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# CLASS – 10

# MATHEMATICS

## Chapter – 5

## Arithmetic Progressions

### Part – 9

### Sum of first n Terms of an AP

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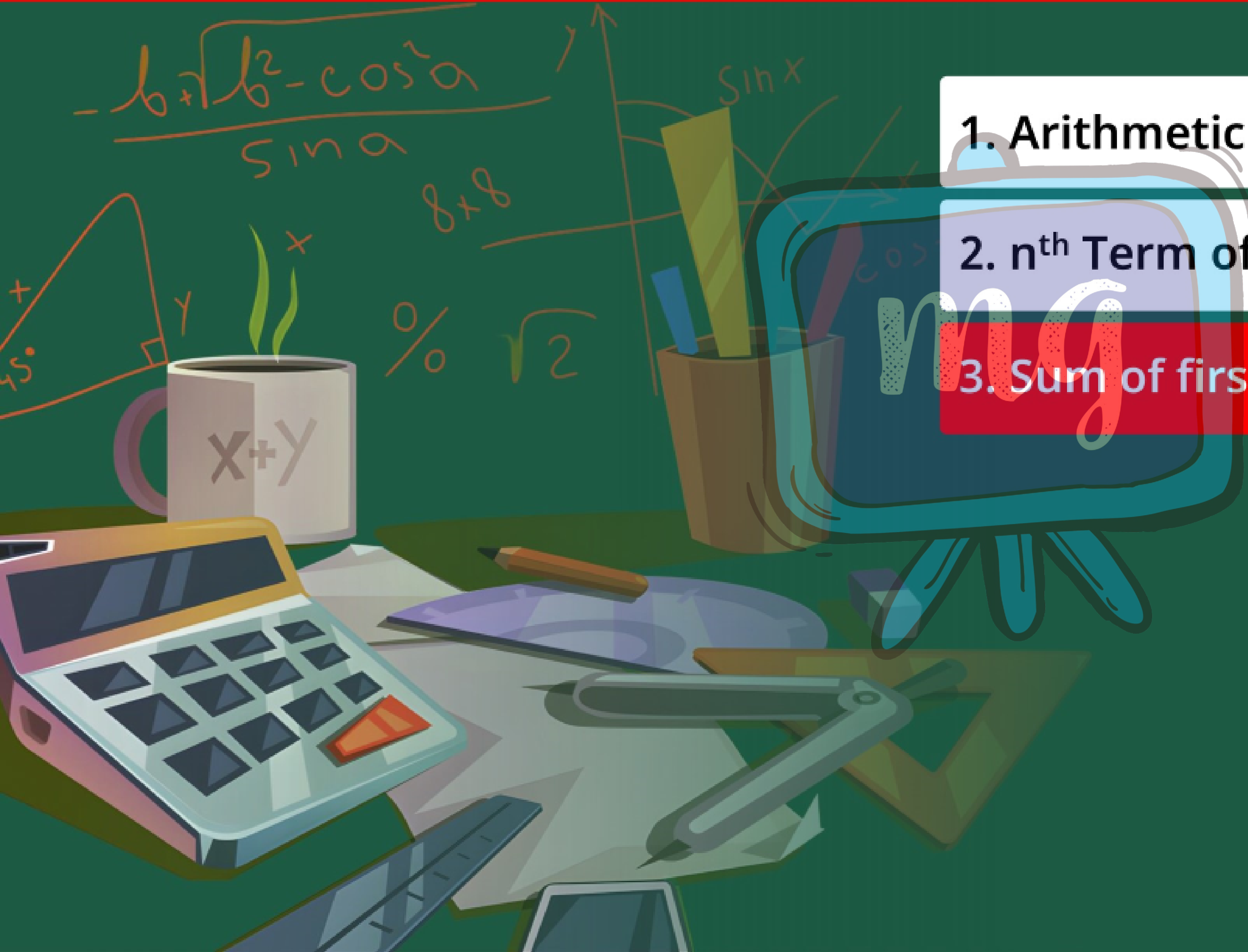
# OVERVIEW



1. Arithmetic Progressions
2. nth Term of an AP
3. Sum of first n Terms of an AP

$$\bigcirc\!\!\!\!1 + 2 + 3 + 4 + \text{-- ——————} + \bigcirc\!\!\!\!\!100$$

$$100 + 99 + 98 + 97 + \text{-- -- —— ——} \quad 1$$

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$$\underbrace{101} + \underbrace{101} + \underbrace{101} + \underbrace{101} + \text{- - —} \quad + 101$$

