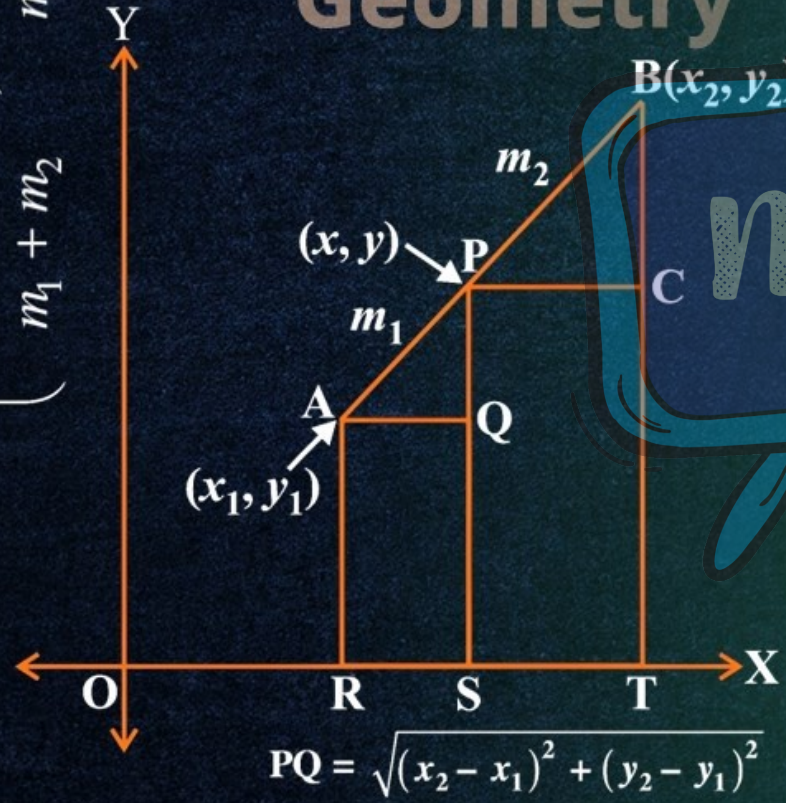
$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Coordinate Geometry



