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CLASS – 10

MATHEMATICS

Chapter – 3

Pair of Linear Equations in Two Variables

Part – 2

Exercise – 3.1

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EXERCISE – 3.1

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1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz : If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Let the No. of Boys = x

'' '' '' Girls = y

$$\boxed{y = x + 4}$$

$$\boxed{x + y = 10}$$

$$y = x + 4$$

x	0	1	2
y	4	5	6

$$x + y = 10$$

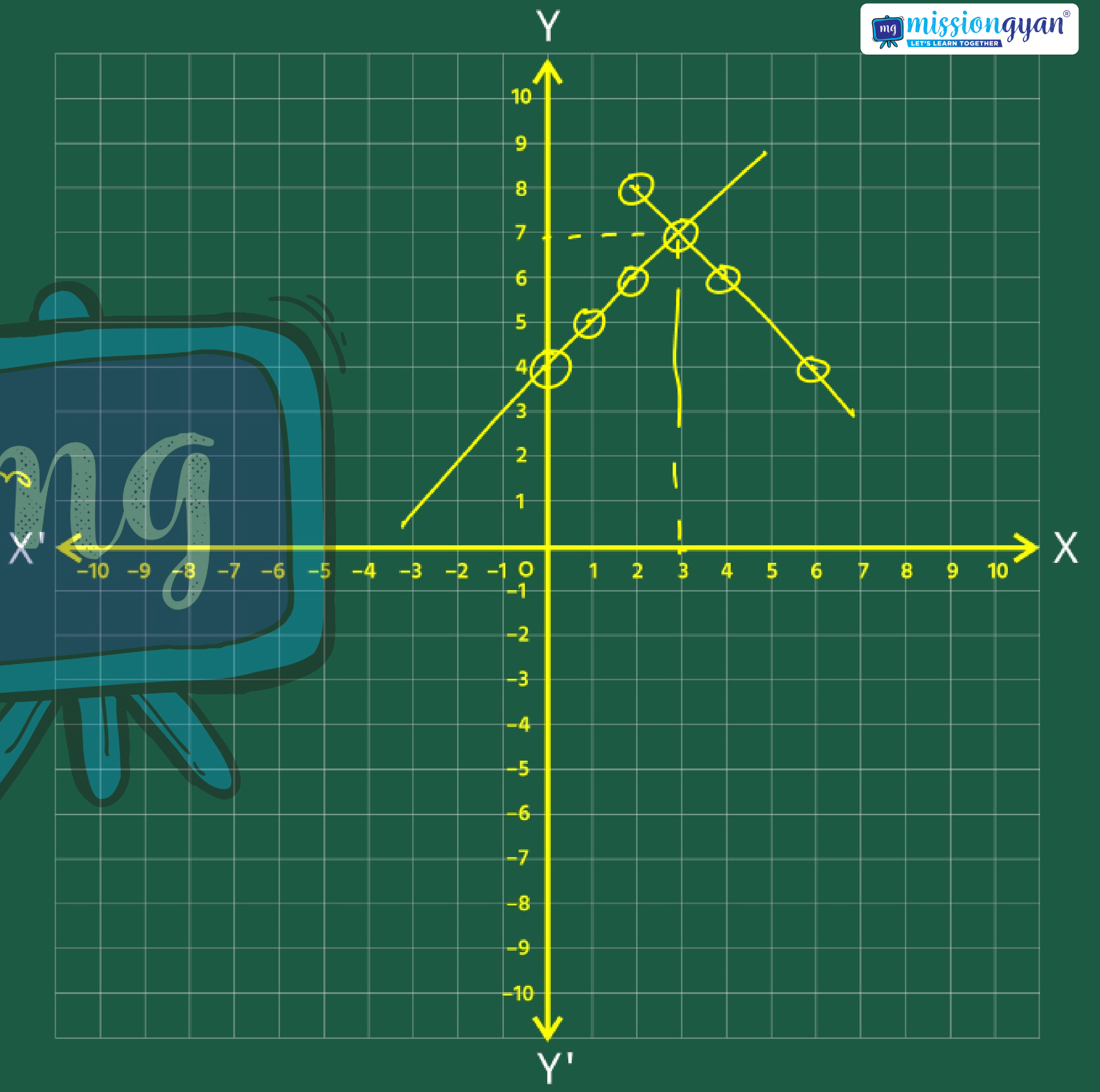
$$y = 10 - x$$

x	2	4	6
y	8	6	4

Hence the lines are intersecting and the intersection Point (3, 7).

So the Solution of the given problem is

$x = 3, y = 7$



Hence the No. of Boys is = 3

" " Girls is = 7

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Let the cost of Pencil = x

" " " Pen = y

$5x + 7y = 50$ — (1)

$7x + 5y = 46$ — (2)

eq (1) + eq (2)

$12x + 12y = 96$

$\boxed{x + y = 8}$ — (3)

eq (2) − eq (1)

$2x - 2y = -4$

$\boxed{x - y = -2}$ — (4)

