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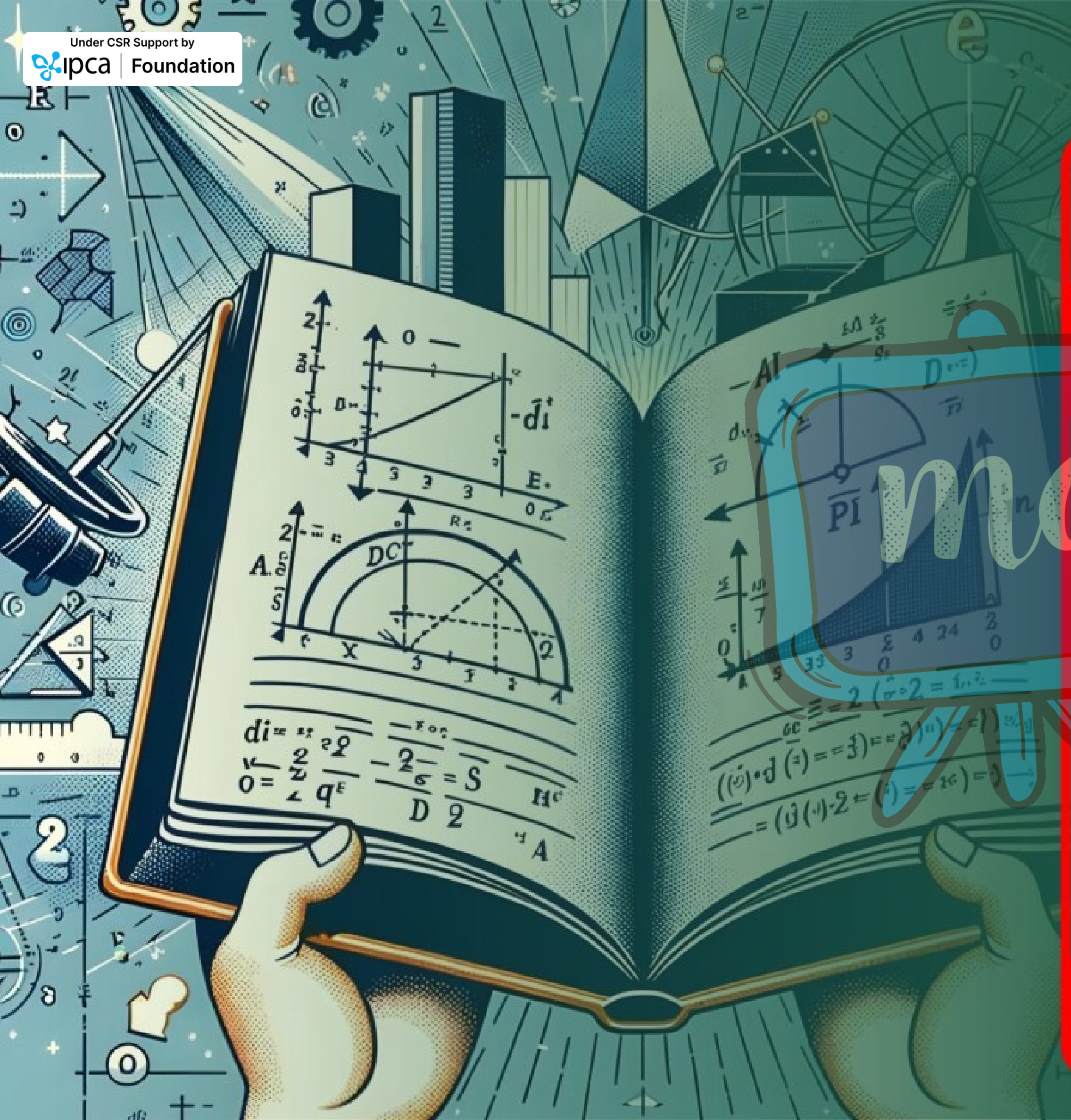


