

# CLASS – 10 MATHEMATICS

## Chapter – 8

### Introduction to Trigonometry

#### Part – 3

### Trigonometric Ratios of Some Specific Angles

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# OVERVIEW

1. Trigonometric Ratios

2. Trigonometric Ratios of Some Specific Angles

3. Trigonometric Identities

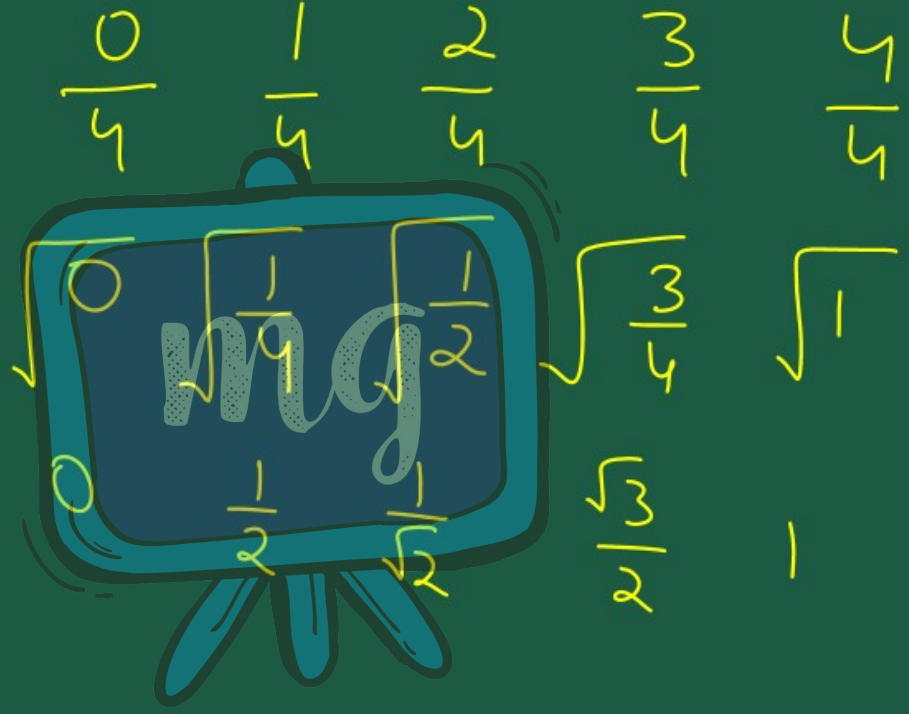
# COMPETENCY BASED LEARNING



Ratio of the sides for some specific angles

Geometric representation of trigonometric ratios

Sinθ

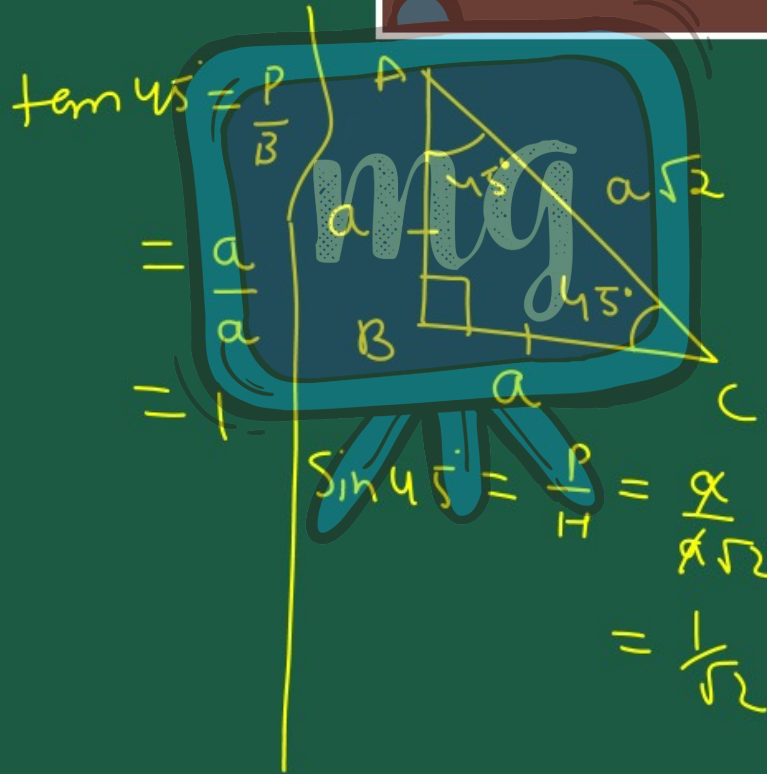


	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
$\cot \theta$	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
$\csc \theta$	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

## TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES

The specific angles that are defined for  
trigonometric ratios are 0°, 30°, 45°, 60°  
and 90°.