$x + y = 8$

$x = 8 - y$

x	6	4	2
y	2	4	6

$x - y = -2$

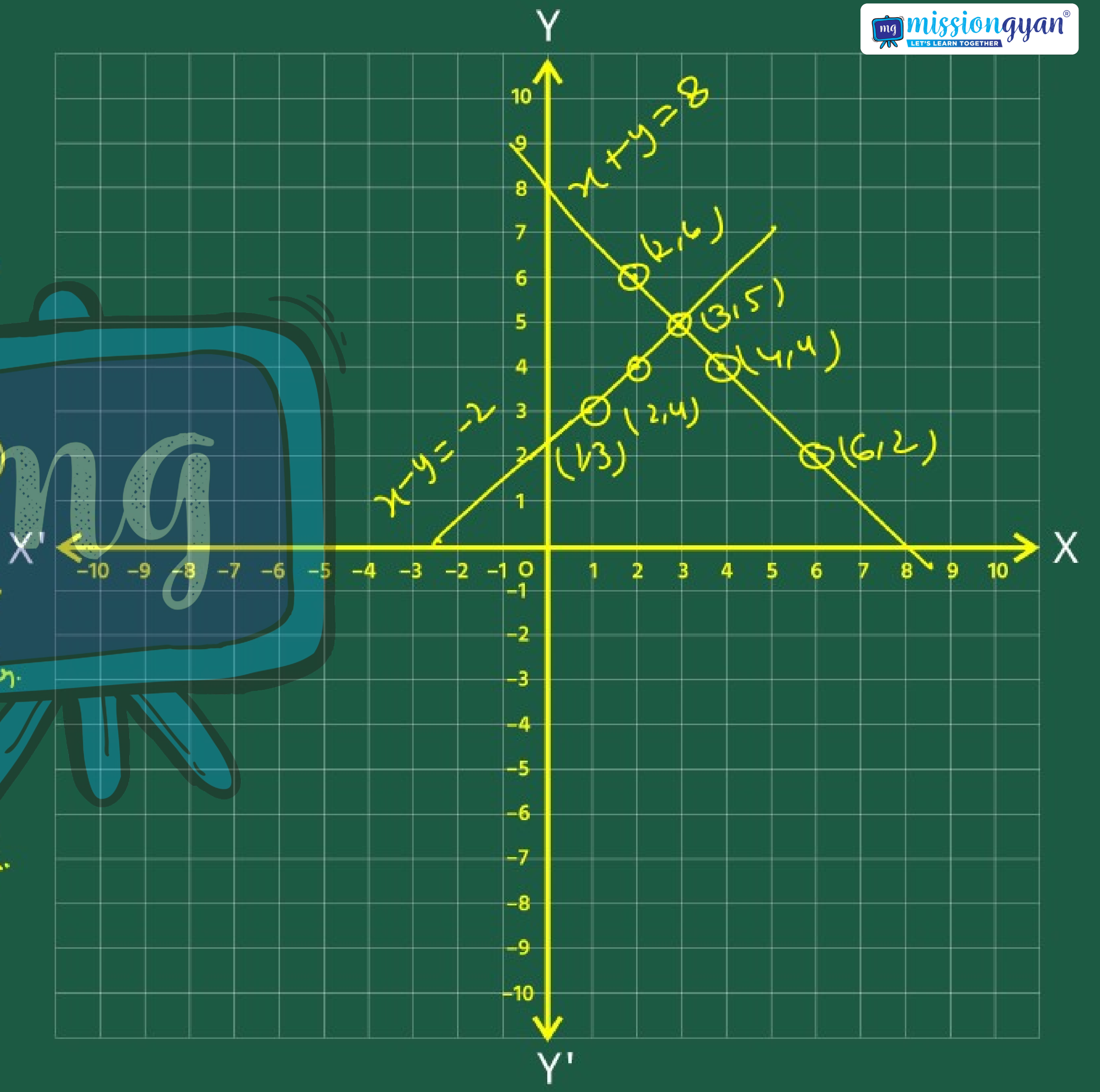
$x = y - 2$

x	1	2	3
y	3	4	5

So the Intersection Point of the given equations is (3,5)

Hence the cost of Pencil = 3 Rs.

" " of Pen = 5 Rs.



2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Let's compare the given eq. by the general eq.

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

On comparing let's find $\frac{a_1}{a_2} = 5/7$

$$\frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$$

$$\frac{c_1}{c_2} = \frac{8}{-4}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence the given equations give intersection at a point upon drawing on a graph.

$\boxed{\begin{matrix} U & N & I \\ 1 & 2 & 3 \end{matrix}}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ → unique solⁿ.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → No solⁿ.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ → Infinitely many solⁿ.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

$\frac{a_1}{a_2} = \frac{9}{18}, \quad \frac{b_1}{b_2} = \frac{3}{6}, \quad \frac{c_1}{c_2} = \frac{12}{24}$

$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{1}{2}$

This shows $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence it will given coincident lines on the graph.

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

$\frac{a_1}{a_2} = \frac{6}{2}, \frac{b_1}{b_2} = \frac{-3}{-1}, \frac{c_1}{c_2} = \frac{10}{9}$