CLASS – 10

MATHEMATICS

Chapter – 8

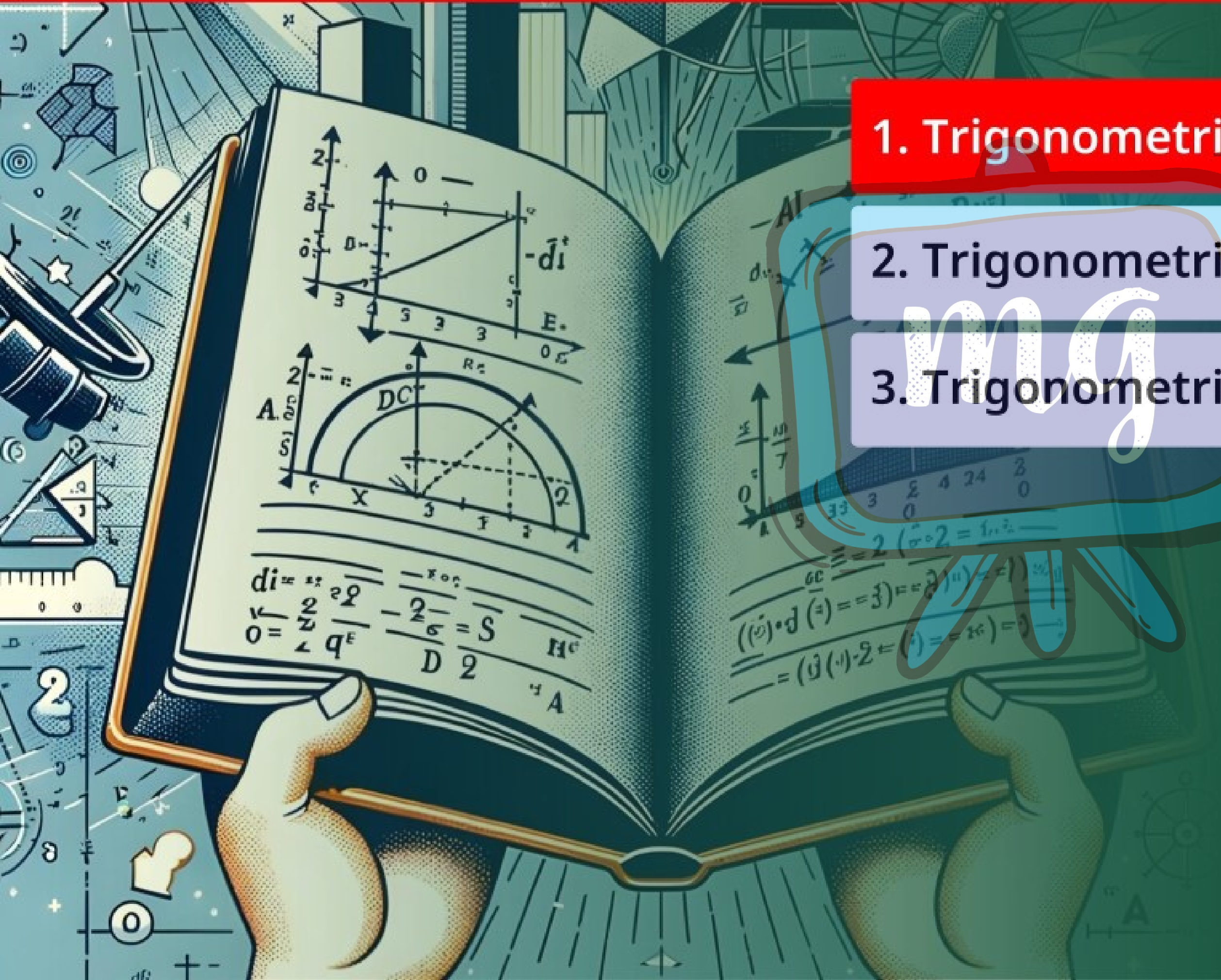
Introduction to Trigonometry

Part – 1

Trigonometric Ratios

Shubham Tiwari

OVERVIEW



1. Trigonometric Ratios
2. Trigonometric Ratios of Some Specific Angles
3. Trigonometric Identities

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COMPETENCY BASED LEARNING

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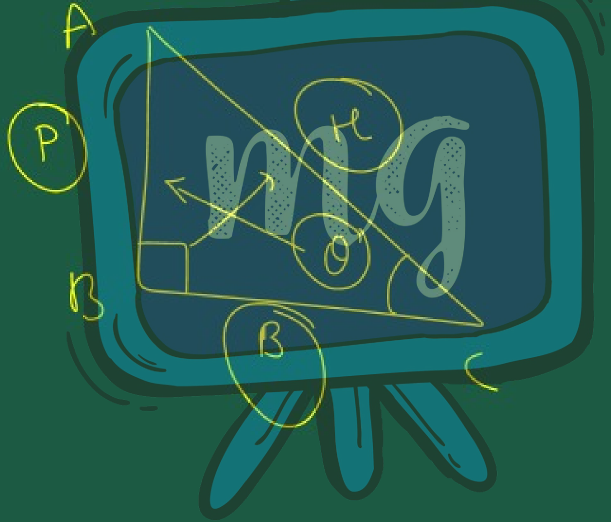
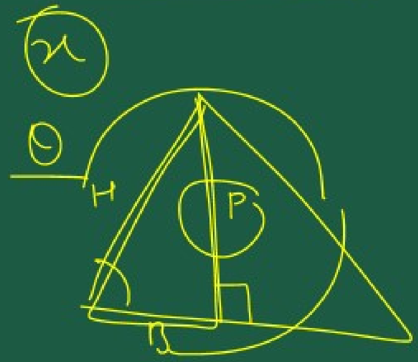
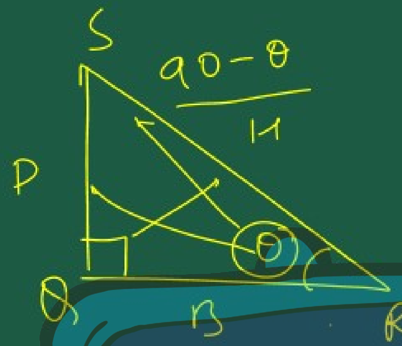
Relation of sides w.r.t. corresponding angles