$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

CLASS – 10

MATHEMATICS

Chapter – 7

Coordinate Geometry

Part – 5

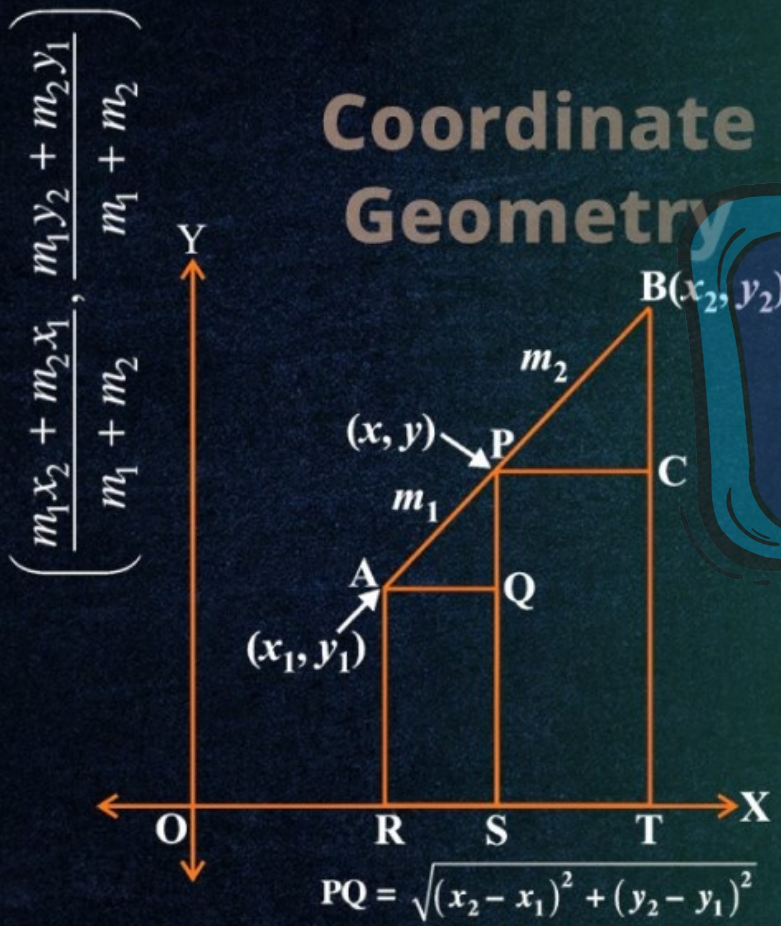
Section Formula

Shubham Tiwari

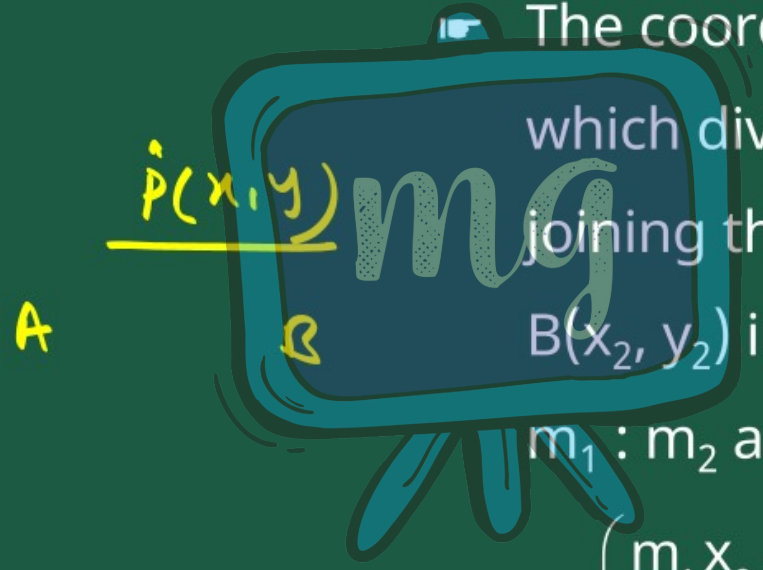
