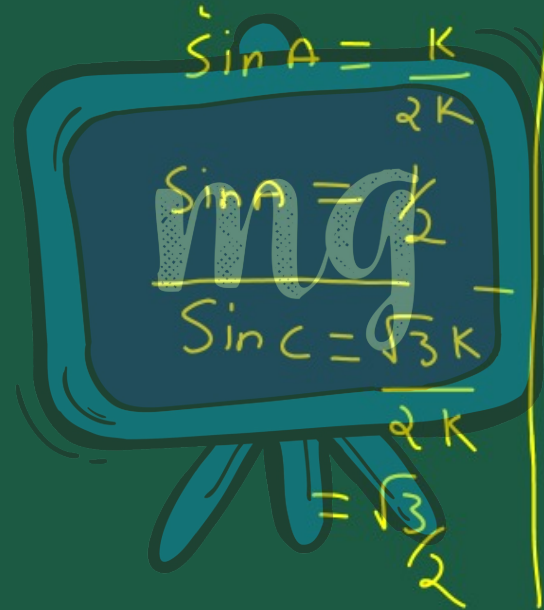
$$P = K$$

$$B = \sqrt{3}K$$

$$H = 2K$$



$$(1) \sin A \cos C + \cos A \sin C$$



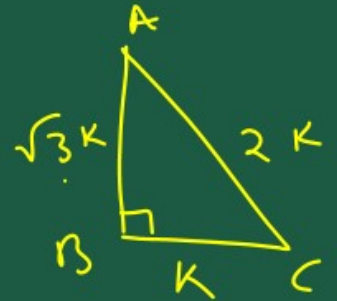
$\sin A = \frac{K}{2K}$   
 $\sin A = \frac{1}{2}$   
 $\sin C = \frac{\sqrt{3}K}{2K}$   
 $= \frac{\sqrt{3}}{2}$

$$\cos A = \frac{\sqrt{3}K}{2K}$$

$$\cos A = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{K}{2K}$$

$$= \frac{1}{2}$$

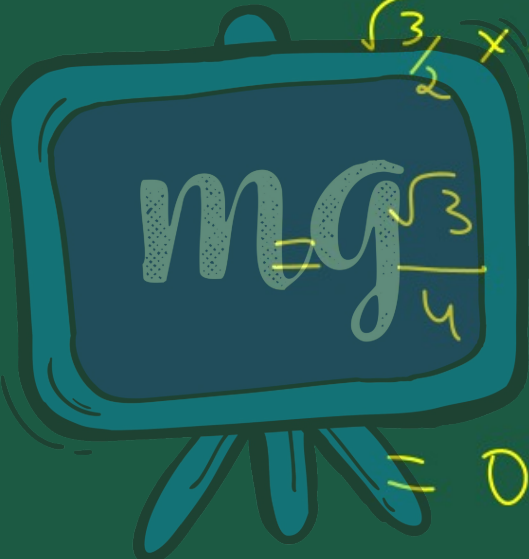


$$\sin A \cos C + \cos A \sin C$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$


$$= \frac{1}{4} + \frac{\sqrt{3} \times \sqrt{3}}{4}$$
$$= \frac{1}{4} + \frac{3}{4}$$

ii. cos A cos C - sin A sin C



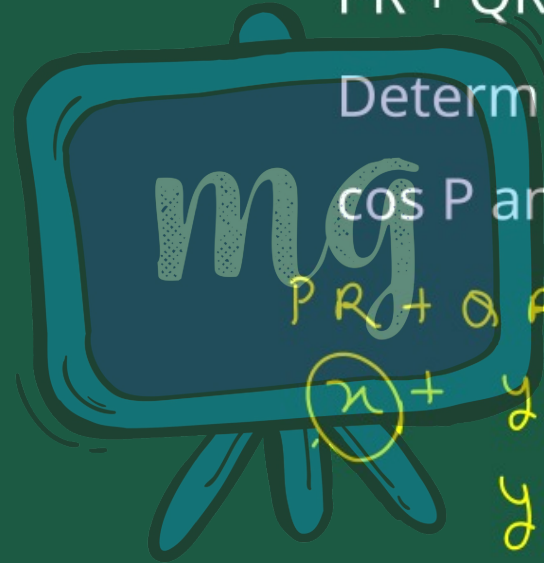
$\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= 0$