$\frac{a_1}{a_2} = 3, \frac{b_1}{b_2} = 3, \frac{c_1}{c_2} = \frac{10}{9}$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$$

This shows the given equation will give parallel lines on the graph

3. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent or inconsistent :

(i) $3x + 2y = 5;\ 2x - 3y = 7$

$\frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = \frac{2}{-3}$, $\frac{c_1}{c_2} = \frac{5}{7}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ | As this shows the condition of intersecting lines hence it is consistent.

(ii) $2x - 3y = 8; \; 4x - 6y = 9$

$\frac{a_1}{a_2} = \frac{2}{4}, \; \frac{b_1}{b_2} = \frac{-3}{-6}, \; \frac{c_1}{c_2} = \frac{8}{9}$

$\frac{a_1}{a_2} = \frac{1}{2}, \; \frac{b_1}{b_2} = \frac{1}{2}, \; \frac{c_1}{c_2} = \frac{8}{9}$

$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$

This shows the condn of Parallel lines hence it is inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7;\ 9x - 10y = 14$

$\frac{a_1}{a_2} = \frac{3/2}{9}, \frac{b_1}{b_2} = \frac{5/3}{-10}, \frac{c_1}{c_2} = \frac{7}{14}$

$\frac{a_1}{a_2} = \frac{\cancel{3}}{2} \times \frac{1}{\cancel{9}_3} = \frac{1}{6}$

$\frac{b_1}{b_2} = \frac{\cancel{5}}{3} \times \frac{-1}{\cancel{10}_2}$

$\frac{a_1}{a_2} = \frac{1}{6}, \frac{b_1}{b_2} = \frac{-1}{6}, \frac{c_1}{c_2} = \frac{1}{2}$

$\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$

This shows intersecting lines.

