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CLASS – 10

MATHEMATICS

Chapter – 6

Triangles

Part – 7

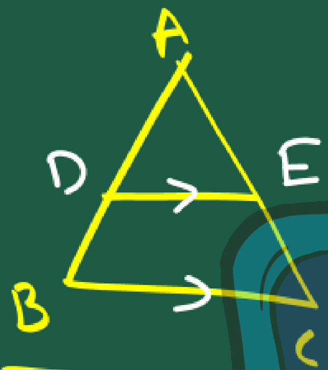
Criteria for Similarity of Triangles

Shubham Tiwari

OVERVIEW

1. Similar Figures
2. Similarity of Triangles
3. Criteria for Similarity of Triangles

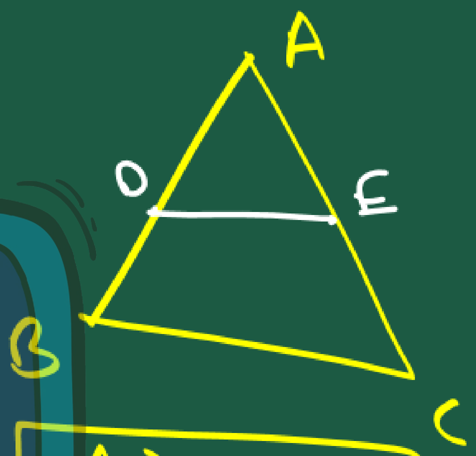
BPT



$DE \parallel BC$

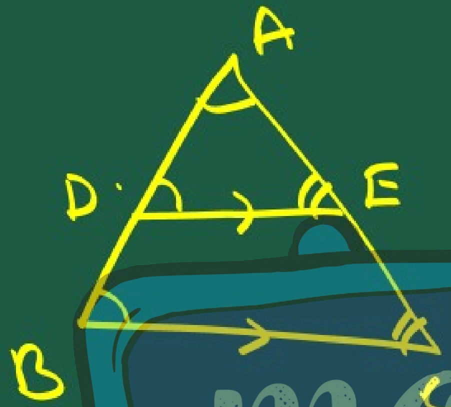
$$\boxed{\frac{AD}{BD} = \frac{AE}{EC}}$$

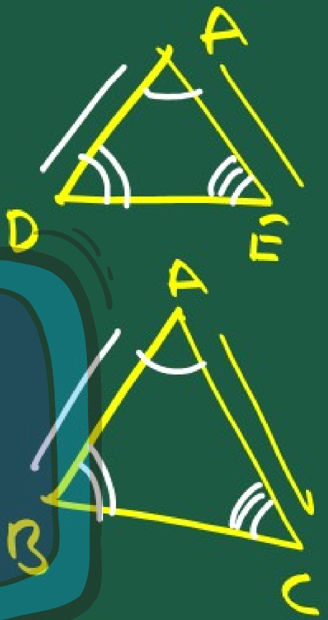
CBPT



$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$

$$\boxed{DE \parallel BC}$$





$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$1 + \frac{BD}{AD} = \frac{EC}{AE} + 1$$

$$\frac{AD+BD}{AD} = \frac{AE+EC}{AE}$$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

SIMILARITY OF TRIANGLES

- Two triangles are similar, if

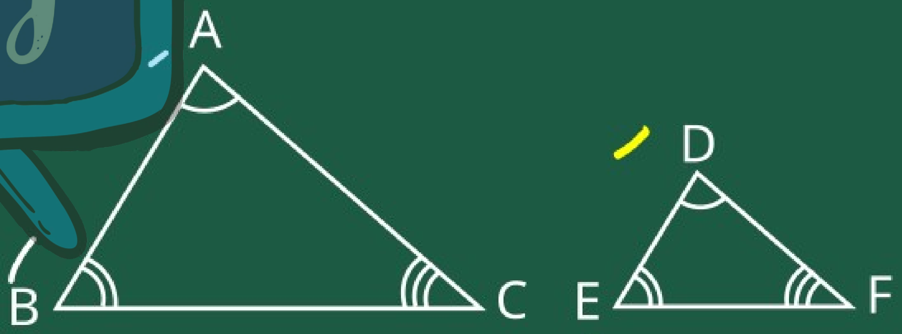
  (i) their corresponding angles are equal and

  (ii) their corresponding sides are in the same ratio (or Proportion).