10. In  $\triangle PQR$ , right-angled at  $Q$ ,  
 $PR + QR = 25$  cm and  $PQ = 5$  cm.

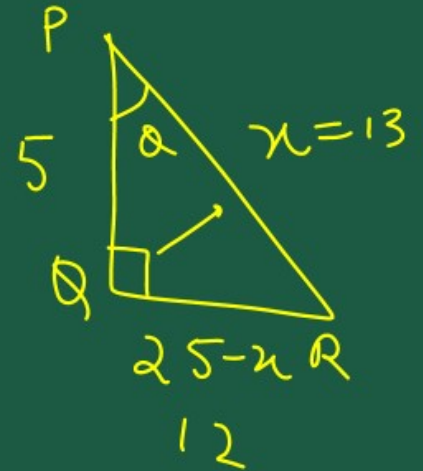
Determine the values of  $\sin P$ ,  
 $\cos P$  and  $\tan P$ .



$$PR + QR = 25$$

$$x + y = 25$$

$$y = 25 - x$$



in  $\triangle POR$ ,  $\angle O = 90^\circ$

$$PR^2 = PO^2 + OR^2$$

$$(x)^2 = 5^2 + (25 - x)^2$$

$$x^2 = 25 + 25^2 + x^2 - 50x$$

$$50x = 25 + 25^2$$

$$2x = 1 + 25$$

$$2x = 26$$

$$x = 13$$

$$\sin P = \frac{P}{H} = \frac{12}{13}$$

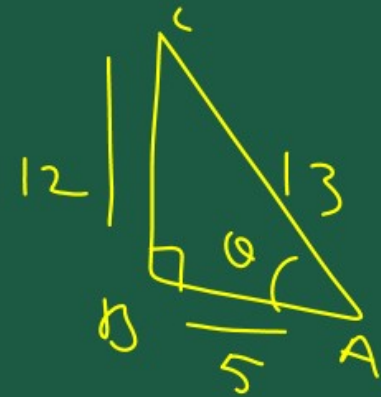
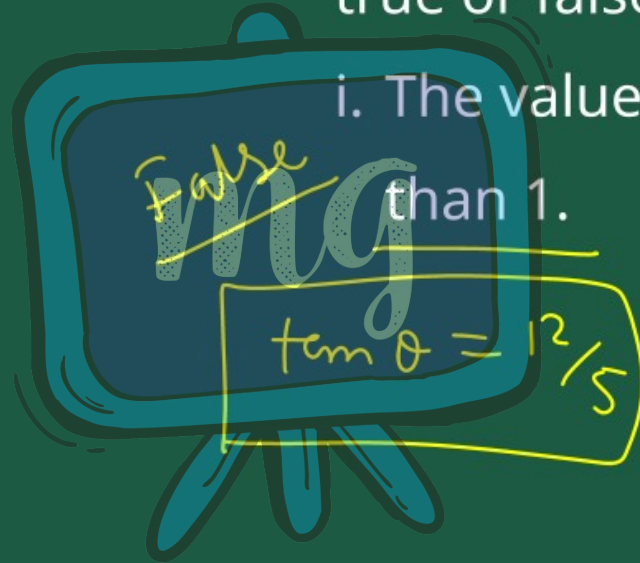


