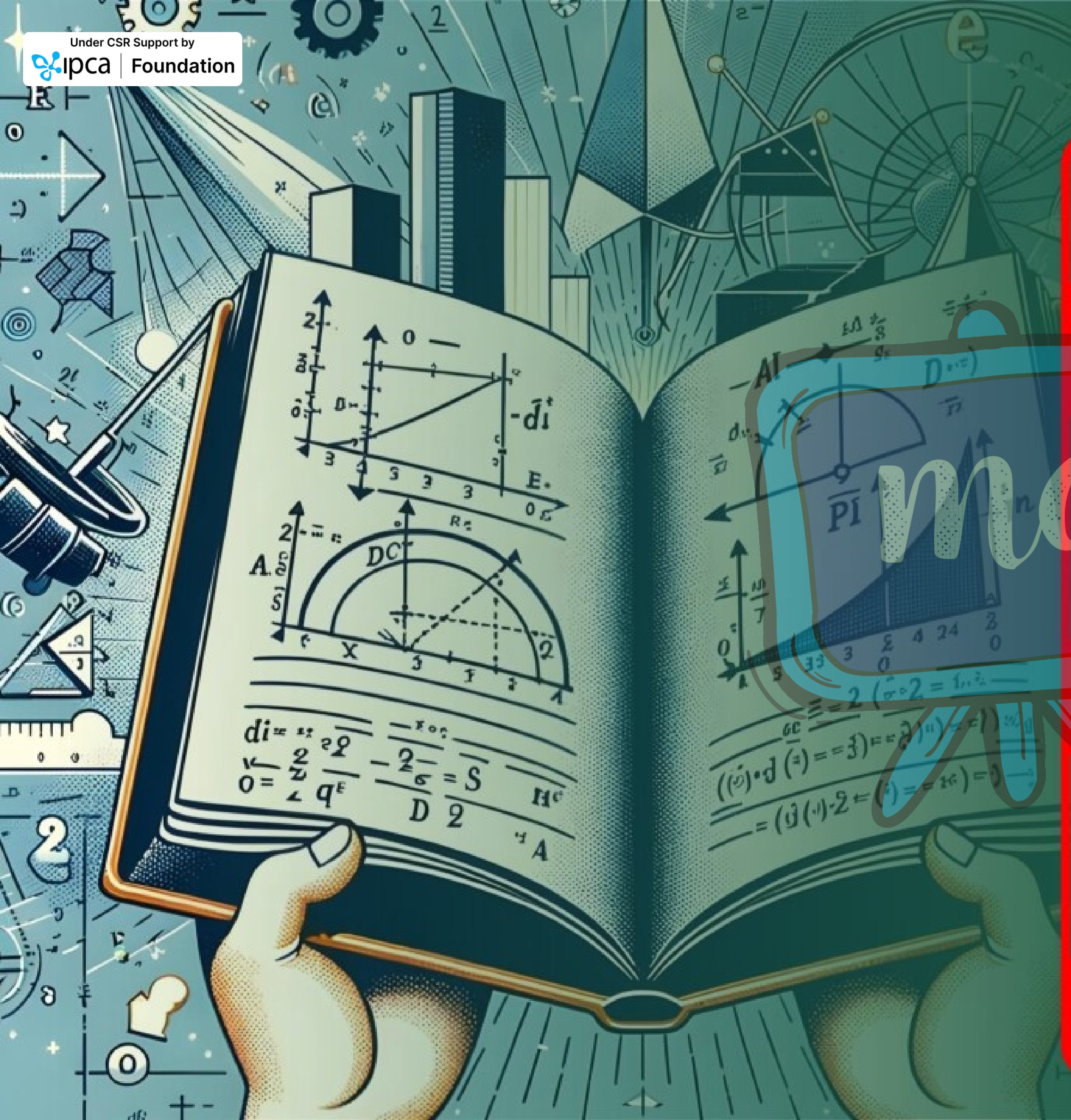

CLASS – 10

MATHEMATICS

CH – 8 : Introduction to Trigonometry

CBSE Board

Previous Year Questions – 3

Shubham Tiwari

1. Prove that

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$$

(CBSE 2023, 2018)