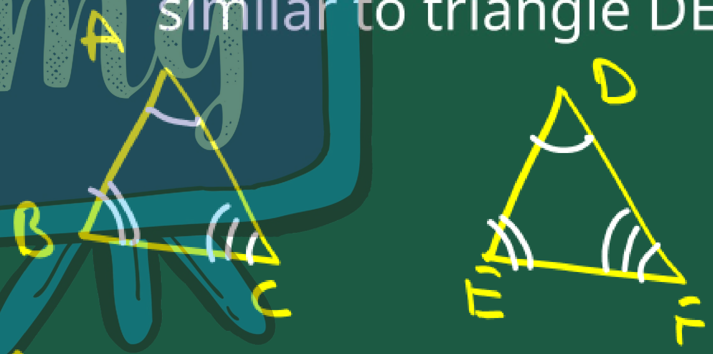
☛ In ΔABC and ΔDEF, if

(i) $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

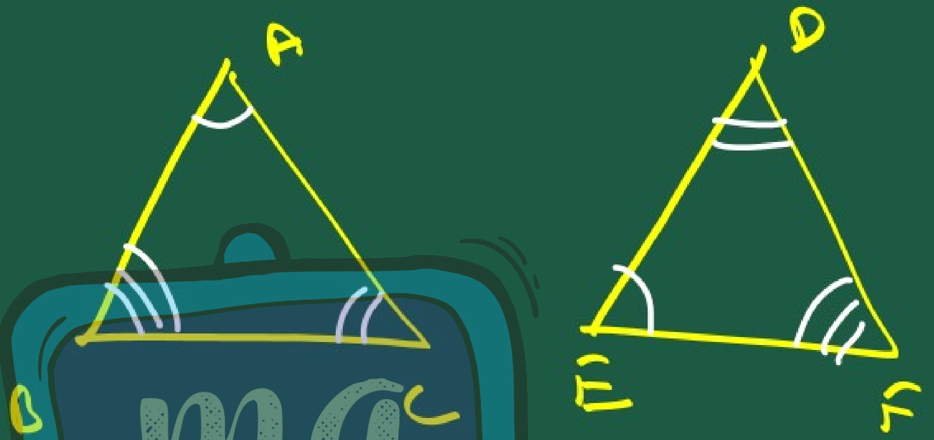
(ii) $\boxed{\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}}$ then the two triangles are similar.



- we write the similarity of these two triangles as '$\boxed{\Delta ABC \sim \Delta DEF}$' and read it as 'triangle ABC is similar to triangle DEF'



$\Delta ABC \sim \Delta DEF$



$\triangle ABC \sim \triangle DEF$

$\boxed{\triangle ABC \sim \triangle EFD}$

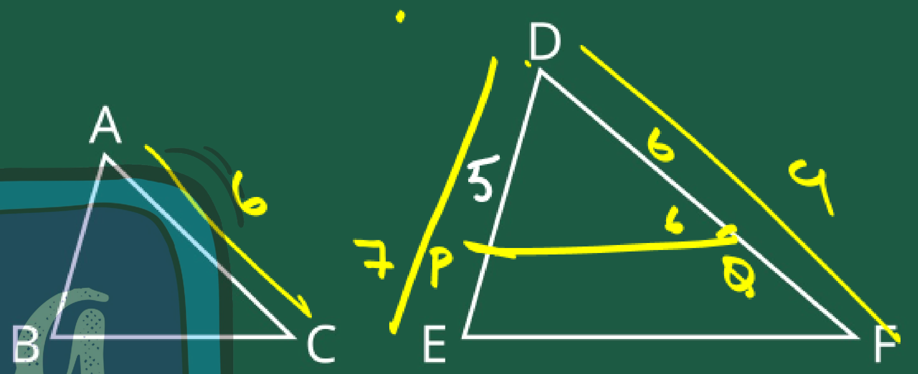
CRITERIA FOR SIMILARITY OF TRIANGLES

- AA Similarity Criterion
- SSS Similarity Criterion
- SAS Similarity Criterion

AAA (ANGLE-ANGLE-ANGLE) SIMILARITY CRITERION

Theorem : 6.3

- If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.



Given: $\angle A = \angle D$

$\angle B = \angle E$

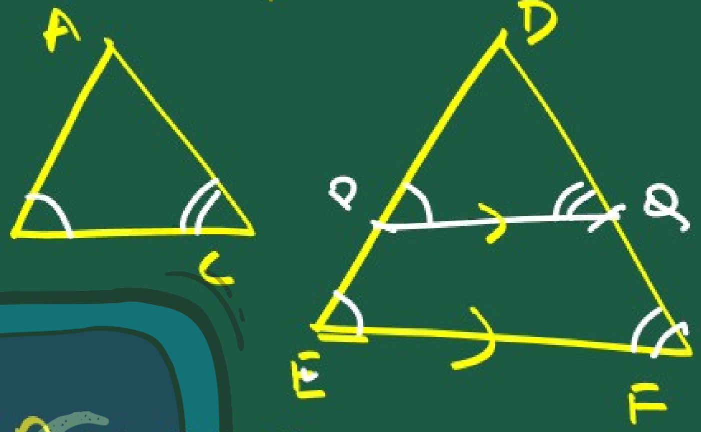
$\angle C = \angle F$

To Prove:

$\Delta ABC \sim \Delta DEF$

Const. Draw $DP = AB$

$DQ = AC$



Proof:
$\angle A = \angle D$ {Given}
$AB = DP$ } {By cons.}
$AC = DQ$ }

in $\triangle ABC$ and $\triangle DPQ$

We can say that

$\triangle ABC \cong \triangle DPQ$

as $\triangle ABC \cong \triangle DPQ$

$$\left.\begin{array}{l}\angle B = \angle P \\ \angle C = \angle Q\end{array}\right\} \text{CPCT}$$

$$\left.\begin{array}{l}\angle B = \angle E \\ \angle C = \angle F\end{array}\right\} \text{Given}$$

Hence

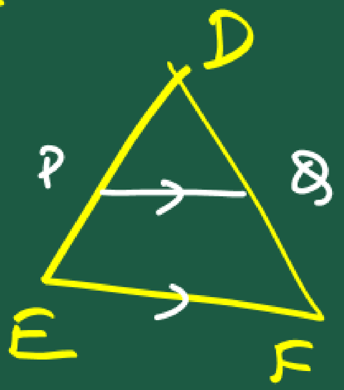
$\angle P = \angle E$

$\angle Q = \angle F$

$\therefore PQ \parallel EF$

By Applying BPT

in ΔDEF



$$\frac{DP}{PE} = \frac{DQ}{QF}$$

$$1 + \frac{PE}{DP} = \frac{QF}{DQ} + 1$$

$$\frac{DP+PE}{DP} = \frac{QF+DQ}{DQ}$$

$$\frac{DE}{DP} = \frac{DF}{DQ}$$

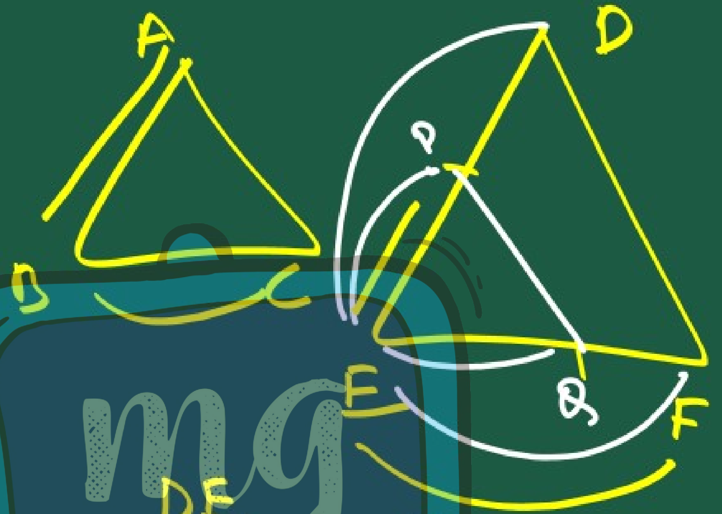
$$\frac{DE}{AB} = \frac{DF}{AC}$$

Similarly $\frac{DE}{AB} = \frac{EF}{BC}$

Hence

$$\boxed{\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}}$$

Hence $\triangle ABC \sim \triangle DEF$



$$\frac{DE}{AB} = \frac{EF}{BC}$$

Note

- If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows :

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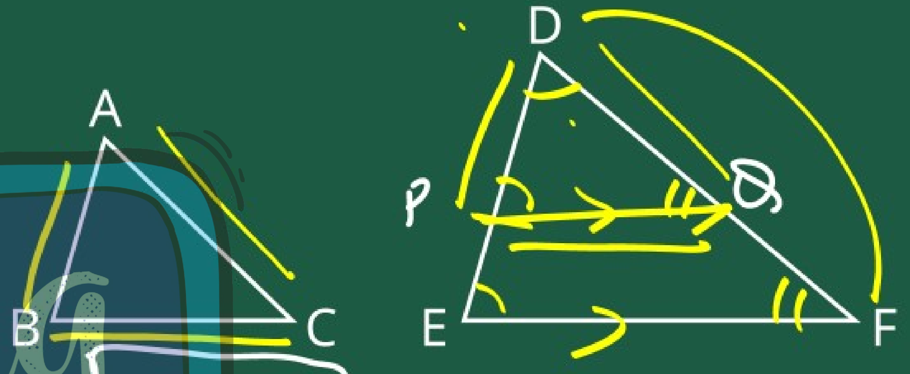
- If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

SSS (SIDE–SIDE–SIDE) SIMILARITY CRITERION

Theorem : 6.4

- If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.



