CLASS – 10

MATHEMATICS

Chapter – 5

Arithmetic Progressions

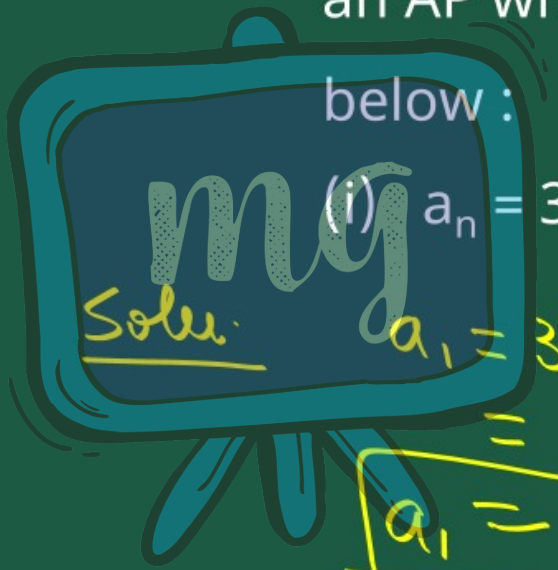
Part – 13

EXERCISE 5.3 (Q.10 – 15)

Shubham Tiwari

10. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below :

(i) $a_n = 3 + 4n$



Solu.

$a_1 = 3 + 4(1)$

$= 3 + 4$

$a_1 = 7$

$a_2 = 3 + 4(2)$

$a_2 = 3 + 8 = 11$

$$d = a_2 - a_1 = 11 - 7 = 4$$

$$S_{15} = \frac{15}{2}\left[2a + (15-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2 \times 7 + 14 \times 4\right]$$

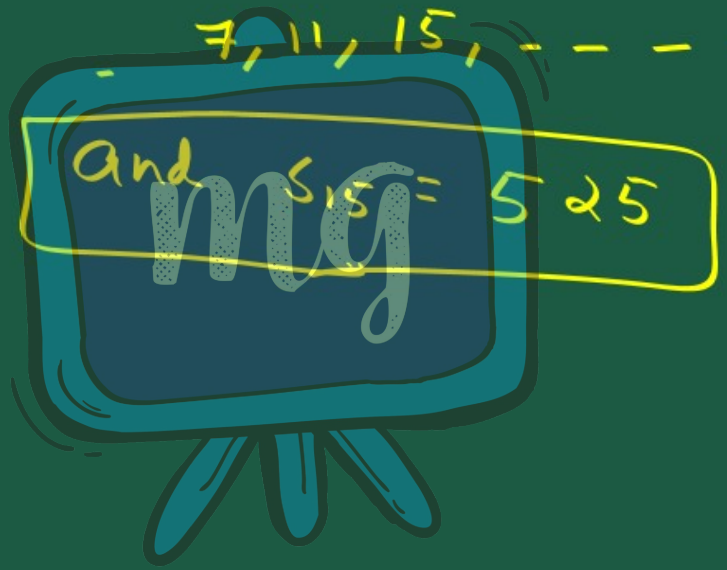
$$= 15\left[7 + 28\right]$$

$$= 15 \times 35$$

$$S_{15} = 525$$

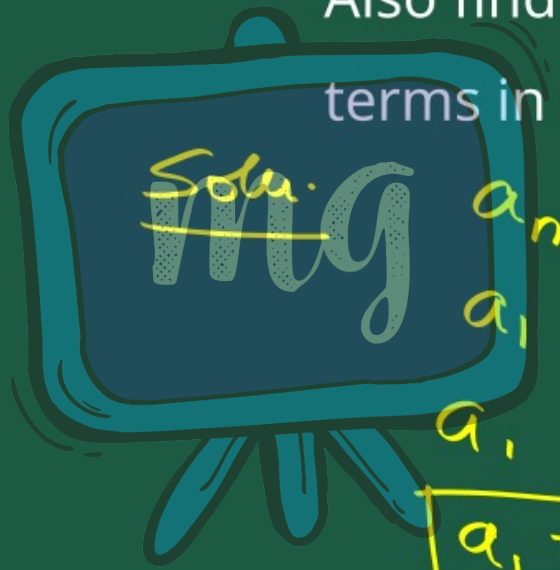
Here the AP is

7, 11, 15, - - - - -



(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.