OVERVIEW

Coordinate Geometry

1. Distance Formula
2. Section Formula

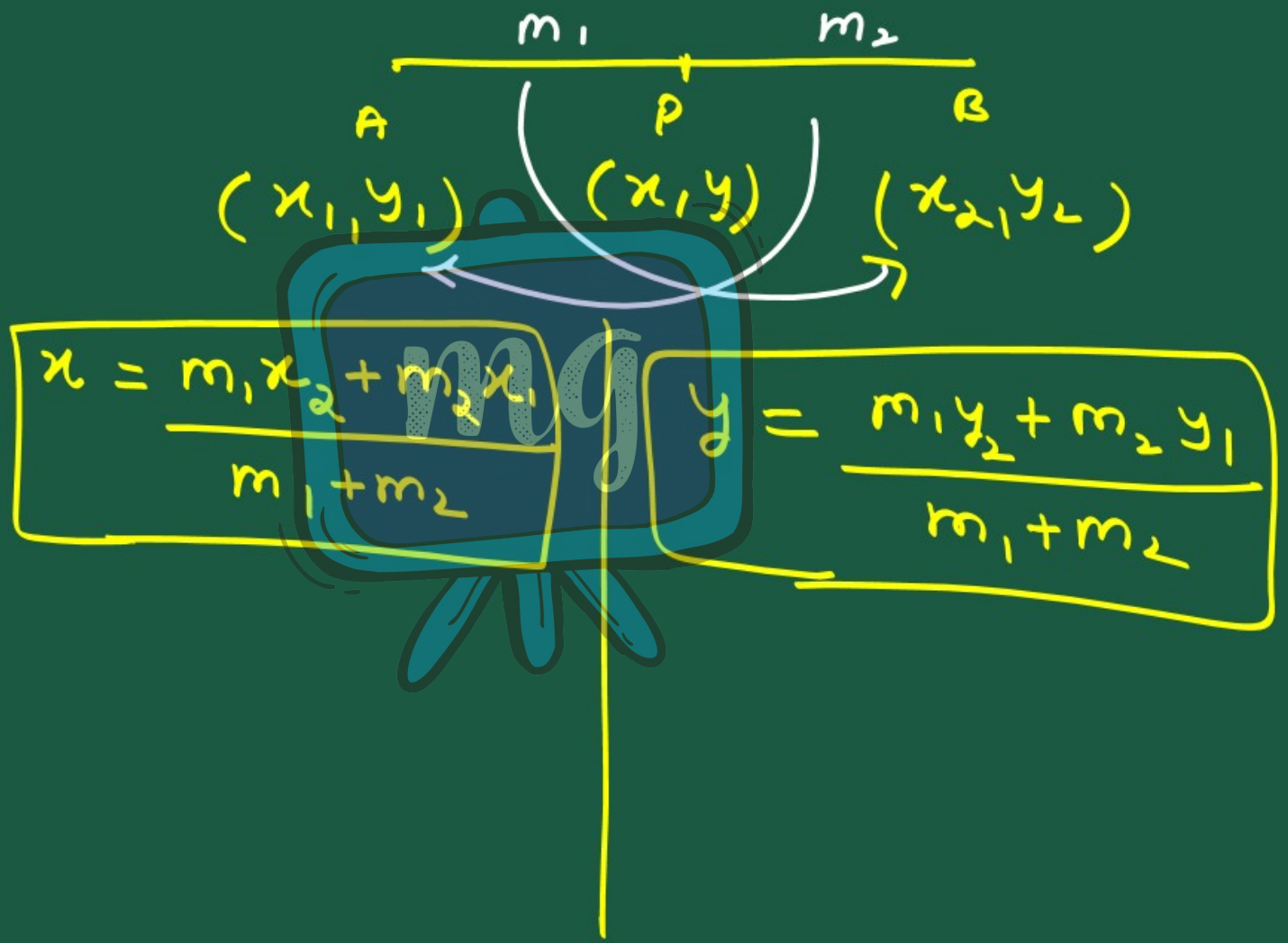


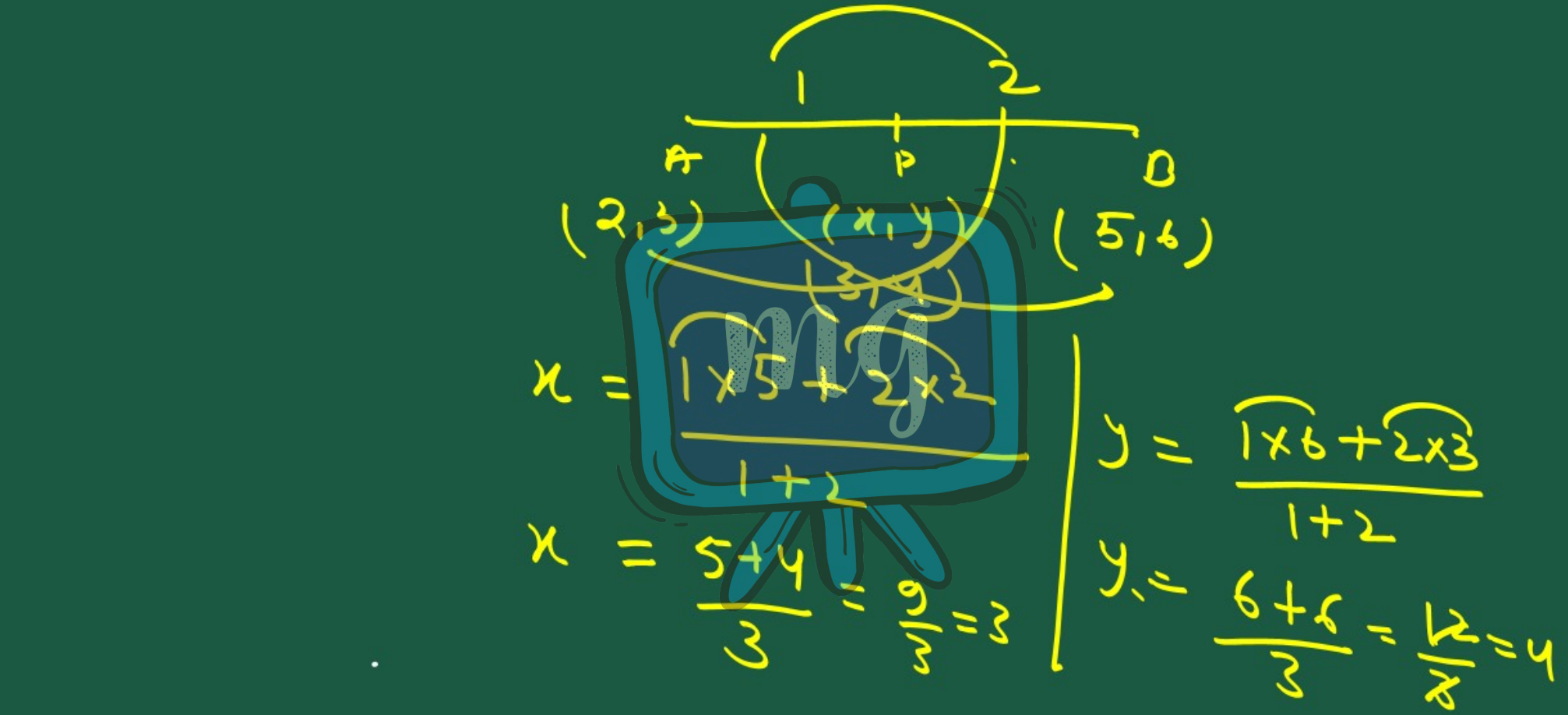
SECTION FORMULA

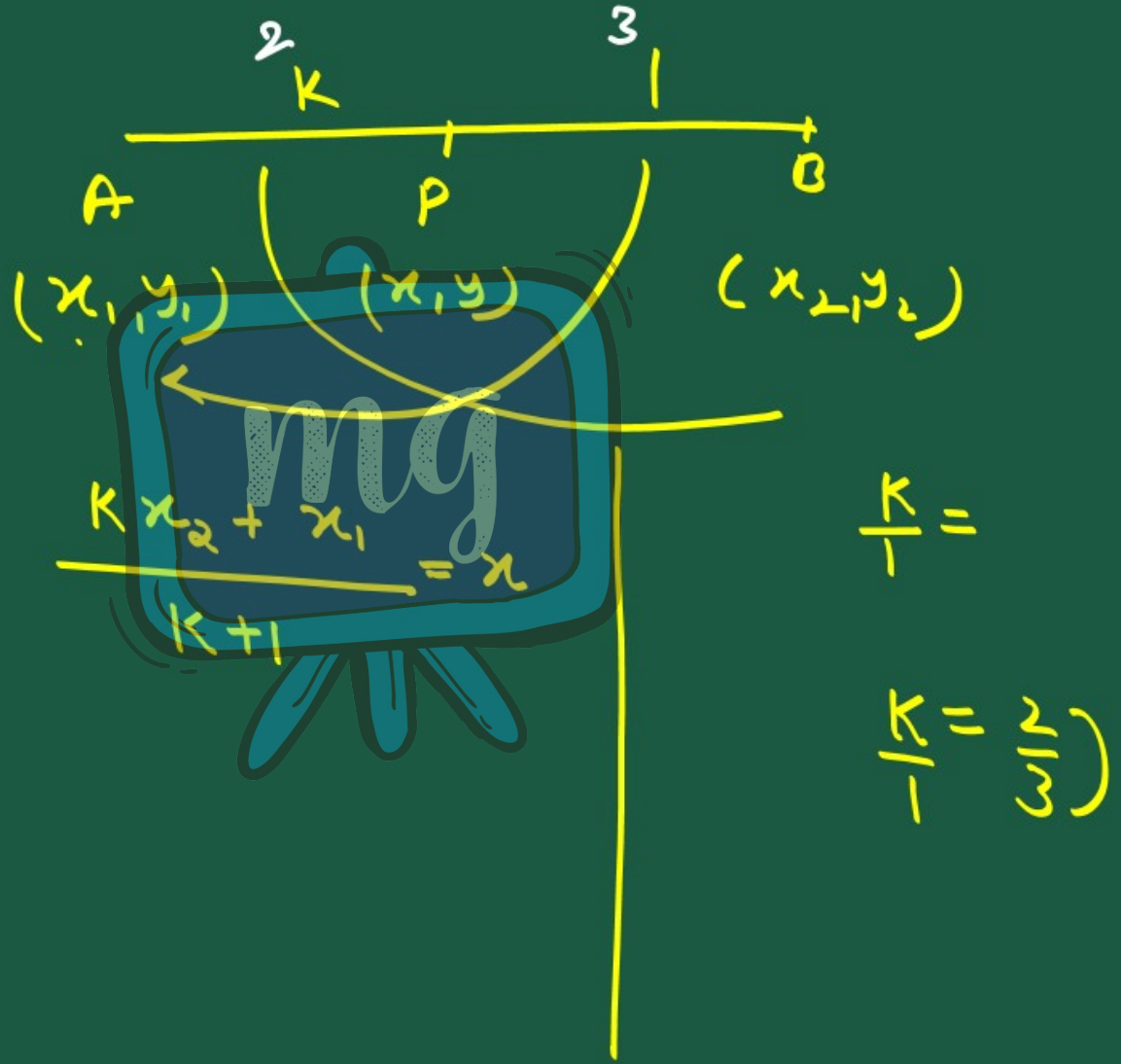


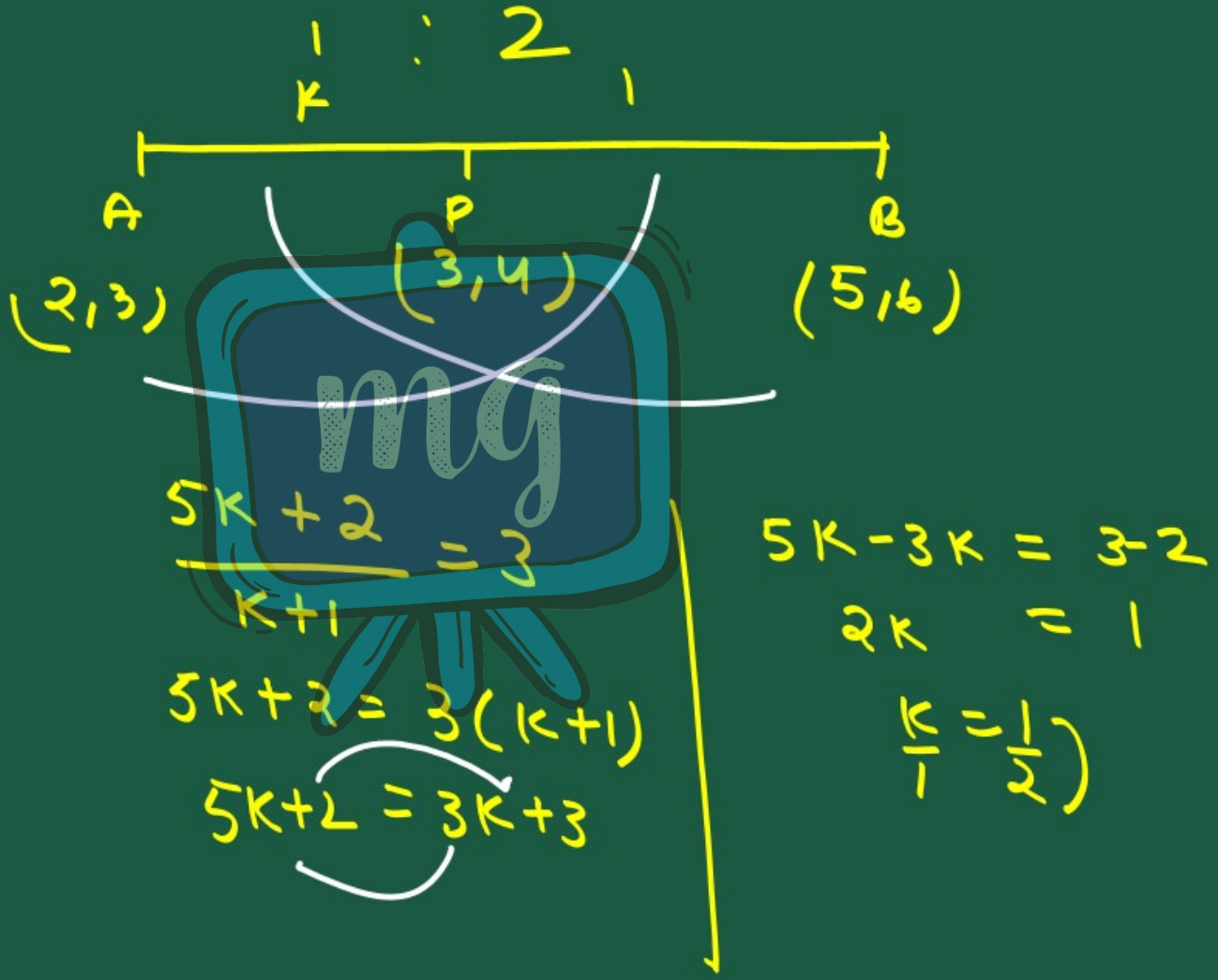
