

CLASS – 10

MATHEMATICS

Chapter – 5

Arithmetic Progressions

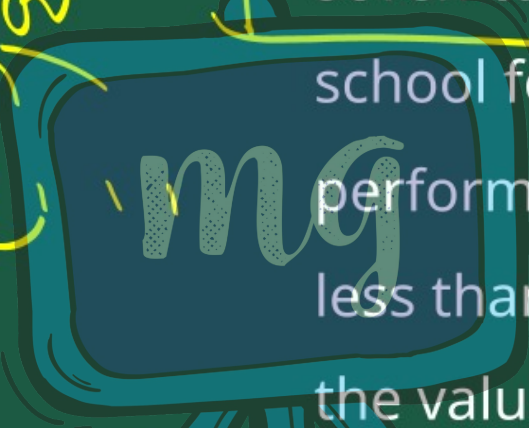
Part – 14

EXERCISE 5.3 (Q.16 – 20)

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16. A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

700



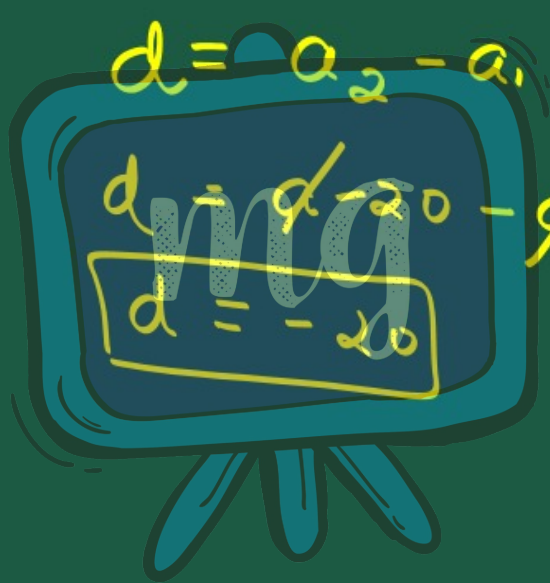
Soln

$$S_7 = 700$$

$$| d = -20$$

$$\underline{n = 7}$$

$$a_1, a_2, a_3, \dots, a_n$$



$$\left\{ \begin{array}{l} a_1 = a \\ a_2 = a - 20 \end{array} \right.$$

$$S_7 = \frac{7}{2} [2a + (7-1)d]$$

$$~~700 = \frac{7}{2} [2a + \frac{3}{2}(-20)]~~$$

$$100 = [a + 3(-20)]$$

$$100 = a - 3 \times 20$$

$$100 = a - 60$$

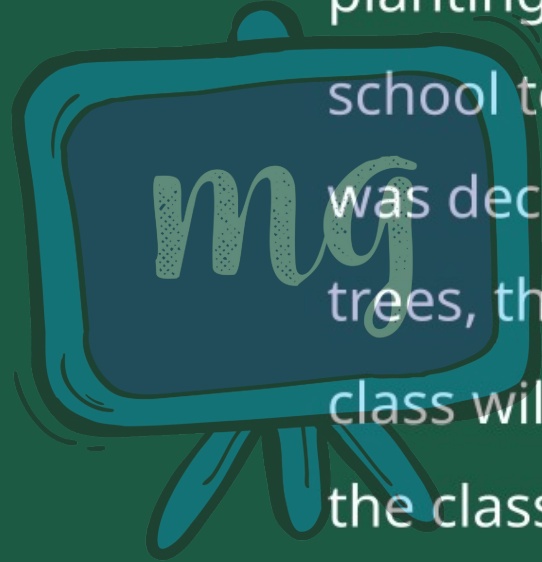
$$\boxed{160 = a}$$

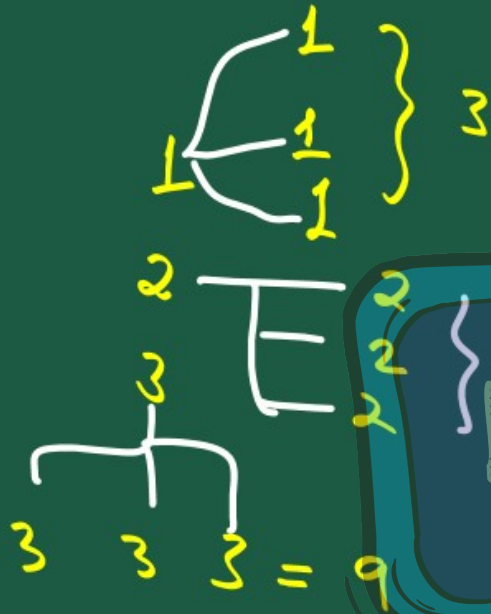
as the First prize is 160 ₱.

then the rest of the prizes are.