$$\cos P = \frac{B}{H} = \frac{5}{13}$$

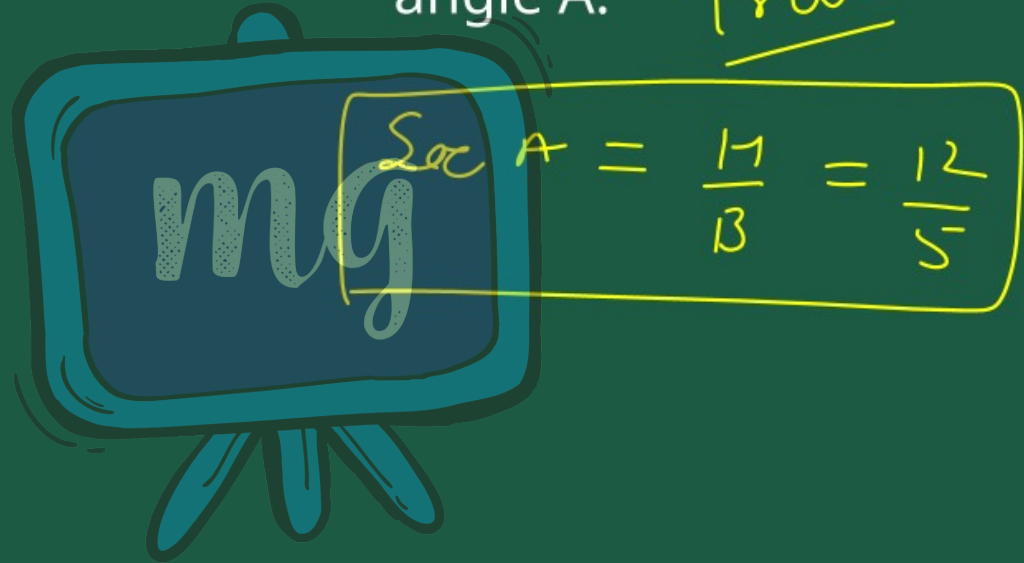
$$\tan P = \frac{P}{B} = \frac{12}{5}$$

11. State whether the following are true or false. Justify your answer.

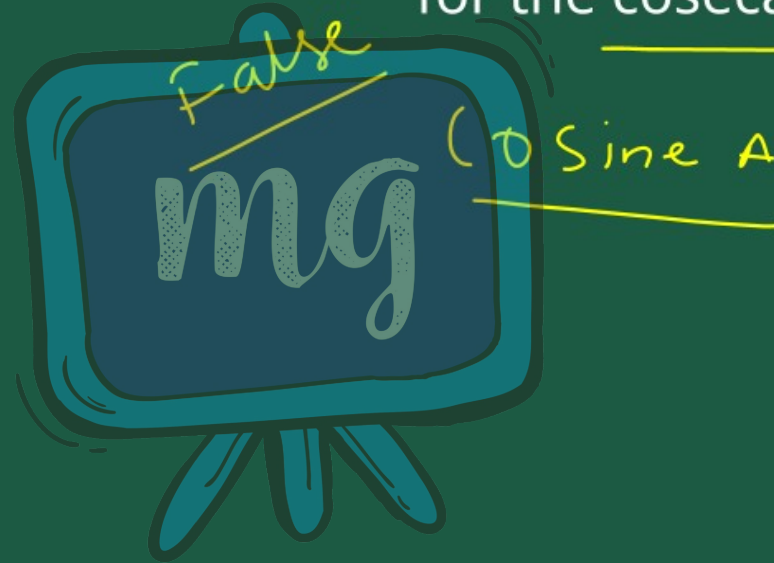
i. The value of  $\tan A$  is always less than 1.



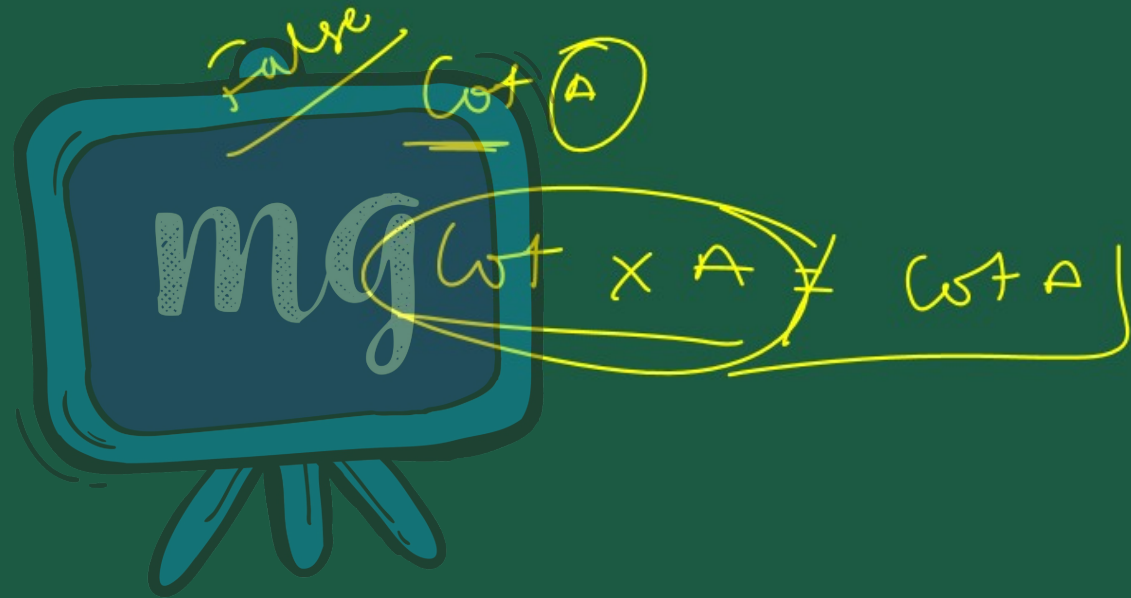
ii.  $\sec A = \frac{12}{5}$  for some value of  
angle A. True



iii. cos A is the abbreviation used  
for the cosecant of angle A.



iv.  $\cot A$  is the product of  $\cot$  and  $A$ .



v.  $\sin\theta = \frac{4}{5}$  for some angle  $\theta$ .