# TRIGONOMETRIC RATIOS OF 45°



in  $\Delta ABC$

$$AC^2 = AB^2 + BC^2$$

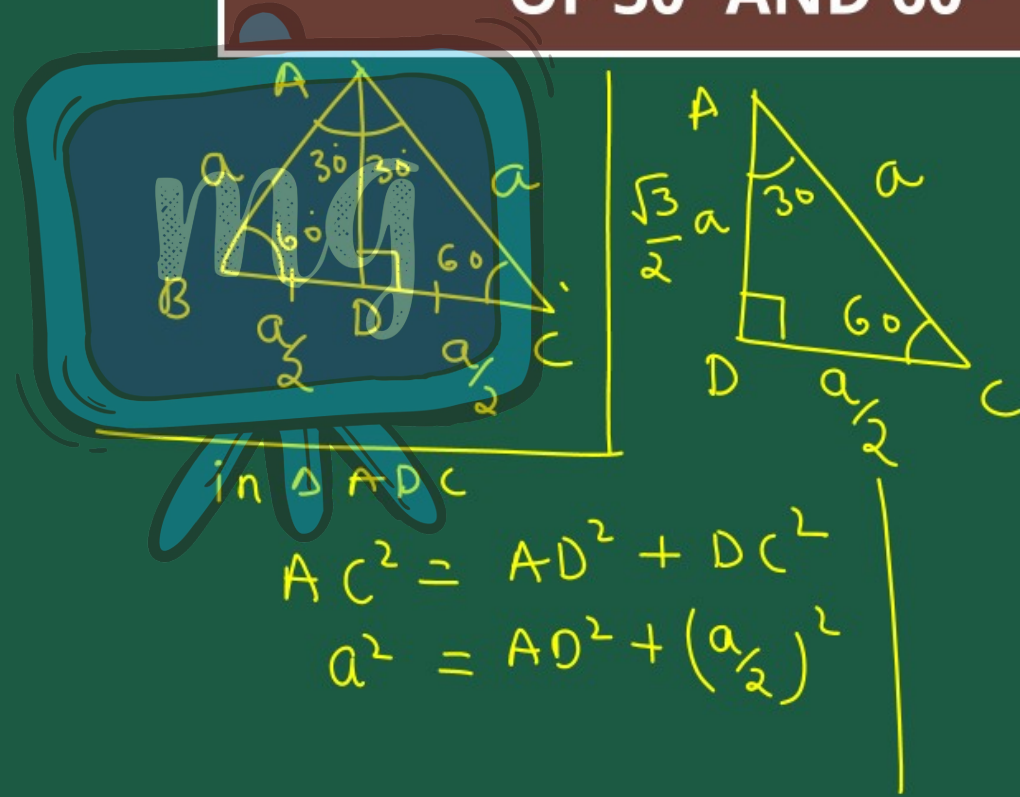
$$AC^2 = a^2 + a^2$$

$$AC^2 = 2a^2$$

$$AC = \sqrt{2a^2}$$

$$AC = \sqrt{2} \times a$$

# TRIGONOMETRIC RATIOS OF 30° AND 60°



$$AC^2 = AD^2 + DC^2$$

$$a^2 = AD^2 + \left(\frac{a}{2}\right)^2$$

$$a^2 - \frac{a^2}{4} = AD^2$$

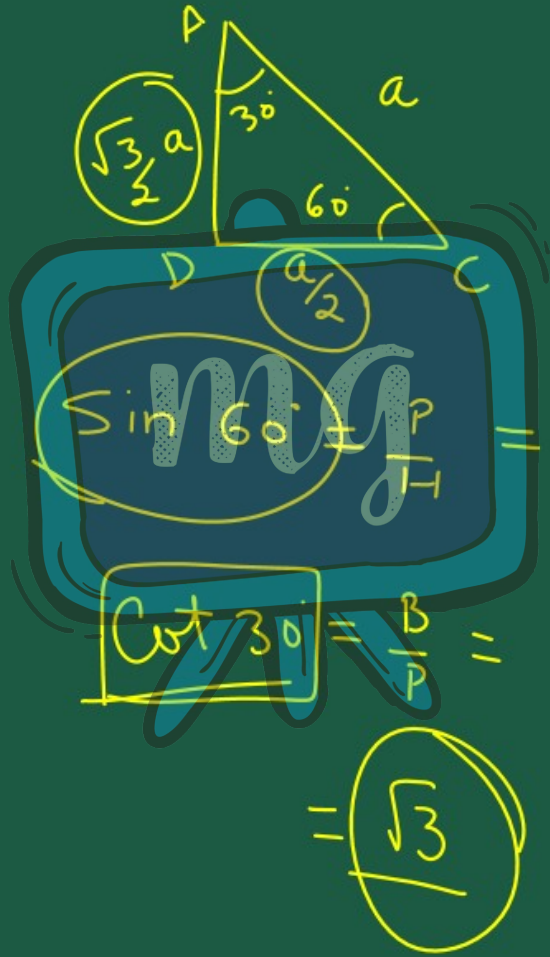
$$\frac{4a^2 - a^2}{4}$$

$$\frac{3a^2}{4} = AD^2$$

$$\frac{\sqrt{3}}{2}a = AD$$

$$AD = \sqrt{\frac{3}{4}a^2}$$

$$AD = \frac{\sqrt{3}}{2}a$$



$$\sin 60^\circ = \frac{AD}{AC}$$

$$\cos 30^\circ = \frac{AD}{AB}$$

$$= \frac{\sqrt{3}}{2}$$

$$\frac{AD}{AC} = \frac{\frac{\sqrt{3}}{2} a}{a} = \frac{\sqrt{3}}{2}$$

$$\frac{AD}{BC} = \frac{\frac{\sqrt{3}}{2} a}{\frac{a}{2}} = \sqrt{3}$$

# TRIGONOMETRIC RATIOS OF $0^\circ$ AND $90^\circ$

$$\sin 90^\circ = \frac{H}{H}$$

$$\sin 90^\circ = 1$$

$$\sin \theta = \frac{P}{H}$$

$$\sin 0^\circ = \frac{0}{H}$$

$$\sin 0^\circ = 0$$

$\cos 0^\circ \approx \frac{1}{1} = \frac{1}{1} = 1$