160	→	1
140	→	2
120	→	3
100	→	4
80	→	5
60	→	6
40	→	7

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g.,





a section of Class I will plant 1 tree,
a section of Class II will plant 2
trees and so on till Class XII. There
are three sections of each class.
How many trees will be planted by
the students?

Solve 3, 6, 9, --- 36

3 + 6 + 9 + --- 36

12 } 36
12 }
12 }
12 }
36

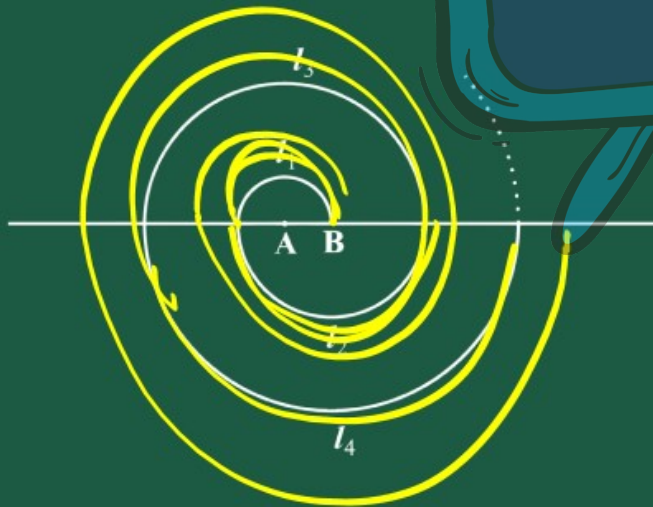
$$S_n = \frac{n}{2} [a + l]$$

$$S_{12} = \frac{12}{2} [3 + 36]$$

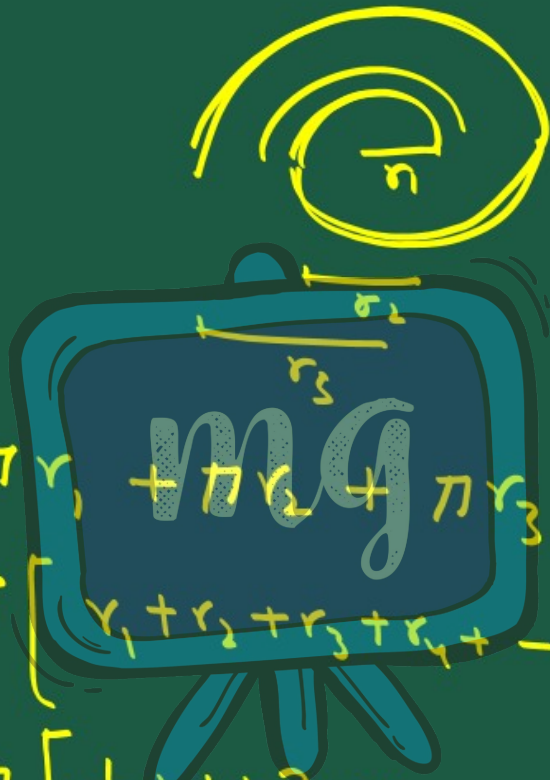
$$= 6 [39]$$

$$S_{12} = 234$$

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. What is the total length of such a spiral made up of thirteen consecutive semicircles?



(Take $\pi = \frac{22}{7}$)



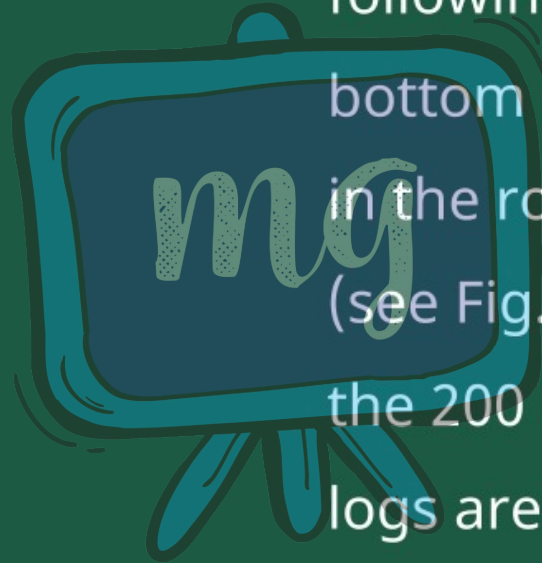
$$\begin{aligned}
 &= \pi r_1 + \pi r_2 + \pi r_3 + \pi r_4 + \dots - \pi r_{13} \\
 &= \pi \left[r_1 + r_2 + r_3 + r_4 + \dots - r_{13} \right] \\
 &= \pi \left[\frac{1}{2} + 1 + \frac{3}{2} + 2 + \dots - r_{13} \right]
 \end{aligned}$$