Tri gon metry

3 sides measurement







$$p^2 + B^2 = H^2$$

$$\angle Q + \angle R + \angle S = 180$$

$$90 + \angle R + \angle S = 180$$

$$\boxed{\angle R + \angle S = 90}$$

$$\theta + \angle S = 90$$

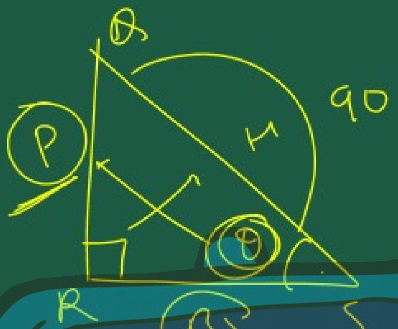
$$\boxed{\angle S = 90 - \theta}$$

A : B

2 : 5

5 : 2

P|Q



$p^2 + B^2 = H^2$

$20 + 5 = 25$

$16 + 9 =$

$15 + 10 =$

$10 + 15 = 25$

$\frac{P}{H} =$ sine θ

$\frac{B}{H} =$ cosine θ

$\frac{P}{B} =$ tangent θ

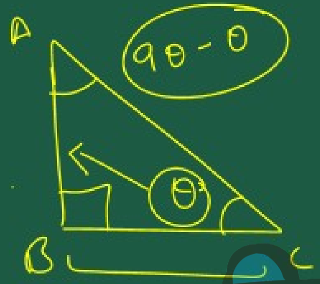
$\sin\theta = \frac{P}{\cancel{H}}$

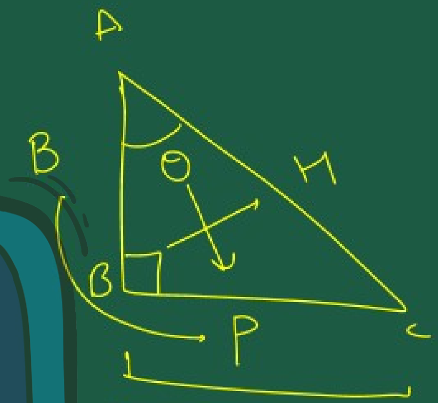
$\cos\theta = \frac{B}{\cancel{H}}$

P/B

$\tan\theta = P/B$

$\boxed{\tan\theta = \sin\theta/\cos\theta}$





$$\boxed{\sin\theta = \frac{AB}{AC}}$$

$$\sin(90-\theta) = \frac{BC}{AC}$$

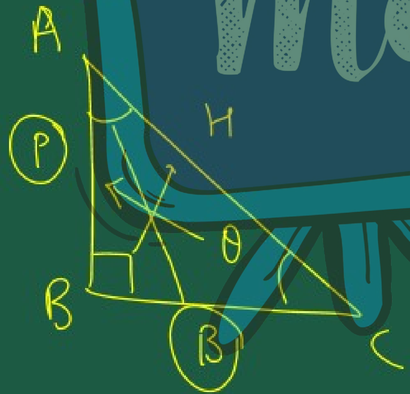
$$\text{Cosine}(\theta) = \frac{BC}{AC}$$

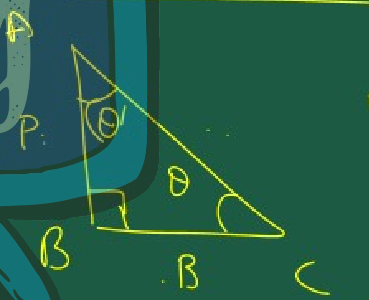
$$\boxed{\sin\theta = \frac{BC}{AC}}$$

$\frac{P}{H} = \sin\theta$ $\qquad$ $H/P = \operatorname{cosec}\theta$

$\frac{B}{H} = \cos\theta$ $\qquad$ $H/B = \sec\theta$

$\frac{P}{B} = \tan\theta$ $\qquad$ $B/P = \cot\theta$





$\tan\theta = \frac{AB}{BC}$

$\tan\theta' = \frac{BC}{AB}$

$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$
$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\frac{\cos\theta}{\sin\theta} = \cot\theta$

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

TRIGONOMETRY

Trigonometry is the branch of mathematics which deals with measurements of angles and the problems related to relationship between sides and angles.