Soln Solving LHS

$$\frac{\sin A\left[1 - 2\sin^2 A\right]}{\cos A\left[2\cos^2 A - 1\right]}$$

$$= \frac{\sin A\left[1 - \sin^2 A - \sin^2 A\right]}{\cos A\left[\cos^2 A + \cos^2 A - 1\right]}$$

$$= \frac{\sin A\left[1 - \sin^2 A - \sin^2 A\right]}{\cos A\left[\cos^2 A - \left[1 - \cos^2 A\right]\right]}$$

$$= \frac{\sin A\left[\cancel{\cos^2 A - \sin^2 A}\right]}{\cos A\left[\cancel{\cos^2 A - \sin^2 A}\right]}$$

$$= \tan A$$

2. Prove that

$$\sec A\,(1 - \sin A)\,(\sec A + \tan A) = 1.$$

(CBSE 2023)

Solu: $(\sec A - \sec A \times \sin A)(\sec A + \tan A)$

$= \left[\sec A - \frac{1}{\cos A} \times \sin A\right](\sec A + \tan A)$

$\left[\sec A - \tan A\right]\left[\sec A + \tan A\right)$

$(\sec^2 A - \tan^2 A) = 1$

3. Prove that

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\cot A + \tan A}$$

(CBSE 2023)

Solu

$$\left[\frac{1}{\sin A} - \sin A\right]\left[\frac{1}{\cos A} - \cos A\right]$$

$$\left[\frac{1 - \sin^2 A}{\sin A}\right]\left[\frac{1 - \cos^2 A}{\cos A}\right]$$

$$\left(\frac{\cos^2 A}{\cancel{\sin A}}\right)\left(\frac{\sin^2 A}{\cancel{\cos A}}\right) = \sin A \cos A$$

LHS = RHS

Solving RHS

$$\frac{1}{\cot A + \tan A} = \frac{1}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$$

$$= \frac{1}{\frac{\cos^2 A + \sin^2 A}{\sin A \cdot \cos A}}$$

$$= \frac{1}{1/\sin A \cdot \cos A} = \sin A \cdot \cos A$$

4. Show that

$$\sin^6 A + 3\sin^2 A\cos^2 A = 1 - \cos^6 A$$

(CBSE 2021)

Soln. from the Identity

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(\sin^2\theta + \cos^2\theta)^3$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$1 = \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cdot\cos^2\theta)$

$1 - \cos^6\theta = \sin^6\theta + 3\sin^2\theta\cdot\cos^2\theta)$

5. Prove that

$$\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$$

(CBSE 2020)

Soln.

$$= \frac{\sec\theta - \tan\theta + 1}{\sec\theta + \tan\theta + 1}$$

$$= \frac{\sec\theta - \tan\theta + (\sec^2\theta - \tan^2\theta)}{\sec\theta + \tan\theta + 1}$$

$$= \frac{(\sec\theta - \tan\theta) + (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\sec\theta + \tan\theta + 1}$$

$$= \frac{(\sec\theta - \tan\theta)\cancel{(1 + \sec\theta + \tan\theta)}}{\cancel{(\sec\theta + \tan\theta + 1)}}$$

$$= \sec\theta - \tan\theta$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)$$

6. Show that

$$\frac{1+\tan A}{2\sin A}+\frac{1+\cot A}{2\cos A}=\operatorname{cosec} A+\sec A$$

(CBSE 2020)

Soln.

$$\frac{1}{2\sin A}+\frac{\cancel{\sin A}/\cos A}{2\cancel{\sin A}}+\frac{1}{2\cos A}+\frac{\cancel{\cos A}/\sin A}{2\cancel{\cos A}}$$

$$\frac{\operatorname{cosec} A}{2}+\frac{1}{2}\sec A+\frac{1}{2}\sec A+\frac{1}{2}\operatorname{cosec} A$$

$$\operatorname{cosec} A\left[\frac{1}{2}+\frac{1}{2}\right]+\sec A\left[\frac{1}{2}+\frac{1}{2}\right]$$

$$\operatorname{cosec} A\,(1)+\sec A\,(1)$$

7. Show that

$$(\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta = 2$$

(CBSE 2020)

