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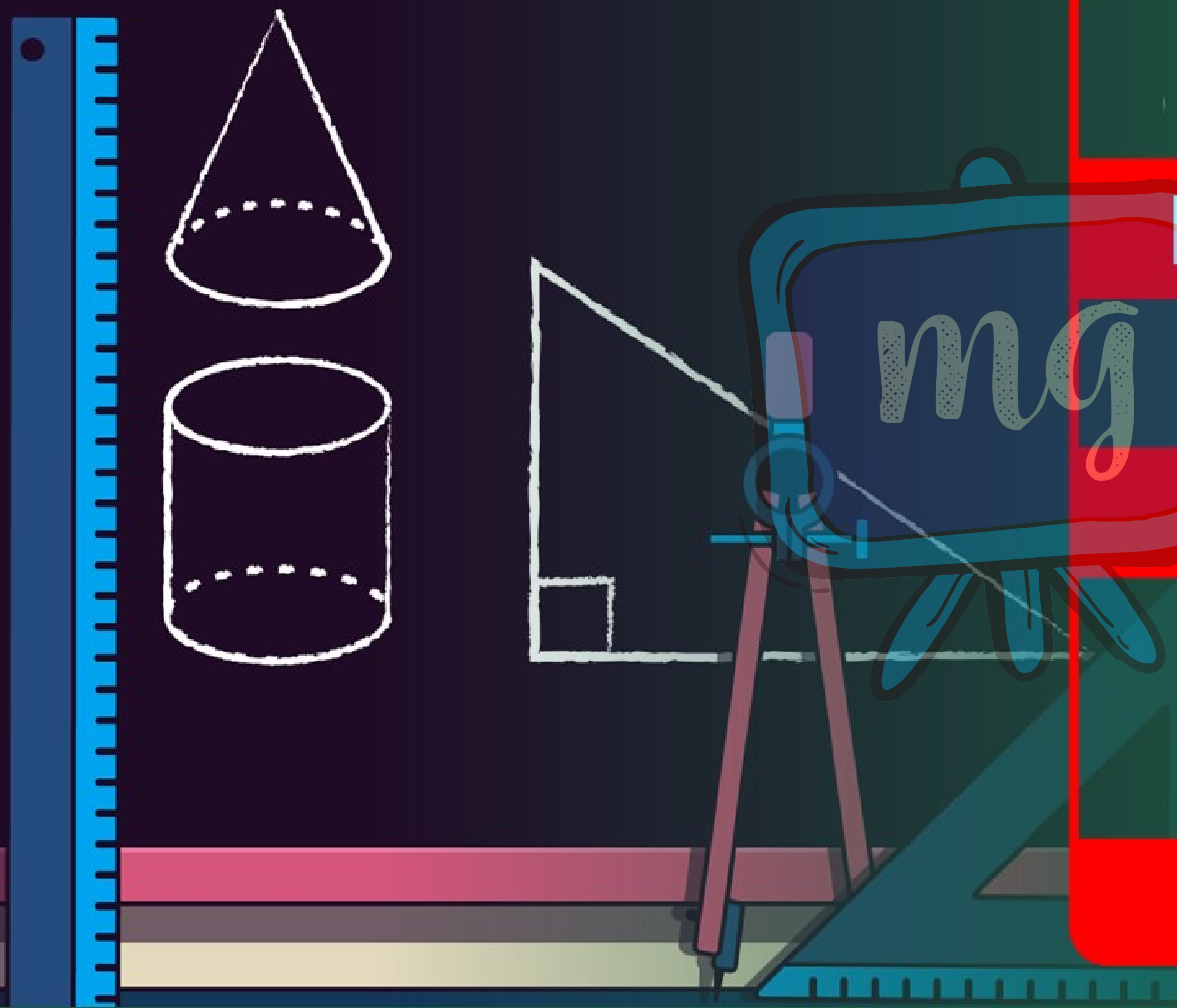