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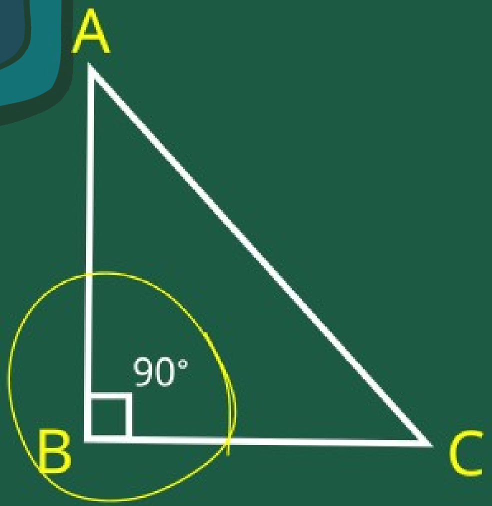
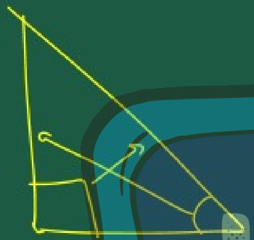
RIGHT ANGLED TRIANGLE

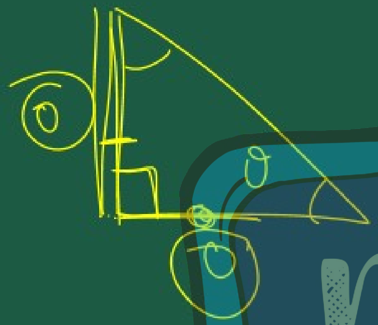
A triangle which has any one of the interior angle equal to 90° is called Right angled triangle



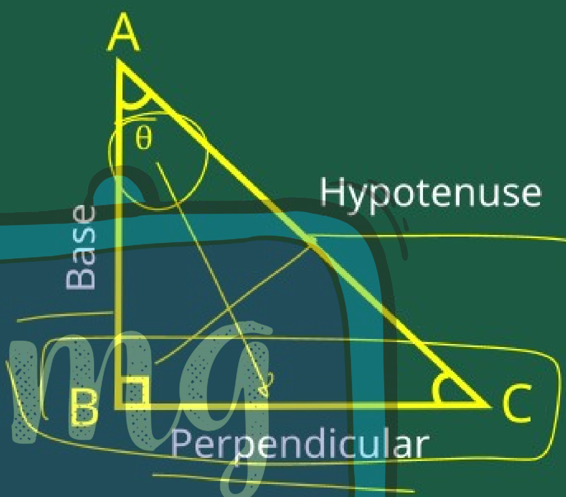


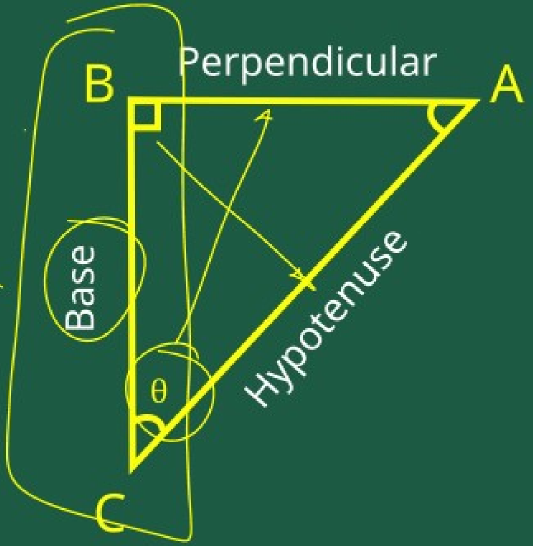
PROPERTIES OF RIGHT ANGLED TRIANGLE

- One angle is always 90° or right angle.
- The side opposite angle of 90° is the hypotenuse.
- The hypotenuse is always the longest side.



- The sum of the other two interior angles is equal to 90°.
- Perpendicular is the side that makes right angle with the base of the triangle.
- A right angle triangle's base is one of the sides that adjoins the 90° angle.



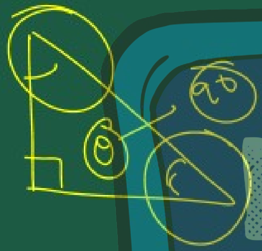


Here θ is the angle into consideration

Angles	Hypotenuse	Perpendicular	Base
for ∠A	AC	BC	AB
for ∠C	AC	AB	BC

TRIGONOMETRIC RATIOS

$\sin\theta$



The ratio of the sides of a right angle triangle with respect to acute angles are called "Trigonometric ratios of the angle".