$$\begin{aligned} &= \pi \left[\frac{1}{2} + 1 + \frac{3}{2} + 2 + \dots + 13 \right] \\ &= \pi \left[S_{13} \right] \\ &= \pi \left[\frac{13}{2} \left[2a + (13-1)d \right] \right] \\ &= \pi \left[\frac{13}{2} \left[2 \times \frac{1}{2} + \left[\frac{6}{12} \right] \times \frac{1}{2} \right] \right] \\ &= \pi \left[\frac{13}{2} \times (1+6) \right] \end{aligned}$$

$$\begin{aligned} &= \pi \left[\frac{13}{2} \times 7 \right] \\ &= \frac{22}{7} \times \frac{13}{2} \times 7 \\ &= 11 \times 13 \\ &= 143 \text{ cm} \end{aligned}$$

length
of spiral

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig.). In how many rows are the 200 logs placed and how many logs are in the top row?



$$S_n = 200 \quad | \quad 20, 19, 18, \dots$$

$$a_1 = 20 \quad | \quad d = 19 - 20$$
$$\boxed{d = -1}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$200 = \frac{n}{2} [2 \times 20 + (n-1)(-1)]$$

$$200 = \frac{n}{2} [40 - n + 1]$$

$$400 = n(41 - n)$$

$$400 = n(41 - n)$$

$$400 = 41n - n^2$$

$$n^2 - 41n + 400 = 0$$

$$n^2 - 25n - 16n + 400 = 0$$

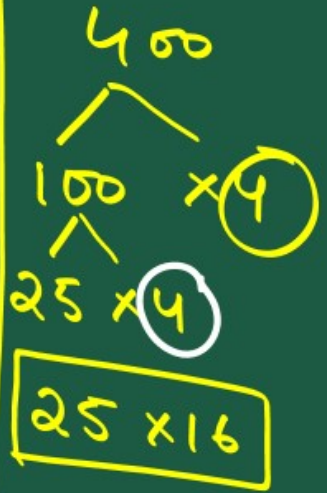
$$n[n - 25] - 16[n - 25] = 0$$

$$(n - 25) = 0$$

$$n = 25$$

$$(n - 16) = 0$$

$$n = 16$$



$$\begin{aligned}a_{16} &= a + 15d \\ &= 20 + 15(-1) \\ &= 20 + (-15) \\ &= 20 - 15\end{aligned}$$

$$a_{16} = 5$$

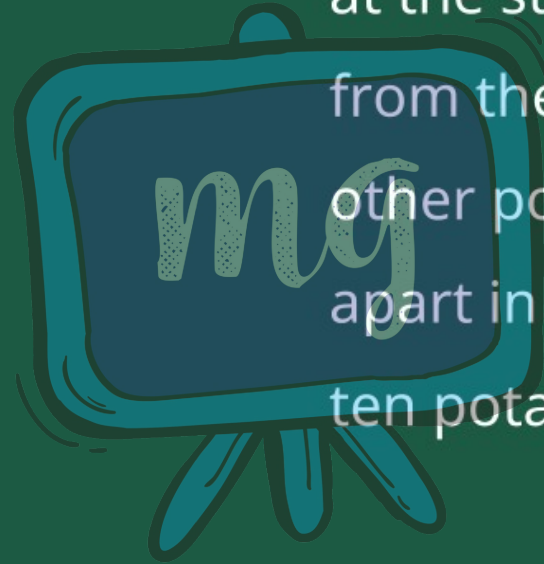
$$\begin{aligned}a_{25} &= a + 24d \\ &= 20 + 24(-1) \\ &= 20 - 24\end{aligned}$$