CLASS – 10

MATHEMATICS

Chapter – 6

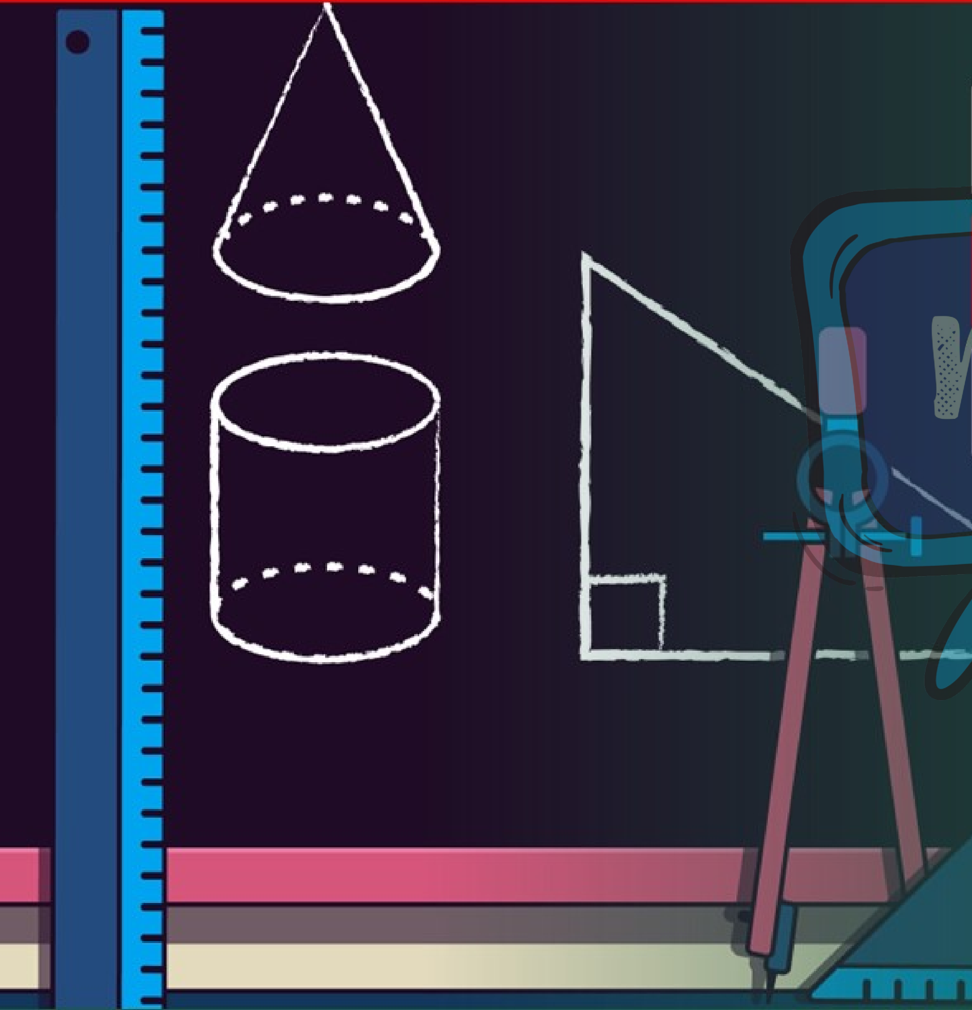
Triangles

Part – 3

Similarity of Triangles

Shubham Tiwari

OVERVIEW



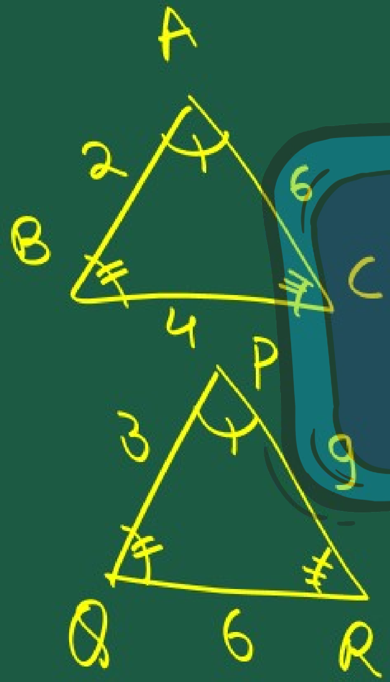
1. Similar Figures
2. Similarity of Triangles
3. Criteria for Similarity of Triangles

SIMILARITY OF TRIANGLES

Two triangles are similar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (or Proportion).



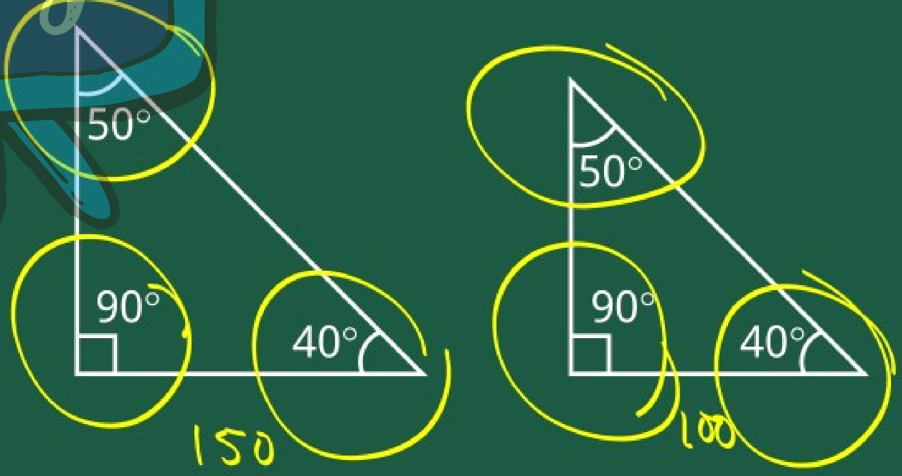
$$\frac{2}{3} = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} = \frac{\cancel{6}^{2}}{\cancel{9}_{3}}$$