1.	sine of θ / $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$
2.	cosine of θ / $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$
3.	tangent of θ / $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

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4.	cotangent of θ / $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$
5.	secant of θ / $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$
6.	cosecant of θ / $\operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

Trick

$\sin\theta$	$\cos\theta$	$\tan\theta$
P	B	P
H	H	B
cosec	sec	cot

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RELATION BETWEEN TRIGONOMETRIC RATIO

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \Rightarrow \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta}$$

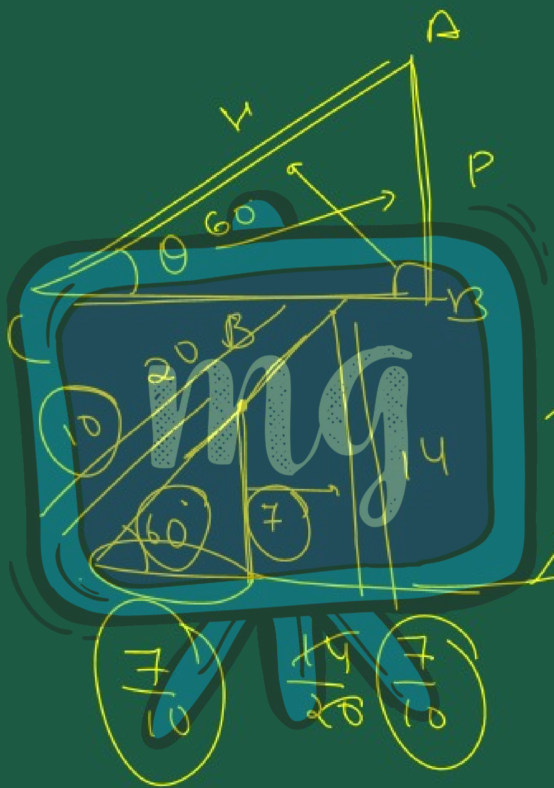
QUOTIENT RELATION

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

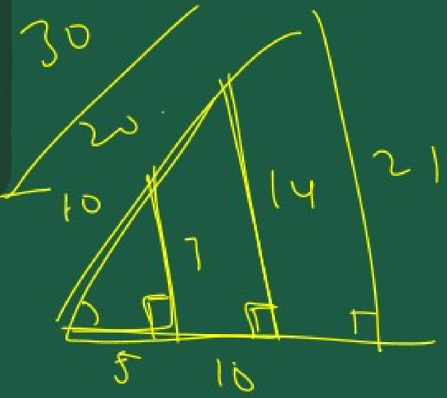
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

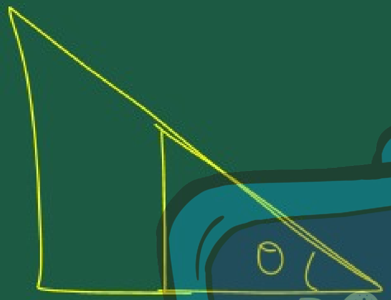
Note

- The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.



$\sin\theta = \frac{P}{H}$





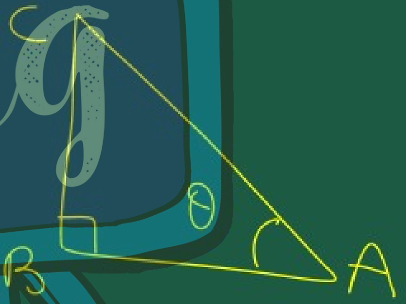
$\sin\theta = \frac{7}{10}$

- The value of each of the trigonometric ratio of an angle does not depend on the size of the triangle. It depends only on the angle.
- Each trigonometric ratio is a real number and has no unit.

Example : 1

Given $\tan A = \frac{4}{3}$, find the other trigonometric ratios of the angle A.

Soln



$\tan A = \frac{4}{3}$

$\frac{P}{B} = \frac{4 \times K}{3 \times K}$

$P = 4K$

$B = 3K$

$\sin\theta = \frac{P}{H} = \frac{4K}{5K} = \frac{4}{5}$

$\cos\theta = B/H = \frac{3\cancel{K}}{5\cancel{K}} = 3/5$

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$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = 3/4$$

$$P^2 + B^2 = H^2$$

$$(4K)^2 + (3K)^2 = H^2$$

$$16K^2 + 9K^2 = H^2$$

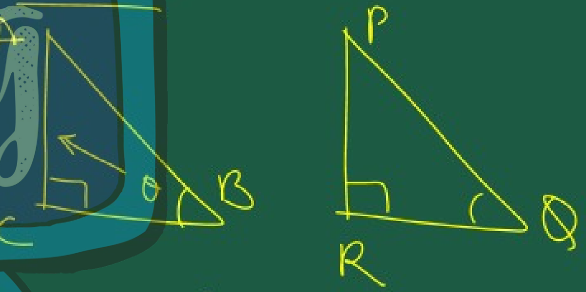
$$25K^2 = H^2$$

$$\boxed{5K = H}$$

Example : 2

If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.