The coordinates of the point P(x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally, in the ratio $m_1 : m_2$ are :

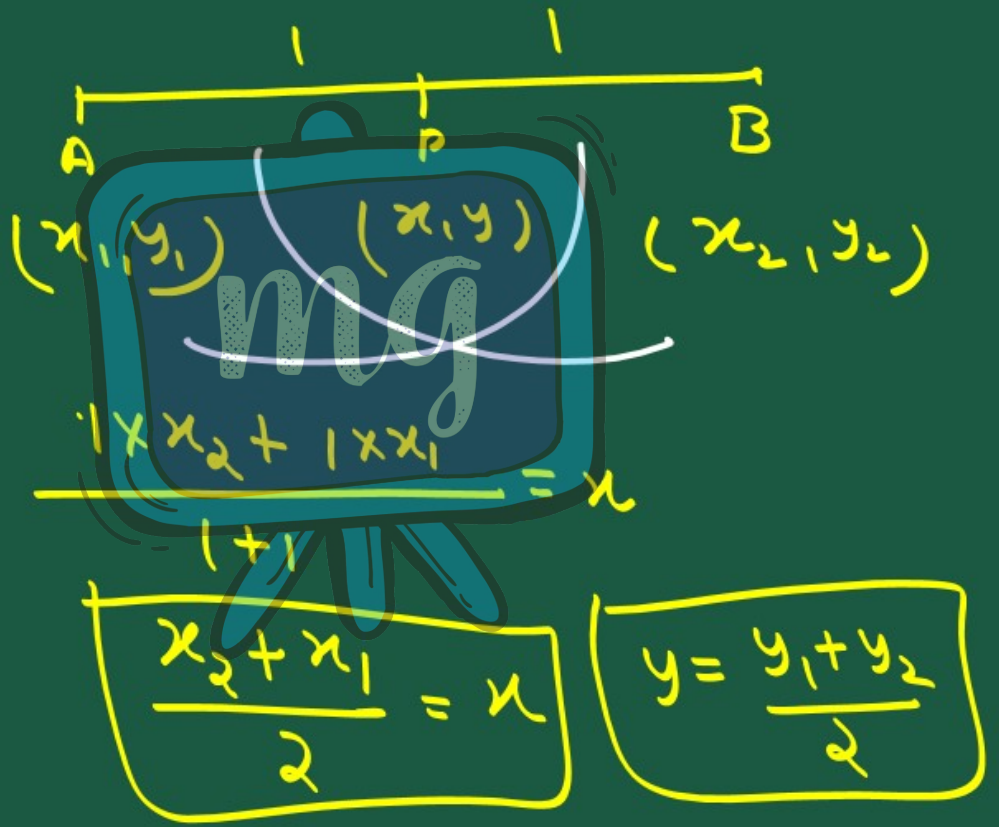
$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