$\cos 0^\circ \approx 1$

$\cos 90^\circ = \frac{0}{1} = 0$

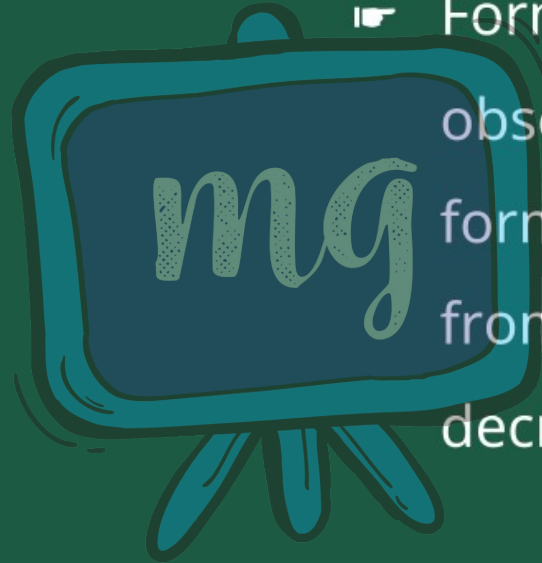
$\cos 90^\circ \approx 0$

# TRIGONOMETRY TABLE

$\angle A$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
<b>sin A</b>	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
<b>cos A</b>	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
<b>tan A</b>	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined
<b>cot A</b>	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
<b>sec A</b>	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined
<b>cosec A</b>	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1



Form the table above you can observe that as  $\angle A$  increases from  $0^\circ$  to  $90^\circ$ ,  $\sin A$  increases from 0 to 1 and  $\cos A$  decreases from 1 to 0.



## Example : 6

In  $\triangle ABC$ , right-angled at B,  $AB = 5$  cm and  $\angle ACB = 30^\circ$  (see Fig.). Determine the lengths of the sides BC and AC.

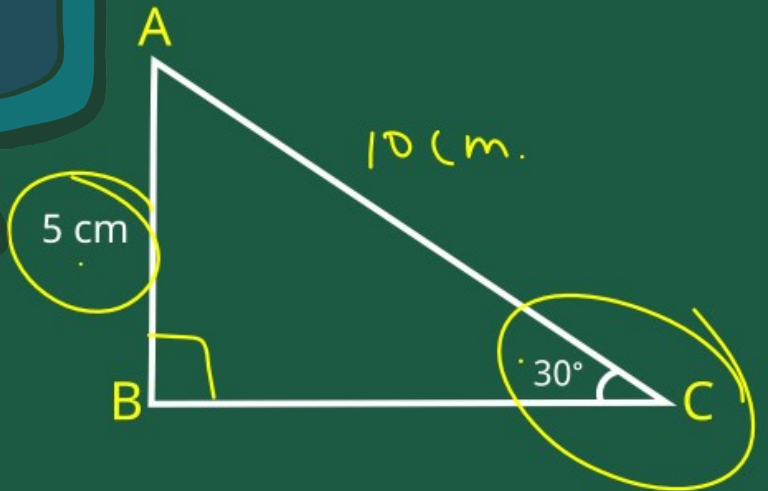
Soln:

$$\sin \theta = \frac{P}{H}$$

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{5}{AC}$$

$$AC = 10$$



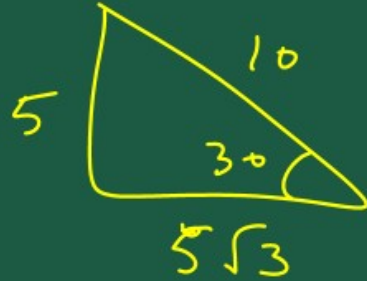
$$\cos \theta = \frac{B}{H}$$

$$\cos 30^\circ = \frac{B}{10}$$

$$\frac{\sqrt{3}}{2} = \frac{B}{10}$$

$$10 \times \frac{\sqrt{3}}{2} = B$$

$$5\sqrt{3} = B$$



## Example : 7

In  $\triangle PQR$ , right-angled at  $Q$ ,  $PQ = 3$  cm and  $PR = 6$  cm. (see Fig). Determine  $\angle QPR$  and  $\angle PRQ$ .

$\angle R$

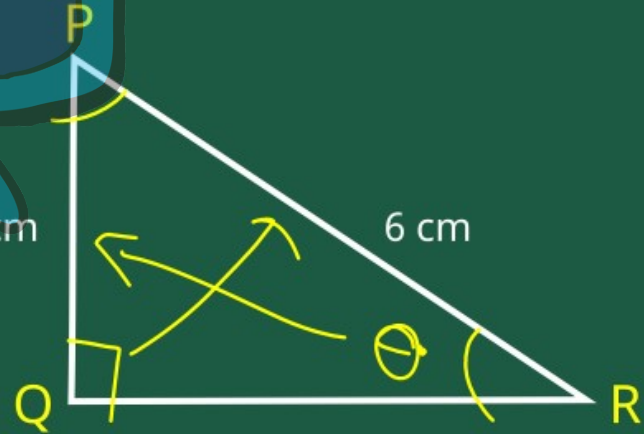
$$\sin \theta = \frac{PQ}{PR}$$

$$\sin \theta = \frac{3}{6}$$

$$\sin \theta = \frac{3}{6} = \frac{1}{2}$$

3 cm

6 cm



$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = \sin 30^\circ$$

$$\theta = 30^\circ$$

$$\angle P R Q = 30^\circ$$

By ASP

$$\angle Q P R = 60^\circ$$

## Example : 8

If  $\sin(A-B) = \frac{1}{2}$ ,  $\cos(A+B) = \frac{1}{2}$ ,

$0^\circ < A+B \leq 90^\circ$ ,  $A > B$ , find A and B.

Solu.

$$A+B=60$$

-(2)

$$\sin(A-B) = \frac{1}{2}$$

$$\sin(A-B) = \sin 30^\circ$$

$$A-B = 30^\circ \quad \text{--- (1)}$$

$$\cos(A+B) = \frac{1}{2}$$

$$\cos(A+B) = \cos 60^\circ$$

$$A - B = 30^\circ$$

$$A + B = 60$$

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$$2A = 90$$

$$A = \frac{90}{2}$$

$$A = 45^\circ \checkmark$$

$$45^\circ + B = 60$$

$$B = 60 - 45^\circ$$

$$B = 15^\circ \checkmark$$

# LEARNING OUTCOMES



**1** | Trigonometric ratios of  $45^\circ$

**2** | Trigonometric ratios of  $30^\circ$  and  $60^\circ$

**3** | Trigonometric ratios of  $0^\circ$  and  $90^\circ$

# ASSESSMENT



1

$$(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ)$$

- A -1
- B 0
- C 1
- D  $\frac{1}{2}$

$$\sin 30^\circ + \cos 30^\circ$$

$$- \sin 60^\circ - \cos 60^\circ$$

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$$

# ASSESSMENT

2

Value of  $\frac{\tan 30^\circ}{\tan 60^\circ}$  is :

A  $\frac{1}{\sqrt{2}}$

B  $\frac{1}{3}$

C  $\frac{\sqrt{3}}{2}$

D 1

$$\frac{1/\sqrt{3}}{\sqrt{3}/1} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$$