ΔABC $\quad$ ΔPQR

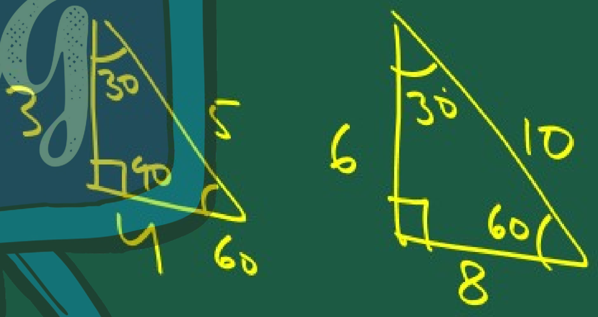
EQUIANGULAR TRIANGLES

- If corresponding angles of two triangles are equal, then they are known as equiangular triangles.

$\frac{\cancel{150}}{\cancel{100}}$ $\frac{3}{2}$



- The ratio of any two Corresponding sides in two equiangular triangles is always the same.



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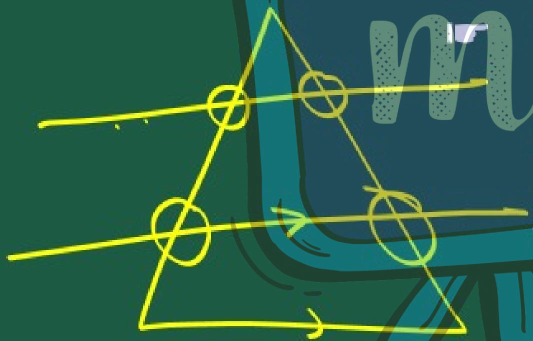
BASIC PROPORTIONALITY THEOREM



THEOREM 6.1

- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

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Given: $DE \parallel BC$

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Constr: $DM \perp AE$

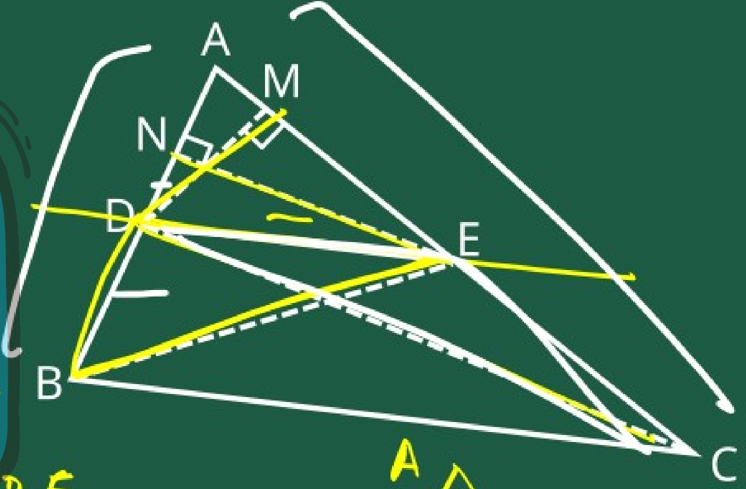
$EN \perp AD$

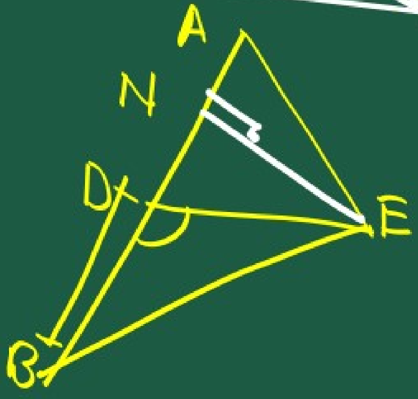
Join D to C and B to E

Proof:

$$\frac{\text{Ar of } \Delta ADE}{\text{Ar of } \Delta DEB} = \frac{\frac{1}{2} B_1 \times h_1}{\frac{1}{2} B_2 \times h_2}$$

$$= \frac{\cancel{\frac{1}{2}} AD \times \cancel{EN}}{\cancel{\frac{1}{2}} BD \times \cancel{EN}} = \frac{AD}{BD} \quad \text{—} (1)$$