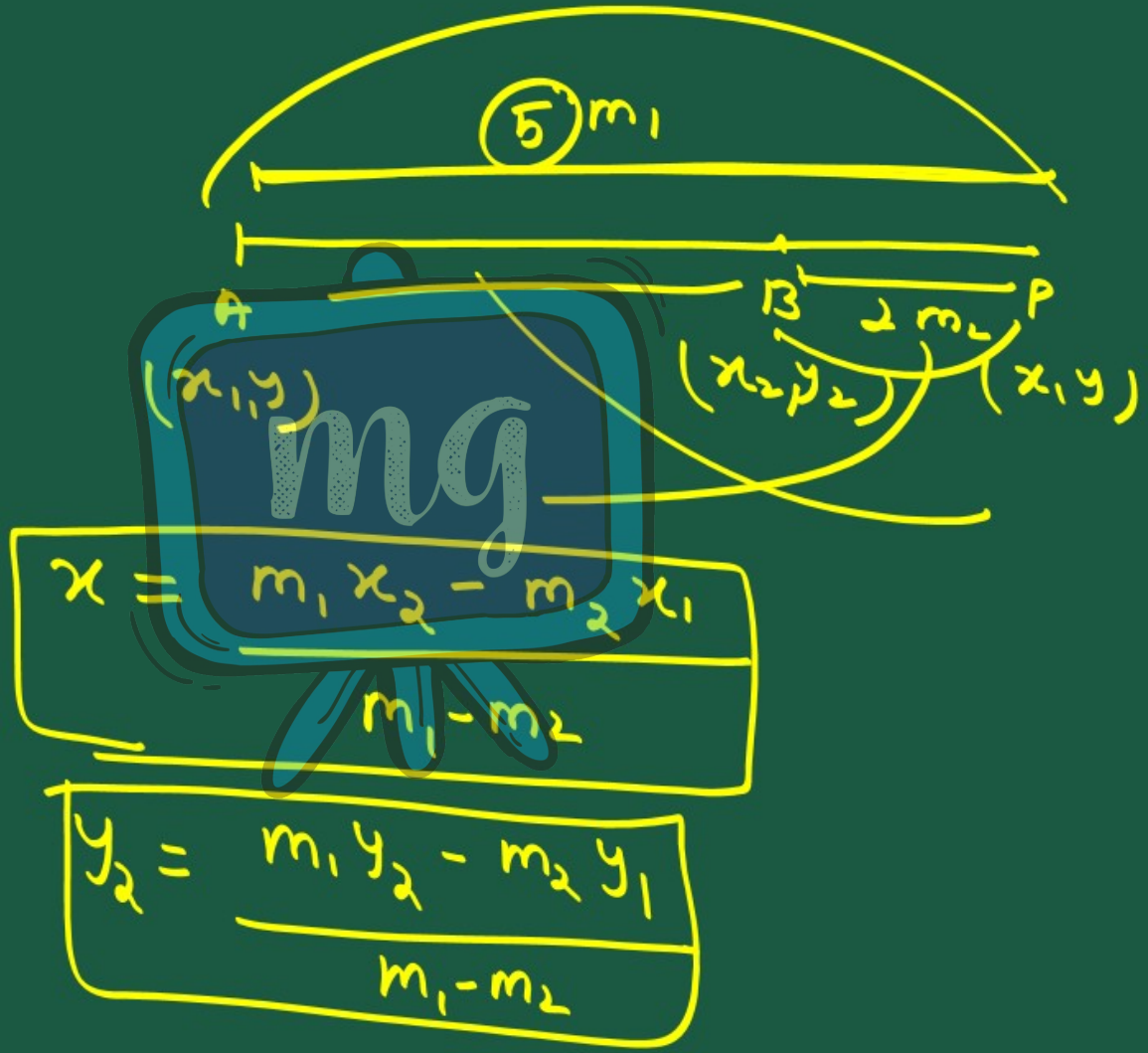


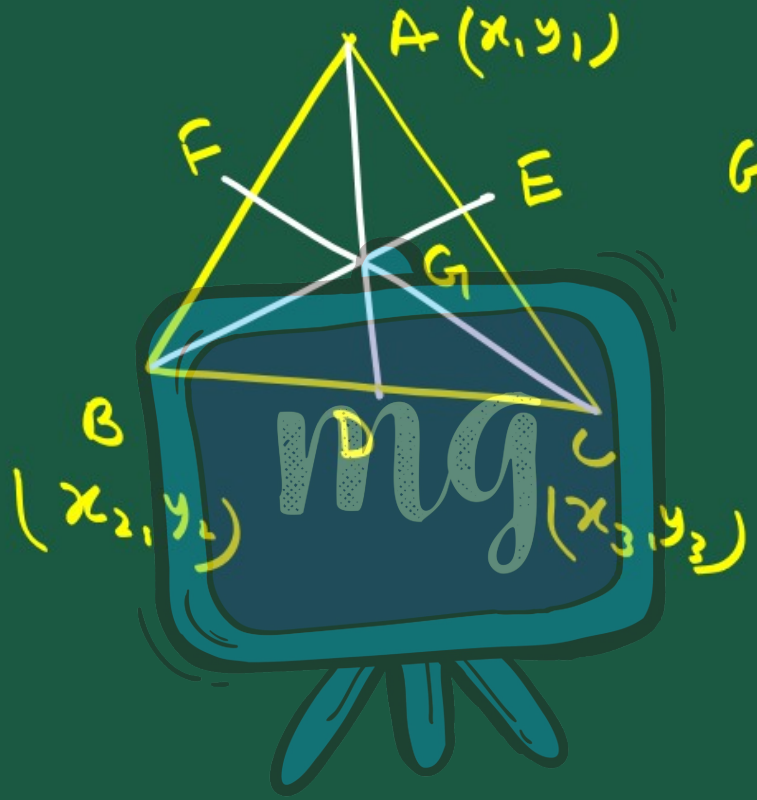












$$G\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$$

☛ Externally :