$$\frac{101 \times \overset{50}{\cancel{100}}}{\cancel{2}} = \boxed{5050}$$

$$S_n = \overset{a}{a_1} + a_2 + a_3 + a_4 + a_5 - - \text{———} + a_n$$

$$S_n = a_n + a_{n-1} + a_{n-2} + a_{n-3} + - - \text{———} a$$

$$(a+a_n) + (a+a_n) + (a+a_n) - - \text{——} (a+a_n)$$

$$a_1 = a$$
$$a_n = a_n$$
$$a + a_n$$

$$a_2 = a \cancel{+d}$$
$$a_{n-1} = a_n \cancel{-d}$$
$$a + a_n$$

$$a_3 = a \cancel{+2d}$$
$$a_{n-2} = a_n \cancel{-2d}$$
$$a + a_n$$

$$2\underline{S_n} = n(a + a_n)$$

$$S_n = \frac{n}{2}\left[a + a_n\right] \Rightarrow$$

$$\boxed{S_n = \frac{n}{2}\left[a + l\right]}$$

$$S_n = \frac{n}{2}\left[\underbrace{a + a}\,+(n-1)d\right]$$

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

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# SUM OF FIRST n TERMS OF AN AP

- The sum of the first n terms of an AP with first term a and common difference d is given by

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

☛ If there are only n terms in an AP, and $a_n = \ell$, the last term then,

$$\boxed{S_n = \frac{n}{2}\left[a + \ell\right]}$$

OR

$$\boxed{S_n = \frac{n}{2}\left[\text{First term} + \text{Last term}\right]}$$

# $n^{th}$ TERM OF AN AP

The $n^{th}$ term of an AP is the difference of the sum to first n terms and the sum to first (n – 1) terms of it.

$$\boxed{S_n = 2n^2 + 5n}$$

$a_{15} =$

$$\boxed{a_n = S_n - S_{n-1}}$$

$a_{15} = S_{15} - S_{14}$

$$\boxed{1+2+3+4+5+6+7+8+9}+10$$

$$\boxed{S_{10} - S_9 = a_{10}}$$

## Note

- ☛ The sum of terms equidistant from the start and end of an AP is constant and is equal to the sum of the first and last term.

- Sum of first n natural numbers, i.e., 1 + 2 + 3 ....... + n

$$\Rightarrow \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

- Sum of first n even natural numbers, i.e., 2 + 4 + 6 + ... + n

$$\Rightarrow \sum_{r=1}^{n} 2r = n(n+1)$$

$$\frac{n(n+1)}{2} - n(n+1)$$

$$n(n+1)\left[\frac{1}{2} - \frac{1}{1}\right)$$

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$$1+2+3+4+---n$$

$$S_n = \frac{n}{2}\left[a+l\right]$$

$$= \frac{n}{2}\left[1+n\right]$$

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$$2+4+6+8+---\underline{n^{th}}$$

$$2\left[\underbrace{1+2+3+4+5+---n}\right]$$

$$\cancel{2} \times \frac{n(n+1)}{\cancel{2}} = n(n+1)$$

- Sum of first n odd natural numbers, i.e., 1 + 3 + 5 +…+ n

$$\Rightarrow \sum_{r=1}^{n}(2r-1) = \mathbf{n}^2$$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[2\times 1+(n-1)2\right]$$

$$= \frac{n}{2}\left[2+2(n-1)\right]$$

$$\frac{n}{\cancel{2}}\left[\cancel{2}(1+(n-1))\right]$$

$$= n(\cancel{1}+n-\cancel{1})$$

$$= \underline{n^2}$$

## Example : 11

Find the sum of the first 22 terms of the AP : $\underbrace{8, 3, -2,}$ . . .

Soln. $S_{22} = ?$ | $a = 8$ | $d = -5$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{22} = \frac{\cancel{22}^{11}}{\cancel{2}}\left[2(8) + (22-1)(-5)\right]$$

$$= 11\left[16 + 21 \times -5\right]$$

$$S_{22} = 11\left[16 + 21 \times -5\right]$$

$$= 11\left[16 + (-105)\right]$$

$$= 11\left[-89\right]$$

$$S_{22} = -979$$

$$\boxed{S_{22} = -979}$$

## Example : 12

If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

Soln. $S_{14} = 1050 \mid a = 10 \mid a_{20} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{14} = \frac{14}{2}\left[2(10) + (14-1)d\right]$$

$$1050 = 7\left[20 + 13d\right]$$

$$\Rightarrow \frac{1050}{7} = 20 + 13d$$

$$\Rightarrow 150 = 20 + 13d$$

$$\Rightarrow 150 - 20 = 13d$$

$$130 = 13d$$

$$\frac{130}{13} = d$$

$$\boxed{d = 10}$$

Hence $a_{20} = a + 19d$

$= 10 + 19 \times 10$

$= 10 + 190$

$\boxed{a_{20} = 200}$

# Example : 13

How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

$$\overbrace{24 + 21 + 18 + 15}^{78}$$

$+ 12 + 9$

$+ 6 + 3 + 0 + -3 - 6$

④ 78

⑬ 78

$S_n = 78 \mid a = 24 \mid d = 21 - 24$

$$\boxed{d = -3}$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$78 = \frac{n}{2}\left[2 \times 24 + (n-1)(-3)\right]$$