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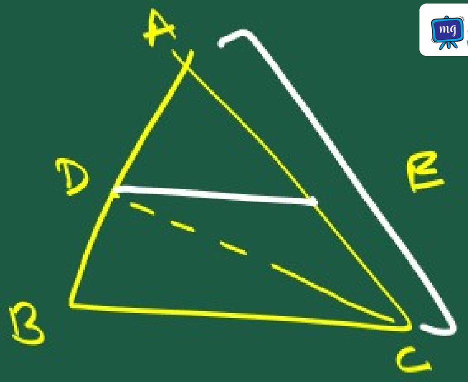
$$\frac{Ar\ \Delta ADE}{Ar\ \Delta EDC} = \frac{\frac{1}{2} B_1 \times h_1}{\frac{1}{2} B_2 \times h_2}$$

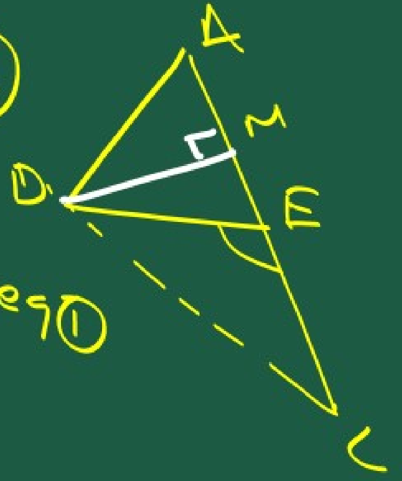
$$= \frac{\cancel{\frac{1}{2}}\ AE \times \cancel{DM}}{\cancel{\frac{1}{2}}\ EC \times \cancel{DM}} = \frac{AE}{EC} \quad \text{--- (2)}$$

Ar of ΔADE is common in eq (1) and (2)

Ar of ΔDEB = Ar ΔEDC

{on the same Base and b/w same Parallel lines}





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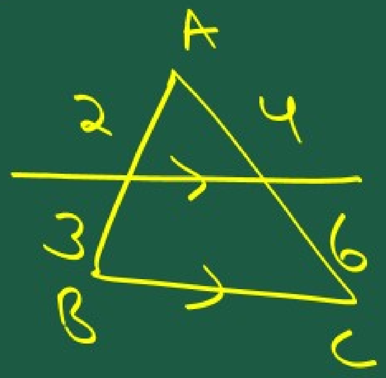
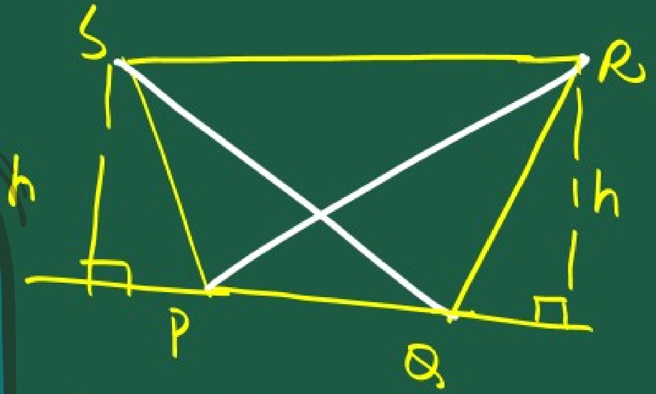
Here LHS of eq 1 = LHS of eq 2

$$\frac{AD}{DB} = \frac{AE}{EC}$$





$$\text{Ar}\,\Delta\, PQS = \tfrac{1}{2}\, PQ \times h$$

$$\text{Ar}\,\Delta\, PQR = \tfrac{1}{2}\, PQ \times h$$

Note

- Basic Proportionality Theorem also known as the Thales Theorem.