Soln



$\sin B = \sin Q$

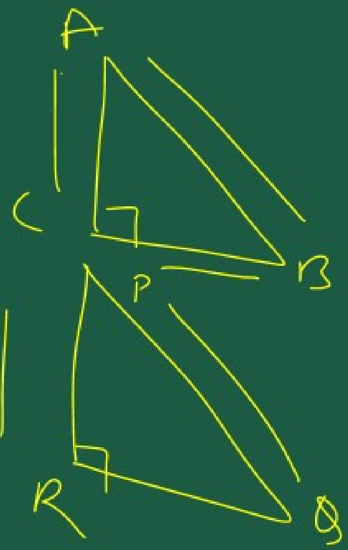
$\frac{AC}{AB} = \frac{PR}{PQ}$

$\left[\frac{AC}{PR} = \frac{AB}{PQ} = K \right]$

$$\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$$

$$\frac{AC}{PR} = K \qquad \frac{AB}{PQ} = K$$

$$AC = K(PR) \qquad AB = K(PQ)$$



$$\frac{BC}{QR} = \sqrt{\frac{AB^2 - AC^2}{PQ^2 - PR^2}}$$

$$= \sqrt{\frac{(KPQ)^2 - (KPR)^2}{PQ^2 - PR^2}}$$

$$\sqrt{\frac{K^2 PQ^2 - K^2 PR^2}{PQ^2 - PR^2}}$$

$$K\sqrt{\frac{\cancel{PQ^2 - PR^2}}{\cancel{PQ^2 - PR^2}}}$$

$$\frac{BC}{QR} = K \times 1 = K$$

$$\left.\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}\right\}$$ By SSS Similarity Rule

$\triangle ABC \sim \triangle PQR$

Hence $\angle B = \angle Q$.

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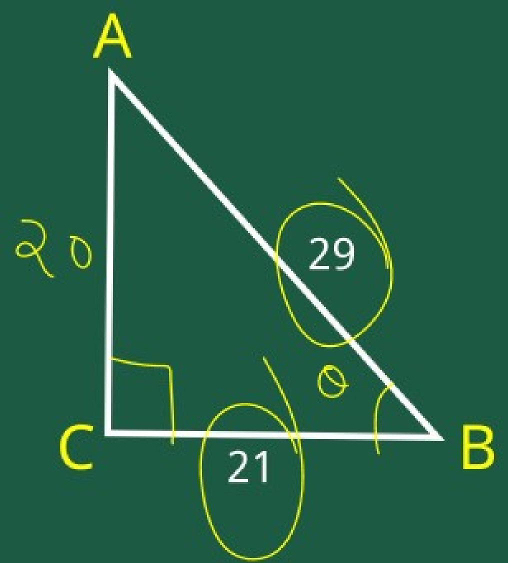
Example : 3

Consider Δ ACB, right-angled at C, in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$ (see Fig.). Determine the values of