Consistent.

(iv) $5x - 3y = 11; \; -10x + 6y = -22$

$\frac{a_1}{a_2} = \frac{5}{-10}, \quad \frac{b_1}{b_2} = \frac{-3}{6}, \quad \frac{c_1}{c_2} = \frac{11}{-22}$

$\frac{a_1}{a_2} = \frac{-1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{-1}{2}$

$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

This shows infinitely many Solⁿ. it is consistent.

(v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

$\frac{a_1}{a_2} = \frac{4/3}{2}, \quad \frac{b_1}{b_2} = \frac{2}{3} \quad \frac{c_1}{c_2} = \frac{8}{12}$

$\frac{a_1}{a_2} = \frac{\cancel{4}^{2}}{3} \times \frac{1}{\cancel{2}}$

$\frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = 2/3, \quad \frac{c_1}{c_2} = \frac{2}{3}$

$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

infinitely many solⁿ.

Consistent.

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically :

(i) $x + y = 5$, $2x + 2y = 10$

$\frac{a_1}{a_2} = \frac{1}{1} = 1$

$\frac{b_1}{b_2} = \frac{1}{1} = 1$

$\frac{c_1}{c_2} = \frac{5}{5} = 1$

$\boxed{x + y = 5}$

$2x + 2y = 10$

$2(x + y) = 10$

$\boxed{x + y = 5}$

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$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

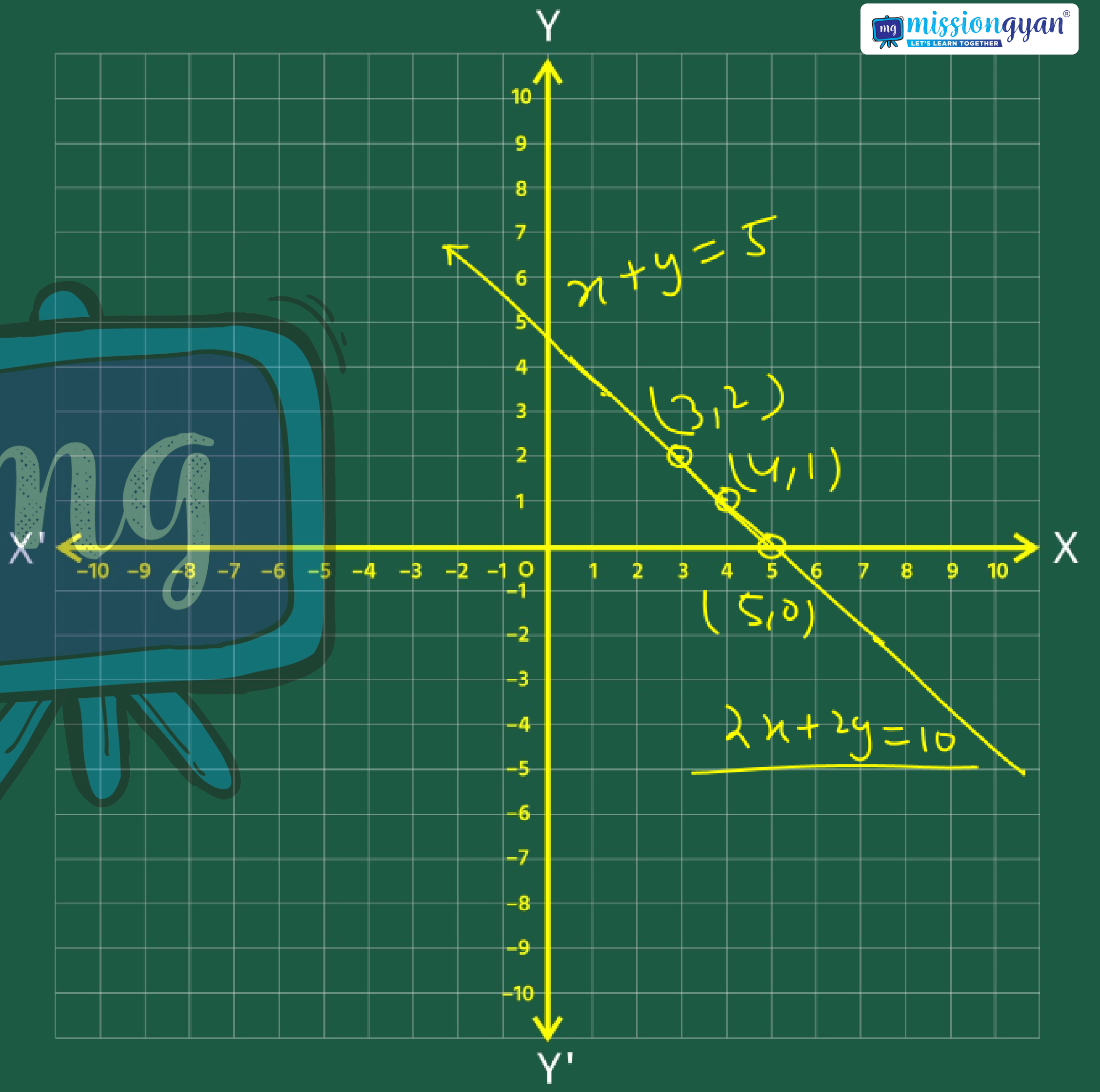
Consistent:

$x + y = 5$

$\boxed{x = 5 - y}$

x	5	4	3
y	0	1	2

Hence This gives Infinite many soln.



(ii) x – y = 8, 3x – 3y = 16

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-1}{-3}$, $\frac{c_1}{c_2} = \frac{8}{16}$

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{1}{2}$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$$

Inconsistent.

(iii) $2x + y - 6 = 0,\ 4x - 2y - 4 = 0$

$\frac{a_1}{a_2} = \frac{2}{4}$, $\frac{b_1}{b_2} = \frac{1}{-2}$, $\frac{c_1}{c_2} = \frac{-6}{-4}$

$\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-1}{2}$, $\frac{c_1}{c_2} = \frac{3}{2}$.

$\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$

Here it is consistent.

$$2x + y = 6$$

$$\boxed{y = 6 - 2x}$$

x	0	1	2
y	6	4	2

$$4x - 2y = 4$$

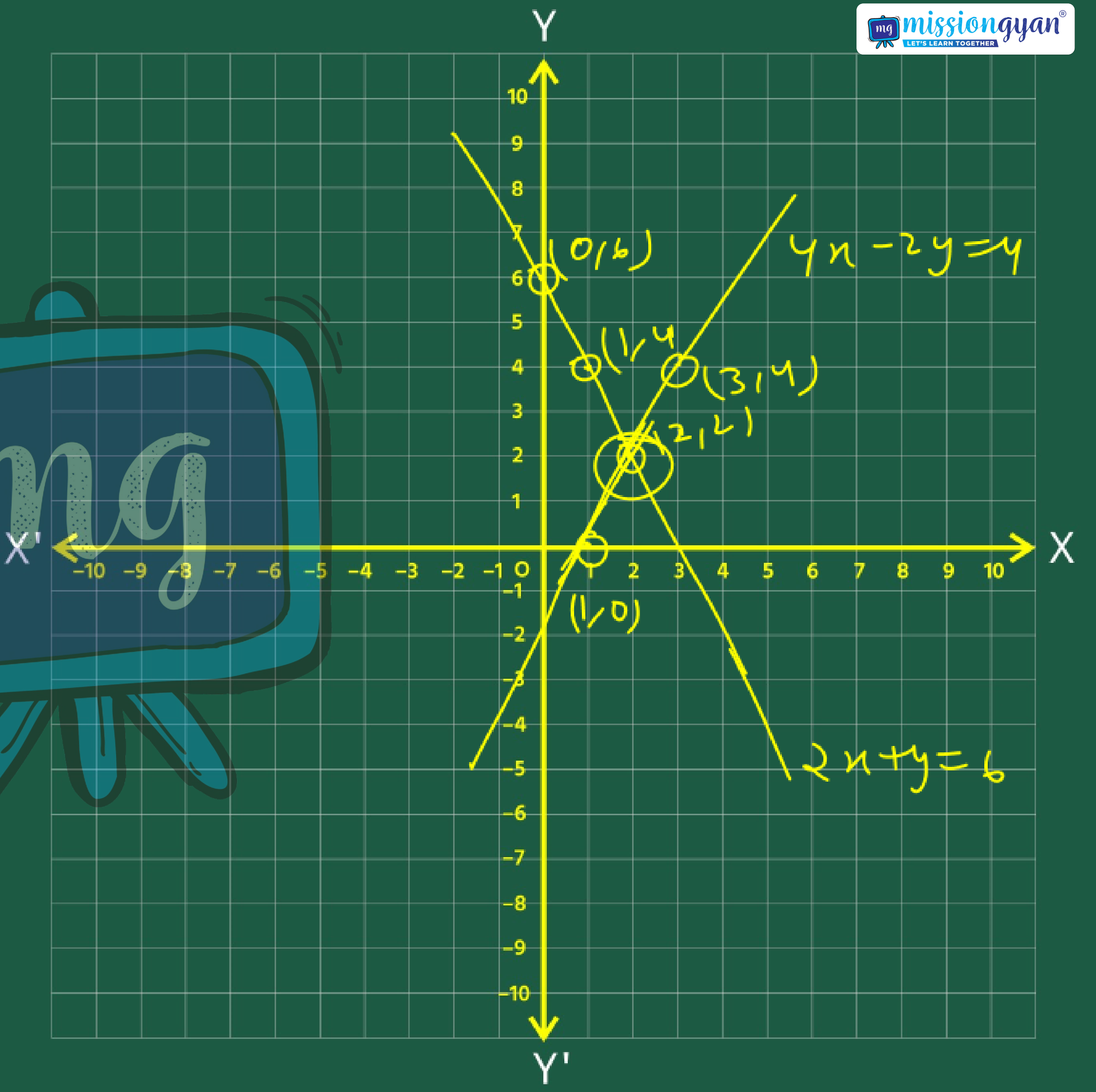
$$2x - y = 2$$

$$\boxed{2x - 2 = y}$$

x	1	2	3
y	0	2	4

Hence the Intersection point $(2,2)$ gives us the required solⁿ.

$$\left.\begin{aligned} x &= 2 \\ y &= 2 \end{aligned}\right\}$$



(iv) $2x - 2y - 2 = 0, \; 4x - 4y - 5 = 0$

$2x - 2y = 2 \quad | \quad 4x - 4y = 5$

$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{2}{5}$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$$

Inconsistent.

5. Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden.

Sol.:

$\frac{P}{2} = 36$ — (1)