$$\left(\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}\right)$$

☛ If the ratio in which P divides AB is $k:1$, then the coordinates of the point P will be :

$$\left(\frac{kx_2 + x_1}{K+1}, \frac{ky_2 + y_1}{K+1}\right)$$

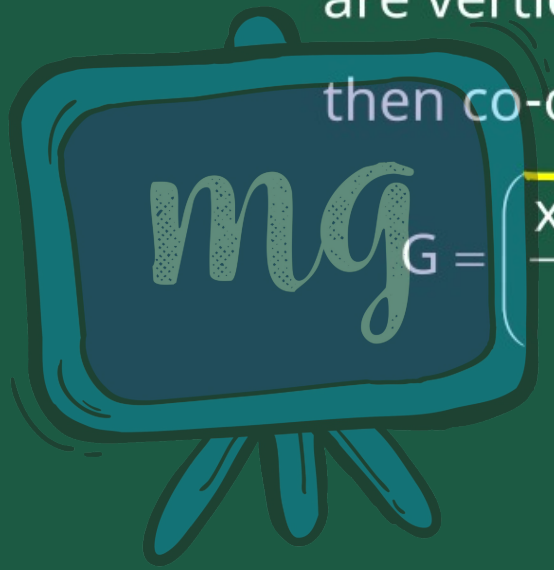
MID POINT FORMULA

The mid–point of a line segment divides the line segment in the ratio 1:1.

Therefore, the coordinates of the mid-point P of the join of the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is :

$$\left(\frac{1.x_1 + 1.x_2}{1+1}, \frac{1.y_1 + 1.y_2}{1+1}\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

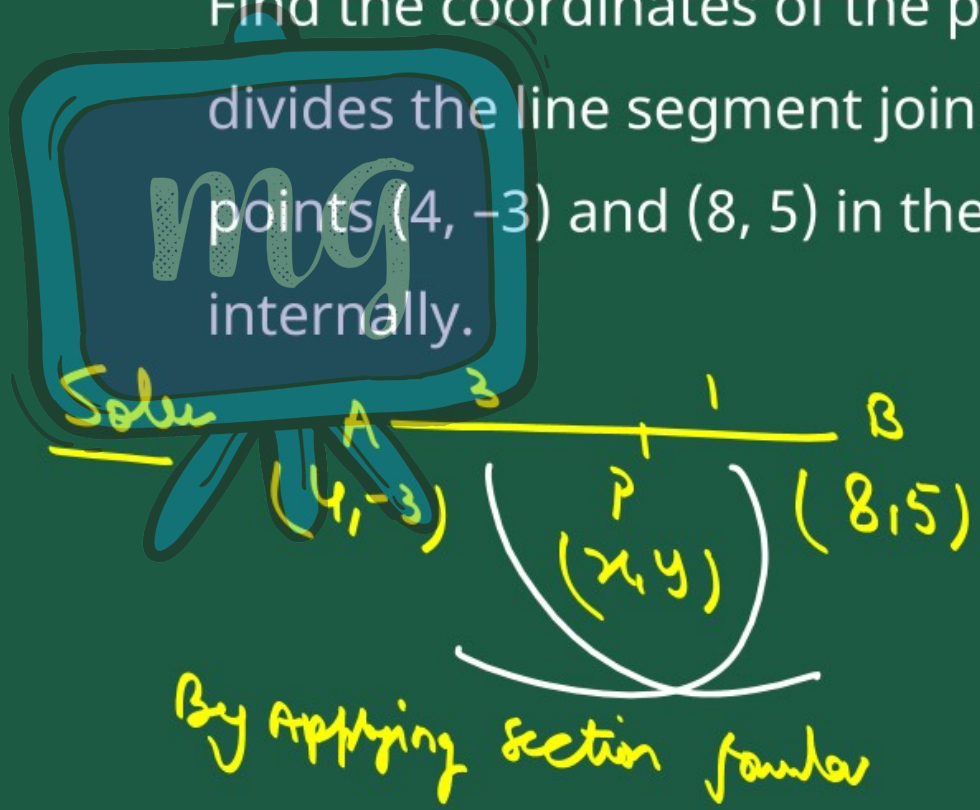
☛ Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of any triangle ABC, then co-ordinates of centroid :



$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Example : 6

Find the coordinates of the point which divides the line segment joining the points (4, –3) and (8, 5) in the ratio 3 : 1 internally.



$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

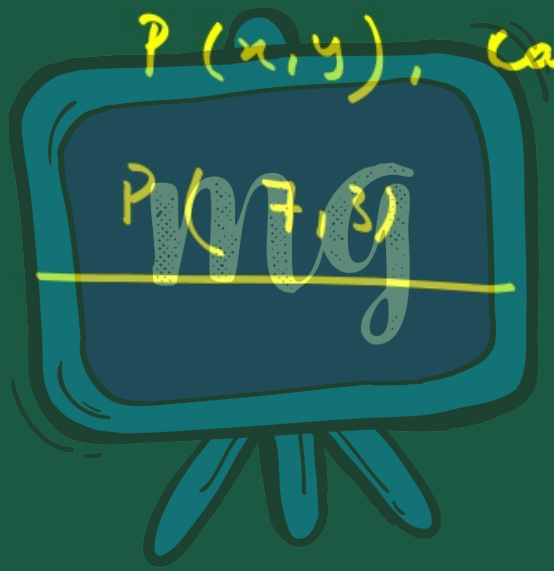
$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$x = \frac{3 \times 8 + 1 \times 4}{3+1} = \frac{24+4}{4} = \frac{28}{4} = 7$$

$$y = \frac{3 \times 5 + 1 \times (-3)}{3+1} = \frac{15-3}{4} = \frac{12}{4} = 3$$

Name the coordinates a.