$$a_{25} = -4$$

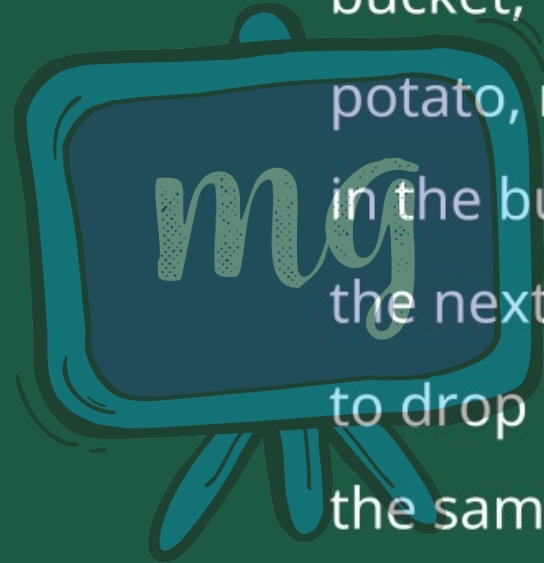
Rejected

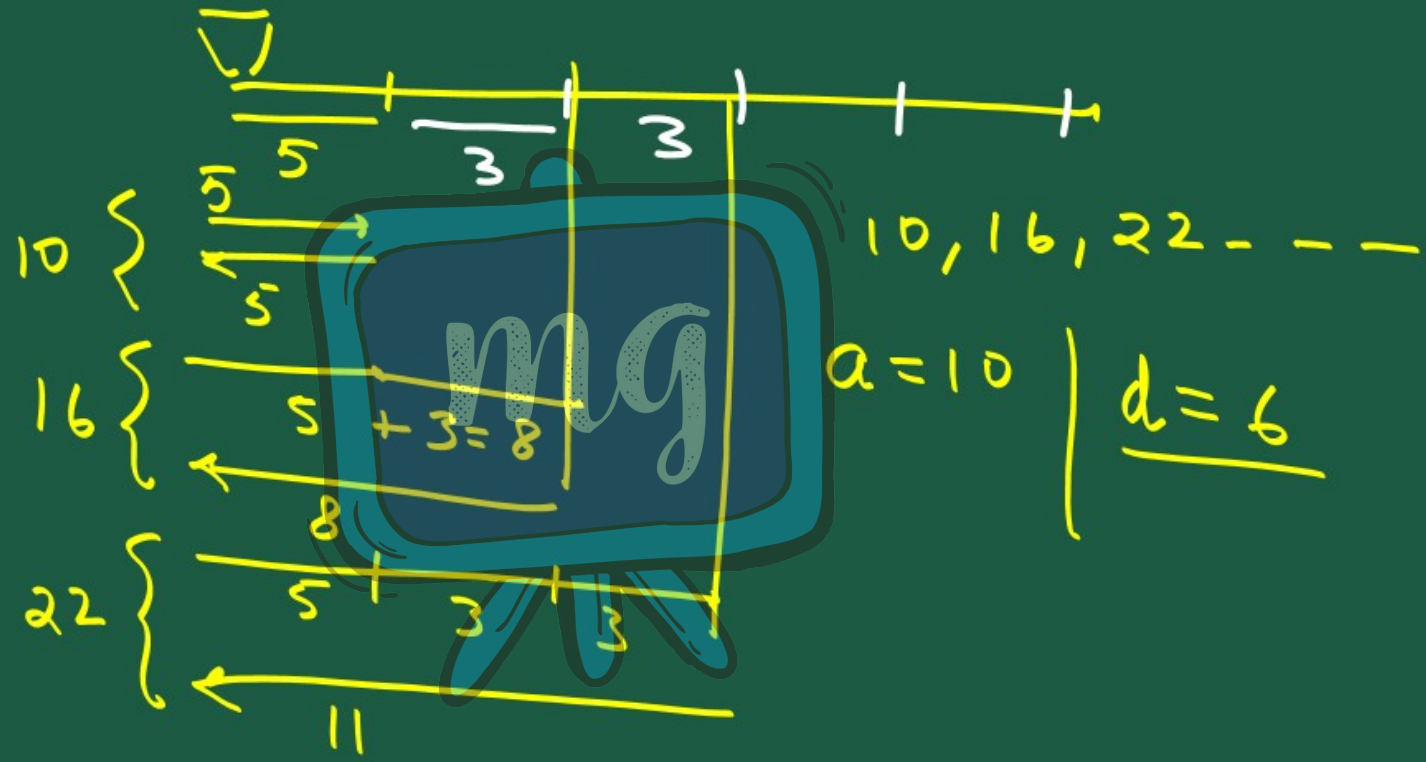
Hence there are total
16 rows and on
16th row, there are 5 logs placed.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig.).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?





$$S_{10}$$

$$S_{10} = \frac{10}{2} [2a + (10-1)d]$$

$$S_{10} = 5 [2(10) + 9 \times 6]$$

$$= 5 [20 + 54]$$

$$= 5 [74]$$

$$S_{10} = 370m$$