Given :- $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Cons: $DP = AB$

$DQ = AC$

To Prove :- $\triangle ABC \sim \triangle DEF$

Proof:

$AB = DP$ | $AC = DQ$

$$\frac{AB}{DE} = \frac{AC}{DF} \quad \boxed{\frac{DP}{DE} = \frac{DQ}{DF}}$$

$$\frac{DP}{DE} = \frac{DQ}{DF}$$

$$\frac{DP}{PE} = \frac{DQ}{QF}$$

Hence from converse of BPT

$PQ \parallel EF$

Hence in $\triangle DPQ$ & $\triangle DEF$

$\angle D = \angle D$ {common}

$\angle P = \angle E$ } Corresponding Angle

$\angle Q = \angle F$ }

By AAA similarity Rule.

$$\triangle DPQ \sim \triangle DEF$$

$$\frac{DP}{DE} = \frac{PQ}{EF} = \frac{DQ}{DF}$$

$$\frac{AB}{DE} = \frac{PQ}{EF} = \frac{AC}{DF}$$

Hence from the above prove and by given.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{PQ}{\cancel{EF}} = \frac{BC}{\cancel{EF}}$$

$PQ = BC$

hence $\triangle ABC \cong \triangle DPQ$

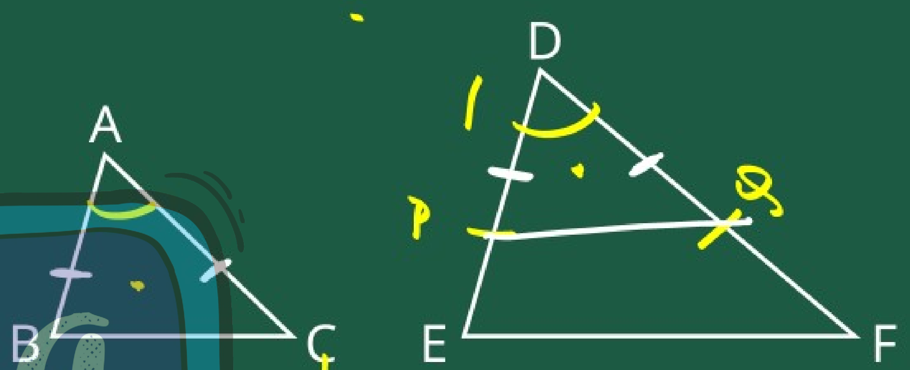
if $\triangle DPQ \sim \triangle DEF$

$\boxed{\text{Then } \triangle ABC \sim \triangle DEF}$

SAS (SIDE–ANGLE–SIDE) SIMILARITY CRITERION

Theorem : 6.5

- If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.



Given $\angle A = \angle D$

$$\frac{AB}{DE} = \frac{AC}{DF}$$

To prove: $\triangle ABC \sim \triangle DEF$

Cons: $AB = DP$

$AC = DQ$

in ΔABC & ΔDPQ

$AB = DP$ } [Const]

$AC = DQ$

$\angle A = \angle D$ [Given]

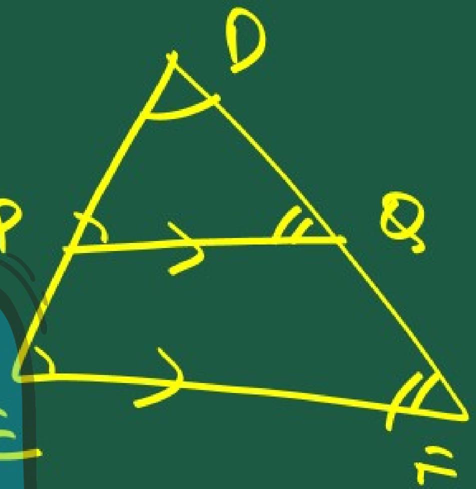
By SAS Congruency Rule

$\Delta ABC \cong \Delta DPQ$

$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$\therefore \frac{DP}{DE} = \frac{DQ}{DF} \text{ \{Sy cny\}}$$

$$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$$



Hence By Converse of BPT

PQ || EF

in ΔDPQ & ΔDEF

$\angle D = \angle D$ — Given

$\angle P = \angle E$ — Corresponding Angle

Here $\Delta DPQ \sim \Delta DEF$

as $\Delta DPQ \cong \Delta ABC$

Therefore $\Delta ABC \sim \Delta DEF$

H.P.

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LEARNING OUTCOMES

1. AAA Similarity Criterian
2. SSS Similarity Criterian
3. SAS Similarity Criterian

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ASSESSMENT

1 If $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ then, ΔABC & ΔPQR are similar according to which test?

A AA test

B SAS test

C SSS test

D None

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ASSESSMENT

2 If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$ then, ΔABC & ΔXYZ are similar according to which test?

A. AAA test

B. AA test

C. SAS test

D. SSS test

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ASSESSMENT

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3 Which of the following is not a similarity criterion for two triangles?

A AA

B SAS

C SSS

D ASA