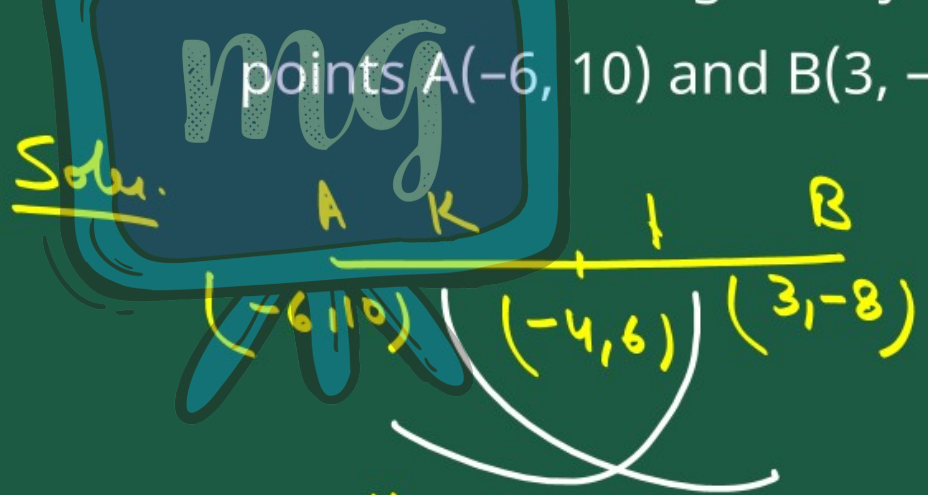
P (x, y), can be written as



Example : 7

In what ratio does the point (–4, 6) divide the line segment joining the points A(–6, 10) and B(3, –8)?

Solu.



$$x = \frac{kx_2 + x_1}{k+1} \qquad y = \frac{ky_2 + y_1}{k+1}$$

As we have assumed the ratio as $K:1$

Let's find out the ratio by section formula

$$x = \frac{Kx_2 + x_1}{K+1}$$

$$-4 = \frac{K(3) + 1(-6)}{K+1}$$

$$\frac{3K-6}{K+1} = -4$$

$$3K-6 = -4(K+1)$$

$$3K-6 = -4K-4$$

$3k - 6 = -4k - 4$

$3k + 4k = -4 + 6$

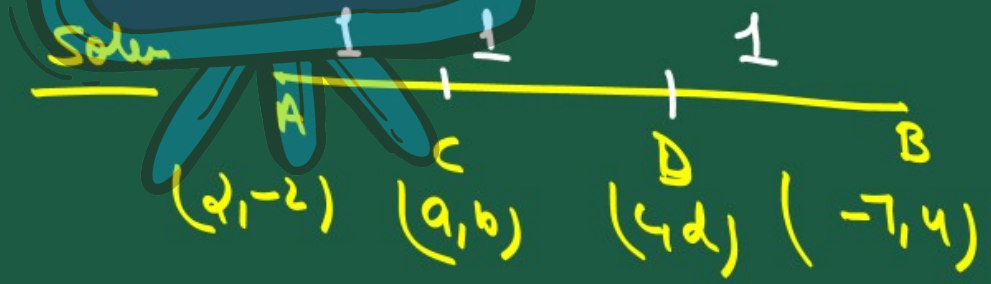
$7k = 2$

$\frac{k}{1} = \frac{2}{7}$

Hence the ratio are 2 : 7.

Example : 8

Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, –2) and B(–7, 4).



(-1, 0)

A (2, -2)

(a, b)

B (-7, 4)

1 2

By applying section Formula

$$a = \frac{1 \times -7 + 2 \times 2}{1+2}$$

$$\frac{-7+4}{3} = \frac{-3}{3} = -1$$

$$b = \frac{1 \times 4 + 2 \times -2}{1+2}$$

$$b = \frac{4-4}{3} = \frac{0}{3} = 0$$

$(2,-2)$ $(-1,0)$ (c,d) $(-7,4)$

By applying midpoint formula.

$$c = \frac{x_1+x_2}{2}, \quad d = \frac{y_1+y_2}{2}$$

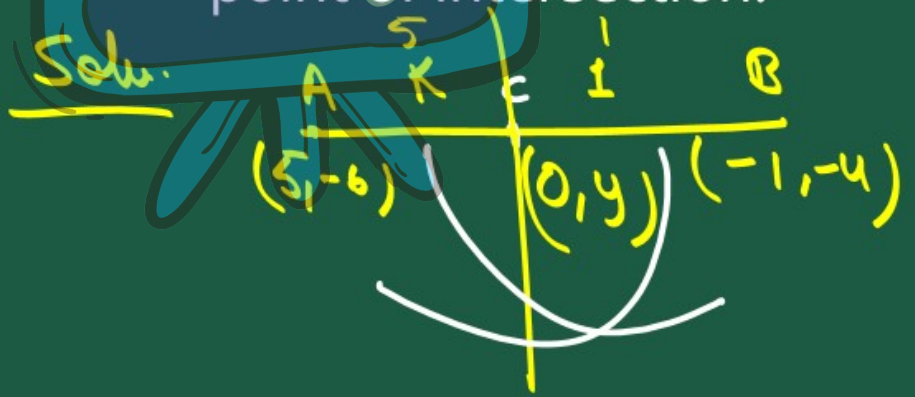
$$c = \frac{-1+-7}{2}, \quad d = \frac{4+0}{2}$$

$$c = \frac{-8}{2}, \quad d = \frac{4}{2}$$

$$(c,d) = (-4,2)$$

Example : 9

Find the ratio in which the y-axis divides the line segment joining the points (5, –6) and (–1, –4). Also find the point of intersection.