Sol.

$a_n = 9 - 5n$

$a_1 = 9 - 5(1)$

$a_1 = 9 - 5$

$a_1 = 4$

$a_2 = 9 - 5(2) = 9 - 10 = -1$

$$d = a_2 - a_1 = -1 - 4 = -5$$

$$S_{15} = \frac{15}{2}\left[2a + (15-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2 \times 4 + (14)(-5)\right]$$

$$S_{15} = 15\left[4 + (-35)\right]$$

$$= 15\left[-31\right]$$

$$S_{15} = -465$$

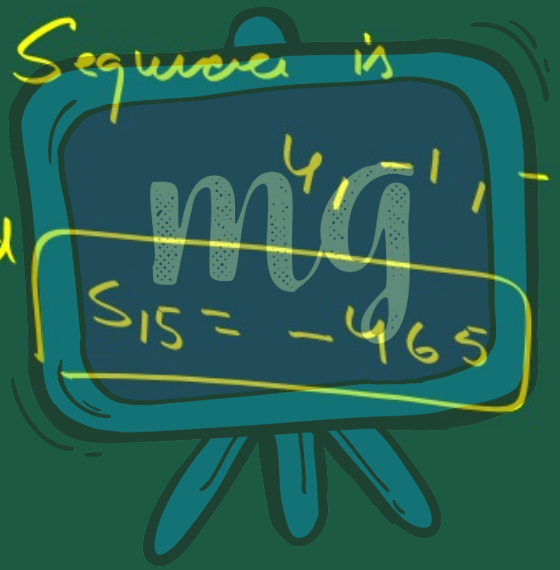
Hence the AP of the given

Sequence is

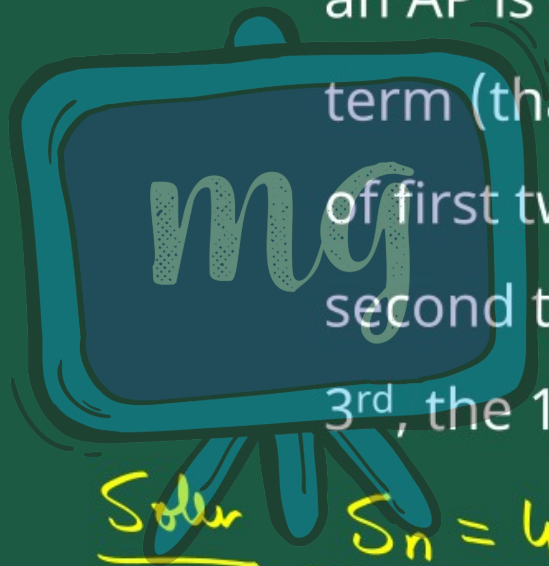
4, −1, −6, ...

and

$S_{15} = -465$



11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1) ? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.