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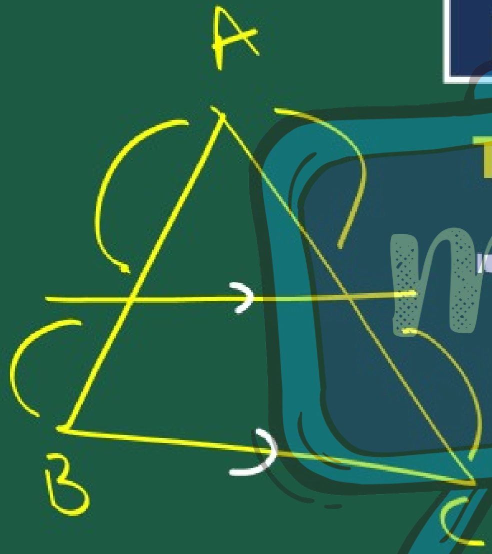
CONVERSE OF BASIC PROPORTIONALITY THEOREM



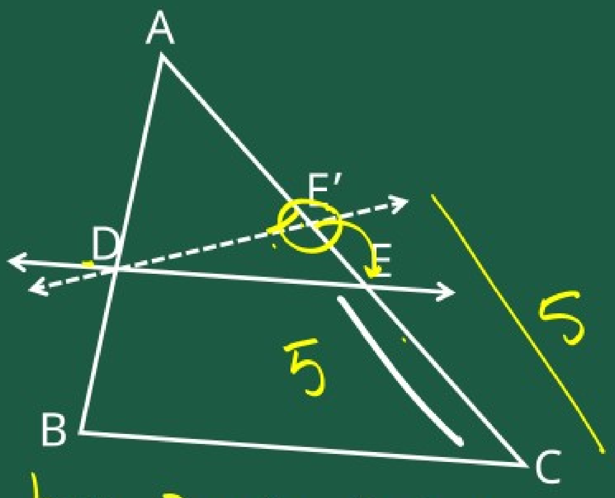
THEOREM 6.2

- If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Given: $\frac{AD}{DB} = \frac{AE}{EC}$

To prove: $DE \parallel BC$

Constr: $DE' \parallel BC$



Proof: $\frac{AD}{DB} = \frac{AE'}{E'C}$ — (1) {by Basic proportionality Theorem}

From Given and eq (1)

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

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$$1 + \frac{AE}{EC} = \frac{AE'}{E'C} + 1$$

$$\frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

$$\frac{\cancel{AC}}{EC} = \frac{\cancel{AC}}{E'C}$$

$$\boxed{EC = E'C}$$

$E'C = EC$ proves that E and E' coincide

then $DE \parallel BC$ $\{DE' \parallel BC\}$

Note

$$\frac{2+3+4}{4+6+8} = \frac{9}{18} = \frac{1}{2}$$

- The ratio of the perimeters of similar triangles is the same as the ratio of their corresponding sides.

LEARNING OUTCOMES

1. Similarity of Triangles
2. Thales theorem (B.P.T.)
3. Converse of B.P.T.

ASSESSMENT

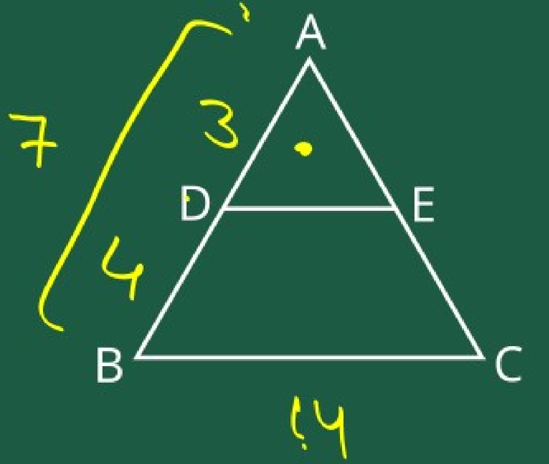
1 In the figure, if DE || BC, AD = 3 cm, BD = 4 cm and BC = 14 cm then DE equals

A 7 cm

B 6 cm

C 4 cm

D 3 cm



$\frac{AD}{DB} = \frac{AE}{EC}$

$1 + \frac{DB}{AD} = \frac{EC}{AE} + 1$

$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$

$\frac{AB}{AD} = \frac{AC}{AE}$

$\frac{\cancel{7}^{1}}{3} = \frac{BC}{DE} = \frac{\cancel{14}^{2}}{DE}$

$\boxed{DE = 6}$

ASSESSMENT

2 The perimeters of two similar triangles ABC, PQR is 64 cm and 24 cm respectively. If PQ is 12 cm what will be the length of AB?

A 30 cm

B 32 cm

C 12 cm

D 16 cm

$$\left.\frac{P_1}{P_2} = \frac{\cancel{64}}{\cancel{24}} = \frac{8}{3}\right\}$$

$$\frac{AB}{PQ} = \frac{P_1}{P_2} = \frac{8}{3}$$

$$\frac{AB}{\cancel{12}_{4}} = \frac{8}{\cancel{3}} \Rightarrow \boxed{AB = 32}$$

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ASSESSMENT

3 The ratio of any two corresponding sides in two equiangular triangles is always the same.

A True

B False