Let assum that the coordinates is $(0, y)$ and the ratios are $k:1$

Now by applying section formula

$$\left[x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \; y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right]$$

$$0 = \frac{k \times (-1) + 1 \times 5}{k + 1} \Rightarrow -k + 5 = 0$$

$$-K + 5 = 0$$

$$\frac{K}{1} = \frac{5}{1}$$

Hence the ratios are $K:1 = 5:1$

once again by section formula

$$y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}$$

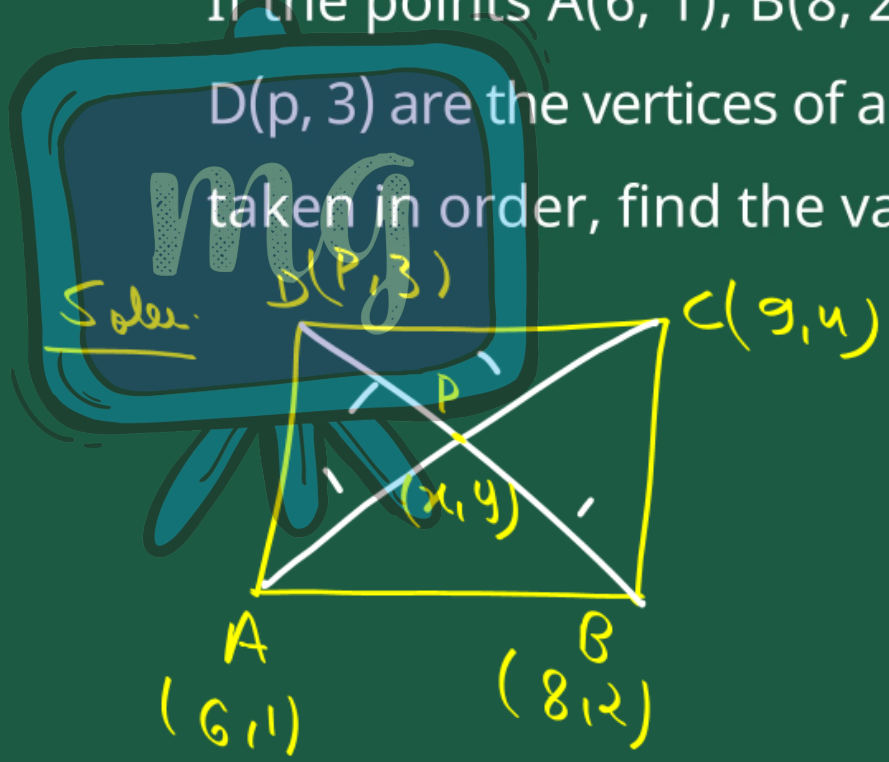
$$y = \frac{5 \times -4 + 1 \times -6}{5+1} = \frac{-20 - 6}{6} = \frac{-26}{6}$$

$$\left(y = \frac{-13}{3} \right)$$

Hence the co-ordinate of the intersection is

Example : 10

If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.



Lets apply midpoint formula in the diagonal AC and BD.

$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

$$x = \frac{6+9}{2} \qquad y = \frac{1+4}{2}$$

$$(x, y) = \left(\frac{15}{2}, \frac{5}{2}\right)$$

Hence from AC the coordinates of P are $(15/2, 5/2)$

Let's apply the mid point in BD.

$$P\left(\frac{P+8}{2}, \frac{3+2}{2}\right)$$

$$P\left(\frac{P+8}{2}, \frac{5}{2}\right)$$

Let's compare the cordinate of P.

$$\frac{P+8}{2} = \frac{15}{2}$$

$$P+8 = 15 \quad | \quad P = 15-8 = 7$$

LEARNING OUTCOMES

1. Section Formula
2. Mid-Point Formula

ASSESSMENT

1 What will be the coordinates of the midpoint of the line segment joining the points (–5, 10) and (15, 2)?

A (–5, –6)

B (–5, 6)

C (5, 6)

D (5, –6)

ASSESSMENT

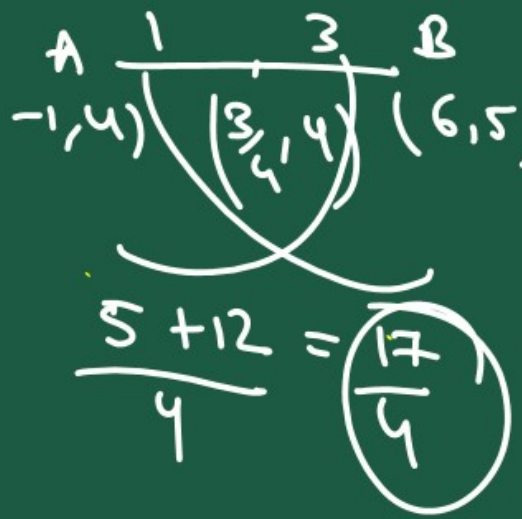
2 What will be the value of y, if the ratio in which the point $(\frac{3}{4}, y)$ divides the line segment joining the points A(–1, 4) and B(6, 5) is 1 : 3 ?

A $y = \frac{17}{4}$

B $y = \frac{5}{2}$

C $y = \frac{9}{2}$

D $y = -\frac{5}{2}$



ASSESSMENT

3 The vertices of a parallelogram in order are A(1, 2), B(4, y), C(x,,6) and D(3, 5). Then (x, y) is

A (6, 3)

B (3, 6)

C (5, 6)

D (1, 4)

$\frac{1+x}{2}$ $\frac{4+3}{2}$

$1+x = 7$

$x = 6$

$\frac{6+2}{2} = \frac{y+5}{2}$

$8 = y+5$