i. $\cos^2\theta + \sin^2\theta$



Soln: $\left(\frac{B}{H}\right)^2 + \left(\frac{P}{H}\right)^2$

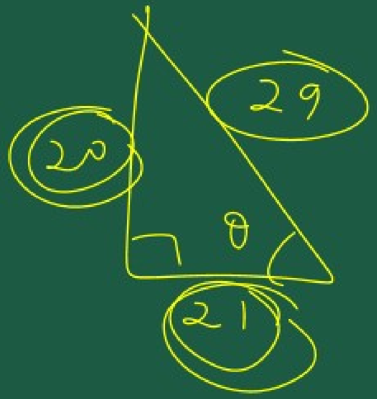
$=$

$$\sin^2\theta + \cos^2\theta$$

$$\left(\frac{20}{29}\right)^2 + \left(\frac{21}{29}\right)^2$$

$$\frac{20^2 + (21)^2}{(29)^2}$$

$$\frac{\cancel{(29)^2}}{\cancel{(29)^2}} = 1$$



$$(29)^2-(21)^2=p^2$$

$$(29-21)(29+21)$$

$$8 \times 50$$

$$400=p^2$$

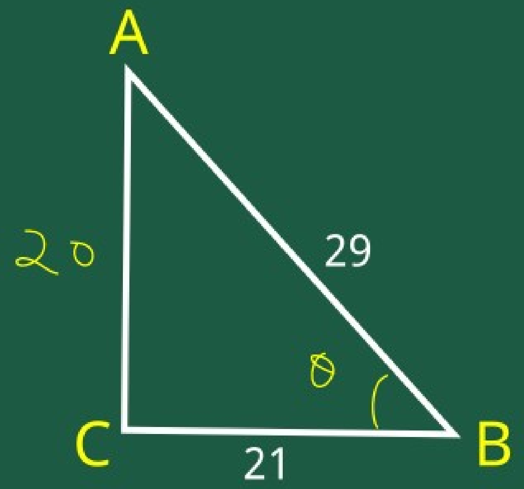
$$\boxed{20=p}$$

ii. $\cos^2\theta - \sin^2\theta$

$\Rightarrow \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2$

$\Rightarrow \frac{(21)^2 - (20)^2}{(29)^2}$

$= \frac{41}{(29)^2}$



Example : 4

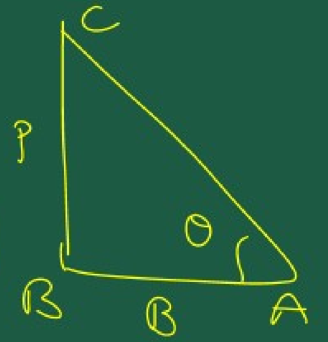
In a right triangle ABC, right-angled at B, if tan A = 1, then verify that $\boxed{2 \sin A \cos A = 1.}$

tan A = 1

$\frac{P}{B} = 1$

$\boxed{P = B}$

BC = AB

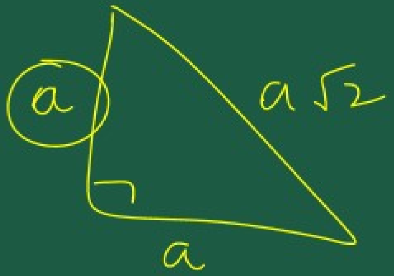


$$a^2 + a^2 = H^2$$

$$2a^2 = H^2$$

$$\sqrt{2a^2} = H^2$$

$$\boxed{a\sqrt{2} = H}$$



$$2\sin A\cos A = 2 \times \frac{\cancel{a}}{\cancel{a}\sqrt{2}} \times \frac{\cancel{a}}{\cancel{a}\sqrt{2}}$$

$$= 2 \times \frac{1}{2} = \frac{1}{1} = 1$$

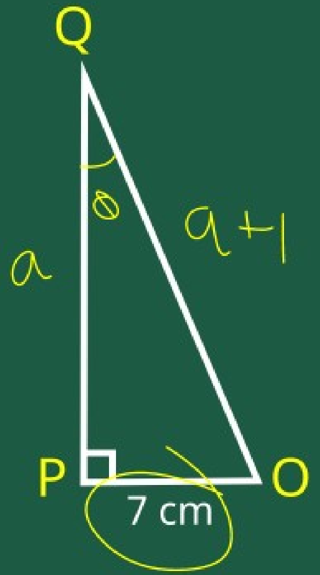
Example : 5

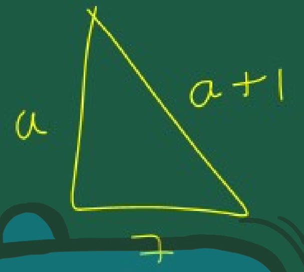
In ΔOPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm (see Fig.) Determine the values of sin Q and cos Q.

Solu = $OQ - PQ = 1$

$OQ = 1 + PQ$

[let $PQ = a$] $OQ = 1 + a$





$$a^2 + 7^2 = (a+1)^2$$

$$\cancel{a^2} + 7^2 = \cancel{a^2} + 2a + 1$$

$$+49 - 1 = 2a$$

$$48 = 2a$$

$$\boxed{24 = a} \Rightarrow \boxed{PQ = 24}$$

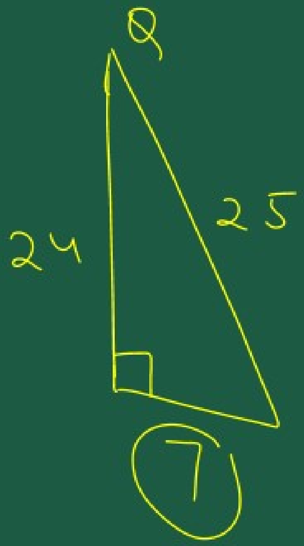
Hence $\boxed{OQ = 25}$

$$\sin\theta = \frac{P}{H}$$

$$\boxed{\sin\theta = \frac{7}{25}}$$

$$\cos\theta = \frac{B}{H}$$

$$\boxed{\cos\theta = \frac{24}{25}}$$



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LEARNING OUTCOMES

1. Right Angle Triangle
2. Trigonometric Ratio
3. Relation between Trigonometric Ratio

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ASSESSMENT

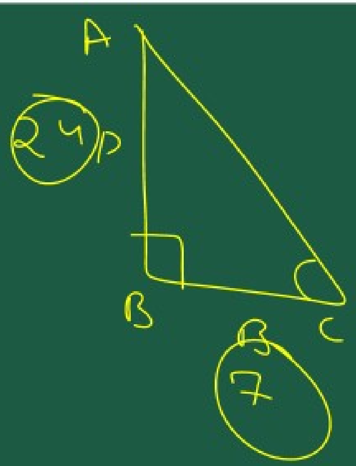
1 In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. The value of tan C is:

A $\frac{12}{7}$

B $\frac{24}{7}$

C $\frac{20}{7}$

D $\frac{7}{24}$



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ASSESSMENT

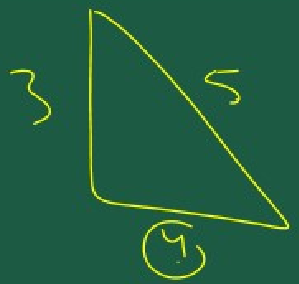
2 If $\sin A = \frac{3}{5}$ then $\cos A = ?$

A $\frac{4}{5}$

B $\frac{3}{4}$

C $\frac{3}{5}$

D $\frac{4}{3}$



$\sin\theta = 3/5 = P/H$

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ASSESSMENT

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3

The value of each of the trigonometric ratios of an angle depends on the size of the triangle and does not depend on the angle.

A True

B False

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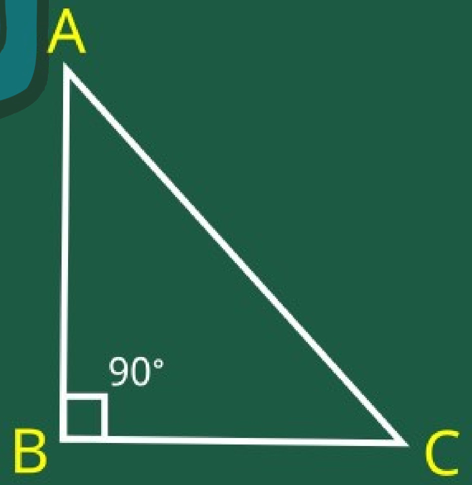
RIGHT ANGLED TRIANGLE

A triangle which has any one of the interior angle equal to 90° is called Right angled triangle



PROPERTIES OF RIGHT ANGLE TRIANGLE

- One angle is always 90° or right angle.
- The side opposite angle of 90° is the hypotenuse.
- The hypotenuse is always the longest side.

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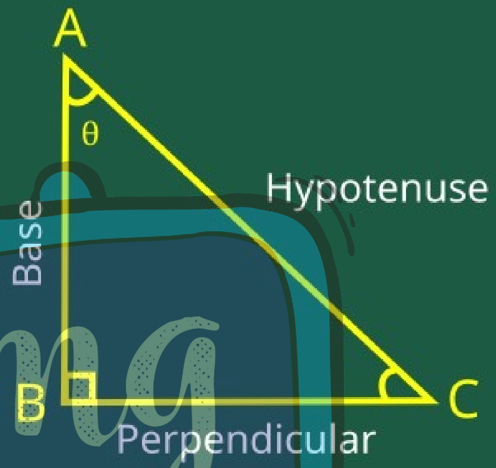
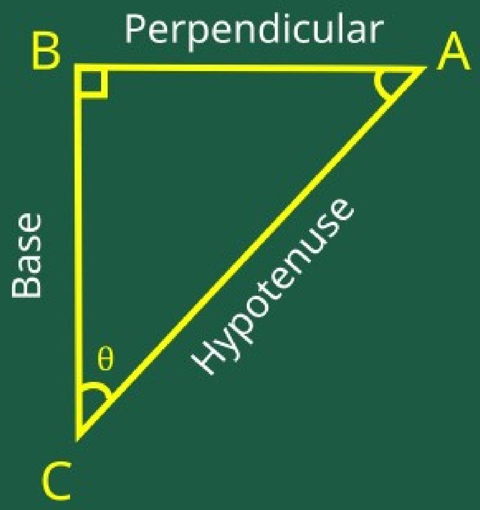
- The sum of the other two interior angles is equal to 90°.
- Perpendicular is the side that makes right angle with the base of the triangle.
- A right angle triangle's base is one of the sides that adjoins the 90° angle.





Here θ is the angle into consideration

Angles	Hypotenuse	Perpendicular	Base
for ∠A	AC	BC	AB
for ∠C	AC	AB	BC

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TRIGONOMETRIC RATIOS

The ratio of the sides of a right angle triangle with respect to acute angles are called "Trigonometric ratios of the angle".

1.	sine of θ / $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$
2.	cosine of θ / $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$
3.	tangent of θ / $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

4.	cosecant of θ / $\operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$
5.	secant of θ / $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$
6.	cotangent of θ / $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

Trick

P	B	P
H	H	B