Solution

$S_n = 4n - n^2$

$S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

$a_1 = 3$

$S_1 = a_1$

$S_2 = a_1 + a_2$

$4 = 3 + a_2$

$4 - 3 = a_2$

$1 = a_2$

$S_2 = 4(2) - (2)^2$

$= 8 - 4$

$\boxed{S_2 = 4}$

$d = a_2 - a_1$

$d = 1 - 3$

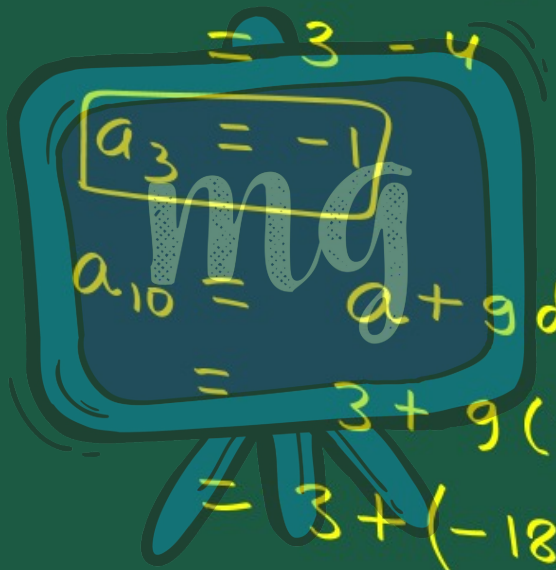
$\boxed{d = -2}$

$a_3 = a + 2d$

$= 3 + 2(-2)$

$= 3 - 4$

$a_3 = -1$



$a_{10} = a + 9d$

$= 3 + 9(-2)$

$= 3 + (-18)$

$a_{10} = 3 - 18 = -15$

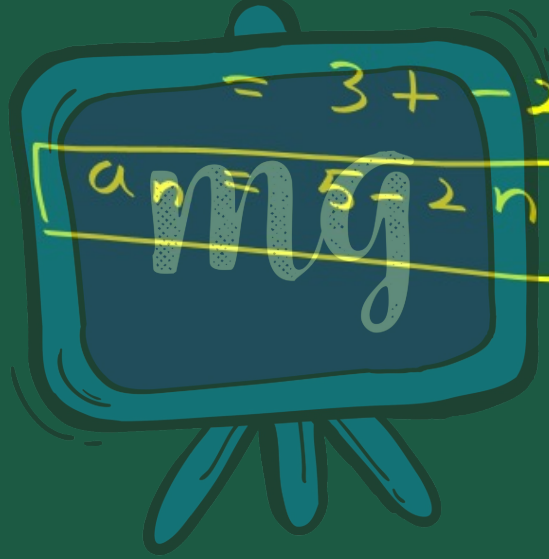
$a_{10} = -15$

$$a_n = a + (n-1)d$$

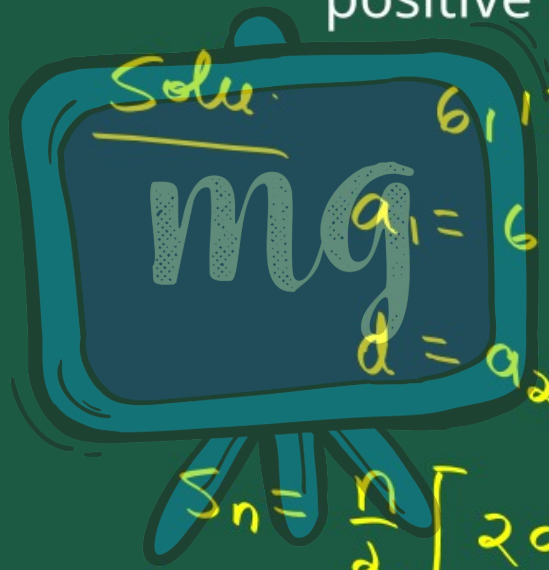
$$= 3 + (n-1)(-2)$$

$$= 3 + -2n + 2$$

$$\boxed{a_n = 5 - 2n}$$



12. Find the sum of the first 40 positive integers divisible by 6.



Sol:- $6, 12, 18, 24 - - - \; 40^{th}$

$a_1 = 6 \quad | \quad a_2 = 12$

$d = a_2 - a_1 = 12 - 6 = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{40} = \frac{40}{2}[2 \times 6 + (39)6]$

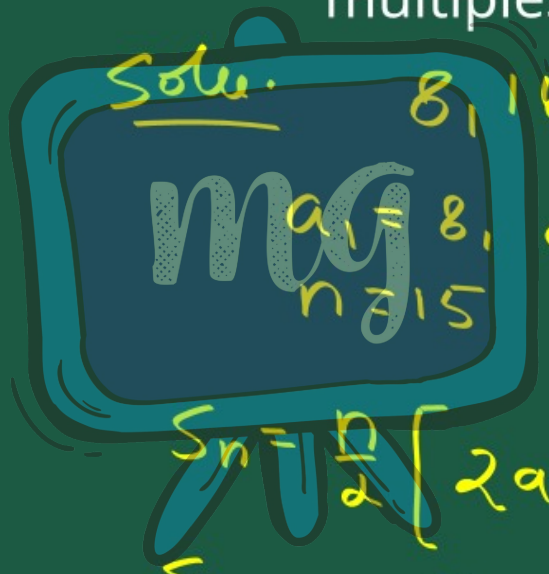
$$S_{40} = 20\left[12 + 39 \times 6\right]$$

$$= 20\left[12 + 234\right]$$

$$= 20\left[246\right]$$

$$S_{40} = 4920$$

13. Find the sum of the first 15 multiples of 8.



Solu. $8, 16, 24 - - - 15^{th}$

$a_1 = 8, \quad a_2 = 16, \quad d = 8$

$n = 15$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2 \times 8 + (15-1)8]$