$$78 = \frac{n}{2}\left[2 \times 24 + (n-1) \cdot -3\right]$$

$$78 \times 2 = n\left[48 + (-3n+3)\right]$$

$$156 = n\left[48 - 3n + 3\right]$$

$$156 = n\left[51 - 3n\right]$$

$$156 = 51n - 3n^2$$

$$\boxed{3n^2 - 51n + 156 = 0}$$

$$3n^2 - 51n + 156 = 0$$

$$\underbrace{3n^2 - 39n}\ \underbrace{-12n + 156} = 0$$

$$3n[n-13] - 12[n-13] = 0$$

$$(3n-12)(n-13) = 0$$

$$3n - 12 = 0 \qquad n - 13 = 0$$

$$n = \frac{12}{3} \qquad \boxed{n = 13}$$

$$\boxed{n = 4}$$

$3 \times 156$

$3 \times 2 \times 78$

$3 \times 2 \times 2 \times 39$

$12 \times 39$

Hence in the given AP.

There are $n = 13$ and $n = 4$

two number of terms are possible

at which we will get $S_n = 78$.

## Example : 14

Find the sum of :

(i) the first 1000 positive integers

$S_n = ?$ | $n = 1000$ | $a = 1$ | $d = 1$

$S_n = \frac{n}{2}[a + l]$

$= \frac{1000}{2}[1 + 1000]$

$1001 \times \frac{1000}{2}$

$1001 \times 500$

$\underline{500500}$

(ii) the first n positive integers

$$S_n = \frac{n}{2}\left[2a + (n-1)\, d\right]$$

$$= \frac{n}{2}\left[2 \times 1 + (n-1)\, 1\right]$$

$$= \frac{n}{2}\left[2 + (n-1)\right]$$

$$\boxed{S_n = \frac{n}{2}(1+n)}$$

## Example : 15

Find the sum of first 24 terms of the list of numbers whose nth term is given by

$$a_n = 3 + 2n$$

Soln

$S_{24} = ?$

$$a_n = 3 + 2n$$

$$a_1 = 3 + 2(1)$$

$$= 3 + 2$$

$$\boxed{a_1 = 5}$$

$$a_n = 3 + 2n$$

$$a_2 = 3 + \underbrace{2 \times 2}$$

$$= 3 + 4$$

$$\boxed{a_2 = 7}$$

$$d = a_2 - a_1 = 7 - 5 = 2$$

$$\boxed{d = 2}$$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{24} = \frac{24}{2}\left[2\times5+(24-1)2\right]$$

$$= 12\left[10+23\times2\right]$$

$$= 12\left[10+46\right]$$

$$= 12\times56$$

$$\boxed{S_{24} = 672}$$

## Example : 16

A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :

(i) the production in the 1st year

$a_3 = 600$

$a_7 = 700$

$a_3 = 600$

$a + 2d = 600$

$a_7 = 700$

$\cancel{a} + 6d = 700$

$\cancel{a} + 2d = 600$

$-\quad\quad -\quad\quad -$

$4d = 100$

$d = \frac{100}{4}$

$\boxed{d = 25}$

$a + 2d = 600$

$a + 2 \times 25 = 600$

$a = 600 - 50$

$\boxed{a = 550}$

Hence the production in the first year was 550 TV sets.

(ii) the production in the 10th year

$$a_{10} = a + 9d$$

$$= 550 + 9 \times 25$$

$$= 550 + 225$$

$$\boxed{a_{10} = 775}$$

in the 10th year production was 775 Tv
sets

(iii) the total production in first 7 years

$$S_7 = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{7}{2}\left[2\times550+(7-1)25\right]$$

$$= \frac{7}{\cancel{2}}\left[\cancel{2}\times550+\overset{3}{\cancel{6}}\times25\right]$$

$$= 7\left[550+75\right]$$

$$S_7 = 7[550+75]$$

$$= 7[625]$$

$$\boxed{S_7 = 4375}$$

Hence the total production till 7 years was 4375 TV sets.

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# LEARNING OUTCOME

Sum of the first n terms of an A.P.

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# ASSESSMENT

1 The sum of the first five multiples of 3 is :

A 45

B 55

C 65

D 75

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# ASSESSMENT

**2** The sum of all odd integers between 2 and 100 divisible by 3 is :

- A 17
- B 867
- C 876
- D 786

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# ASSESSMENT

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**3** Find the sum of 12 terms of an A.P. whose $n^{th}$ term is given by $a_n = 3n + 4$

A 262

B 272

C 282

D 292