Soln

$$\left[(\sin^4\theta - \cos^4\theta) + 1\right]\operatorname{cosec}^2\theta)$$

$$\left[(\sin^2\theta)^2 - (\cos^2\theta)^2 + 1\right]\operatorname{cosec}^2\theta$$

$$\left[(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) + 1\right]\operatorname{cosec}^2\theta$$

$$\left[(\sin^2\theta - \cos^2\theta) + 1\right]\operatorname{cosec}^2\theta)$$

$$= (\sin^2\theta - \cos^2\theta + 1]\ \mathrm{cosec}^2\theta$$

$$= \cancel{\sin^2\theta \times \mathrm{cosec}^2\theta} - \cos^2\theta \times \mathrm{cosec}^2\theta + \mathrm{cosec}^2\theta$$

$$= 1 - \cos^2\theta \times \frac{1}{\sin^2\theta} + \mathrm{cosec}^2\theta]$$

$$\left[1 \ \underbrace{\boxed{- \cot^2\theta + \mathrm{cosec}^2\theta}}_{1}\right]$$

$$(1 + 1)$$

$$= 2$$

8. Prove that

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

(CBSE 2020)

Sol. $= \sqrt{\frac{1+\sin A}{(1-\sin A)} \times \frac{1+\sin A}{(1+\sin A)}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$$= \sqrt{\left(\frac{1+\sin A}{\cos A}\right)^{\cancel{2}}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \boxed{\sec A + \tan A}$$

9. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\boxed{\tan\theta + \cot\theta = 1.}$

(CBSE 2020)

Solu.

$\sin\theta + \cos\theta = \sqrt{3}$

$\tan\theta + \cot\theta = 1$

$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$

$\underbrace{\sin^2\theta + \cos^2\theta}_{1} + 2\sin\theta\cdot\cos\theta = 3$

$1 + 2\sin\theta\cos\theta = 3$

$2\sin\theta\cdot\cos\theta = 3-1$

$= 2$

$2\sin\theta \cdot \cos\theta = 2$

$\boxed{\sin\theta \cdot \cos\theta = 1}$

$\tan\theta + \cot\theta$

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}$

$= \frac{1}{\sin\theta \cdot \cos\theta}$

$= 1/1 = 1$

10. If $1+\sin^2\theta = 3\sin\theta\cos\theta$ then prove that $\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$

(CBSE 2019)

Solu.

$$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cdot\cancel{\cos\theta}}{\cos^2\theta}$$

$$\sec^2\theta + \tan^2\theta = 3\tan\theta$$

$$1+\tan^2\theta + \tan^2\theta - 3\tan\theta = 0$$

$$2\tan^2\theta - 3\tan\theta + 1 = 0$$

Let $\boxed{\tan\theta = a}$

$$2a^2 - 3a + 1 = 0$$

$$2a^2 - 2a - a + 1 = 0$$

$$2a[a-1] - 1[a-1]$$

$$(a-1)(2a-1)$$

$a = 1$ | $a = 1/2$

Hence $\tan\theta = 1$ | $\tan\theta = 1/2$

11. Prove that

$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0.$$

(CBSE 2023)

Solu

$$(\sin^2\theta + \cos^2\theta)^3 =$$

$$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cdot\cos^2\theta$$

$$1 = \underbrace{\sin^6\theta + \cos^6\theta}_{} + 3\sin^2\theta\,\cos^2\theta\,(\sin^2\theta+\cos^2\theta)$$

$$1 - 3\sin^2\theta\cdot\cos^2\theta = \sin^6\theta + \cos^6\theta \quad \text{---} \; (1)$$

$$(\sin^2\theta + \cos^2\theta)^2$$

$$1 = (\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta$$

$$1 = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$$

$$1 - 2\sin^2\theta\cdot\cos^2\theta = \sin^4\theta + \cos^4\theta \quad \text{--- (2)}$$

By placing the values of eq (1) and eq (2), we get

$$2(1 - 3\sin^2\theta\cdot\cos^2\theta) - 3(1 - 2\sin^2\theta\cos^2\theta) + 1$$

$$2 \cancel{- 6\sin^2\theta\cos^2\theta} - 3 \cancel{+ 6\sin^2\theta\cos^2\theta} + 1$$

$$2 - 3 + 1$$

$$3 - 3 = 0$$

6. Show that

$$\frac{1+\tan A}{2\sin A}+\frac{1+\cot A}{2\cos A}=\operatorname{cosec} A+\sec A$$

(CBSE 2020)

7. Show that

$$(\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta = 2$$

(CBSE 2020)

8. Prove that

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

(CBSE 2020)

9. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

(CBSE 2020)

10. If $1+\sin^2\theta = 3\sin\theta\cos\theta$ then prove that $\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$

(CBSE 2019)

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11. Prove that

$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0.$$

(CBSE 2023)