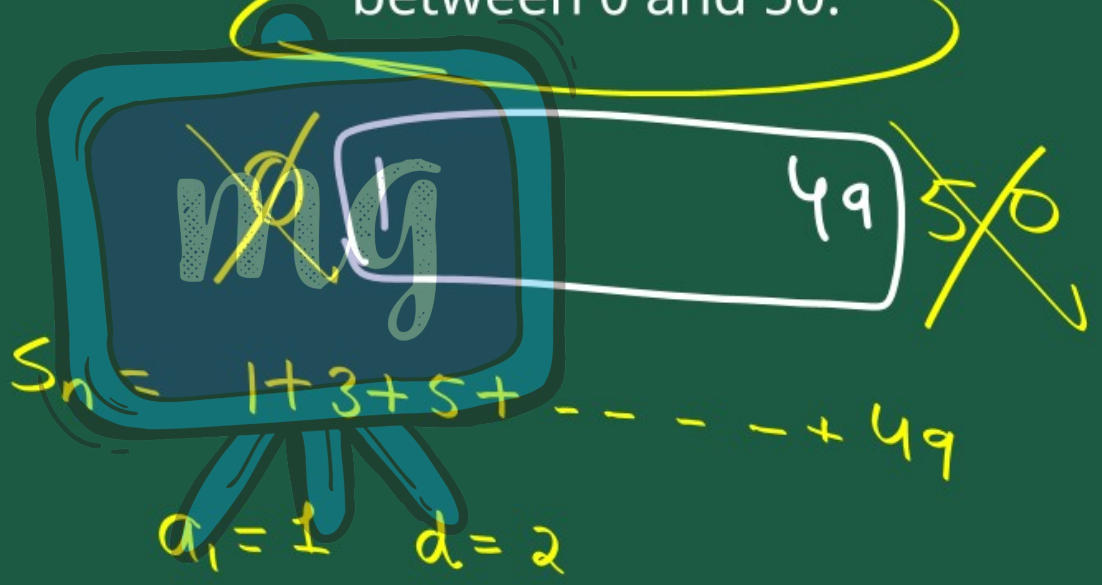
$$S_{15} = 15\left[8 + 14 \times 4\right]$$

$$= 15\left[8 + 56\right]$$

$$= 15\left[64\right]$$

$$S_{15} = 960$$

14. Find the sum of the odd numbers between 0 and 50.



$$a_n = 49$$

$$a+(n-1)d = 49$$

$$1+(n-1)2 = 49$$

$$(n-1)2 = 48$$

$$(n-1) = 24$$

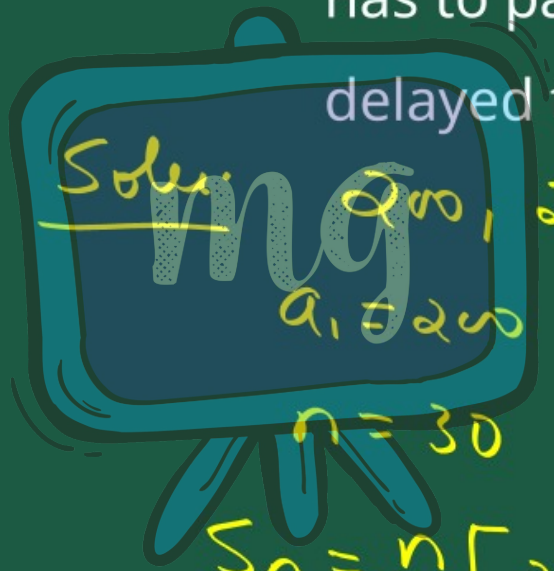
$$\boxed{n = 25}$$

$$S_n = \frac{n}{2}[a+l] = \frac{25}{2}[1+49] = \frac{25}{\cancel{2}_1}[\overset{25}{\cancel{50}}]$$

$$\boxed{S_n = 625}$$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day.

How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.



Sol: 200, 250, 300 - - -

$a_1 = 200 \mid a_2 = 250 \mid d = 50$

$n = 30$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{30} = \frac{30}{2}\left[2 \times 200 + (29)\,50\right]$$

$$= 15\left[400 + 1450\right]$$

$$= 15\left[1850\right]$$

$$S_{30} = ₹\,27,750$$