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CLASS – 10

MATHEMATICS

Chapter – 4

QUADRATIC EQUATIONS

Part – 5

Nature of Roots

Shubham Tiwari

OVERVIEW

1. Quadratic Equations
2. Solution of a Quadratic Equation by Factorisation
3. Nature of Roots

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QUADRATIC FORMULA

The roots of a Quadratic equation $ax^2 + bx + c$ are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is called the quadratic formula or Shridharacharya's formula.

$$x^2 + 5x + 6 = 0$$

$a = 1$ | $b = 5$ | $c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ → Discriminant

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2 \times 1}$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm 1}{2}$$

$$x = \frac{-5 \pm 1}{2}$$

α | β

$$x = \frac{-5+1}{2} = \frac{-4}{2} = -2$$

$$x = \frac{-5-1}{2}$$

$$x = -\frac{6}{2} = -3$$

☛ $ax^2 + bx + c = 0$ has two roots α and β, given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

NATURE OF ROOTS

Let the given equation be

$ax^2 + bx + c = 0$, where $a \neq 0$.

Then, the discriminant is given by

$$D = b^2 - 4ac$$

and the roots of the given equation

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

Case : I

When D > 0

In this case, the roots are real and distinct. These roots are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

Case : II

When D = 0

In this case, the roots are real and equal. Each root is $\frac{-b}{2a}$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b}{2a}$$

Case : III

When $D < 0$

In this case, the roots are imaginary, and we say that the given equation has no real roots.

Note

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If D is a perfect square then roots are rational and if D is a non–perfect square then roots are irrational.

- If $a, b, c \in Q$ and $p + \sqrt{q}$ is one root of quadratic equation then the other root must be the conjugate i.e., $p - \sqrt{q}$ and vice-versa. Here p and q are real numbers.

$p + \sqrt{q}$

$p - \sqrt{q}$

$5 + \sqrt{3}$

$5 - \sqrt{3}$

Example : 7

Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Soln. :

$2x^2 - 4x + 3 = 0$

$D = b^2 - 4ac$

$D = (-4)^2 - 4(2)(3)$

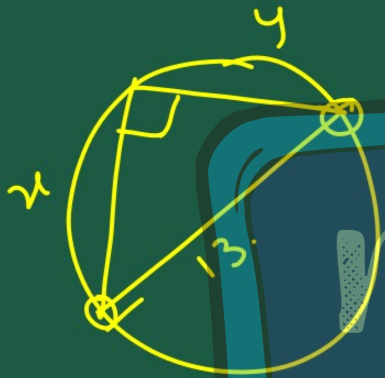
$= 16 - 24$

$D = -8$

Hence the Nature of roots is imaginary Or Non-Real.

Example : 8

A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres.



$x - y = 7$

$\boxed{x = y + 7}$

$$x^2 + y^2 = 13^2$$

$$(y+7)^2 + y^2 = 13^2$$

$$y^2 + 14y + 49 + y^2 = 13^2$$

$$2y^2 + 14y + 49 - 169 = 0$$

$$2y^2 + 14y - 120 = 0$$

$$2(y^2 + 7y - 60] = 0$$

$$y^2 + 7y - 60 = 0$$

$$y^2 + 7y - 60 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-60)}}{2 \times 1}$$

$$= \frac{-7 \pm \sqrt{49 + 240}}{2}$$

$$y = \frac{-7 \pm \sqrt{289}}{2}$$

$$y = \frac{-7 \pm 17}{2}$$

$$y = \frac{-7 + 17}{2}$$

$$y = \frac{10}{2}$$

$$y = 5$$

$$\boxed{y = 5}$$

$$x = y + 7$$

$$\boxed{x = 12}$$

$$y = \frac{-7 - 17}{2}$$

$$y = \frac{-24}{2}$$

$$y = -12$$

−12 can't be the distance

there the distance from two gates are 12 m and 5 m.

Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

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Example : 9

Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

Sol.

$$3x^2 - 2x + \frac{1}{3} = 0$$

$$\frac{9x^2 - 6x + 1}{3} = 0$$

$$9x^2 - 6x + 1 = 0$$

$$9x^2 - 6x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2 \times 9}$$

$$x = \frac{6 \pm \sqrt{36 - 36}}{18}$$

$$x = \frac{6 \pm 0}{18} = \frac{1}{3}$$

then the roots of the given eq are $\frac{1}{3}, \frac{1}{3}$.

LEARNING OUTCOMES

1. Quadratic Formula
2. Nature of roots

ASSESSMENT

1 The equation $(x + 1)^2 - 2(x + 1) = 0$ has

A Two real roots

B No real roots

C One real root

D Two equal root

$$x^2 + 1 + 2x - 2x - 2 = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$x = 1$$

$$x = -1$$

ASSESSMENT

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2 A quadratic equation $ax^2 + bx + c = 0$ has no real roots, if

A $b^2 - 4ac > 0$

B $b^2 - 4ac = 0$

C $b^2 - 4ac < 0$

D $b^2 - ac < 0$

ASSESSMENT

3 The roots of quadratic equation $2x^2 + x + 4 = 0$ are :

A Positive and Negative

B Both Positive

C Both Negative

D No real roots