$P = 72$

$2(l+w) = 72$

$l+w = 36$

$\boxed{x+y=36}$

$l = 4+w$

$l = x$

$w = y$

$\boxed{x = y+4}$

$$x + y = 36$$

$$x = 36 - y$$

x	20	24	30
y	16	12	6

$$x = y + 4$$

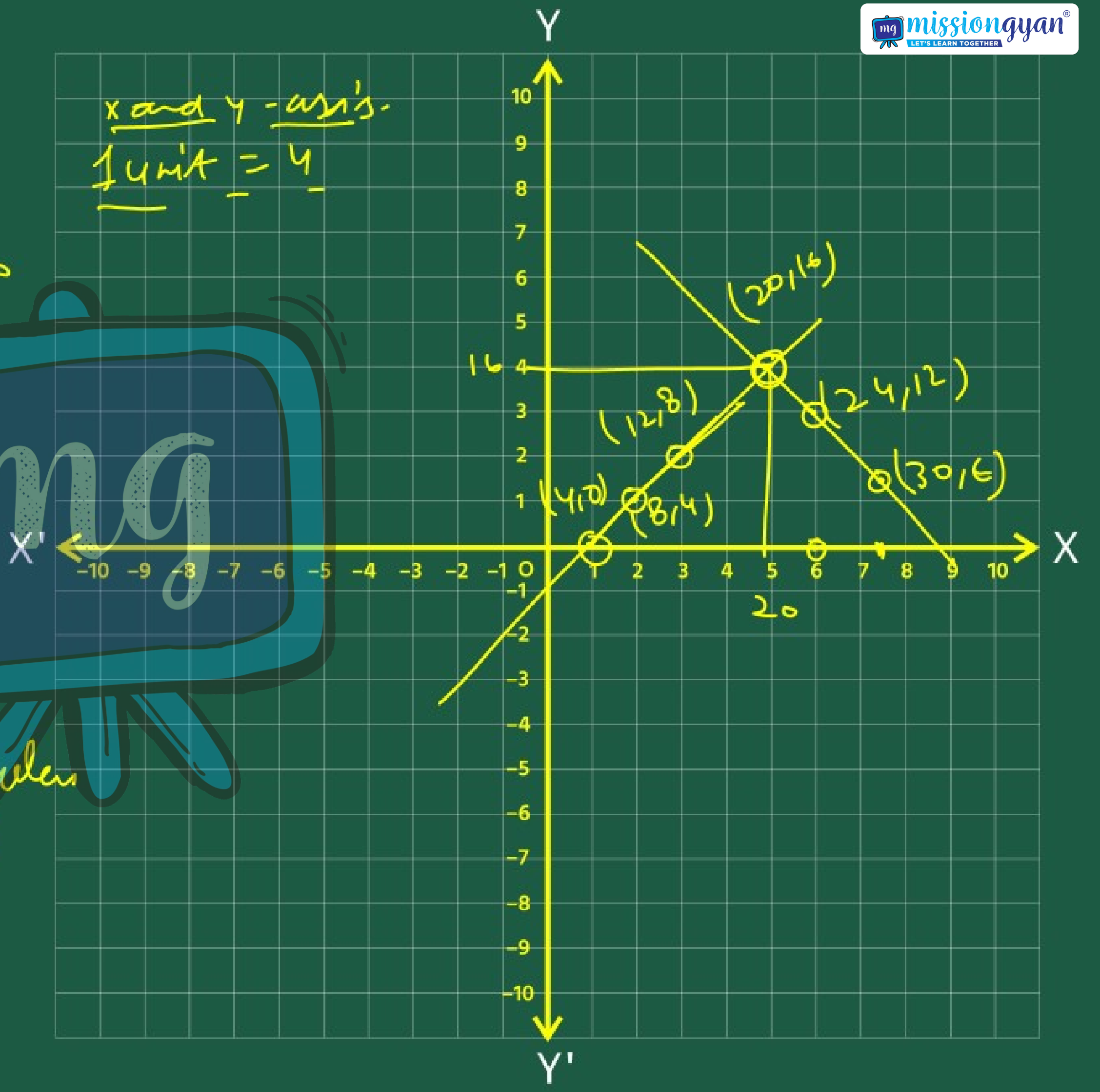
x	4	8	12
y	0	4	8

The intersection point is (20, 16)

$x = 20$

$y = 16$

Hence the length and breadth of the Rectangular garden is 20 m and 16 m.



6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(i) intersecting lines

$2x + 3y - 8 = 0$

$a_1 = 2,\ b_1 = 3,\ c_1 = -8$

by taking $a_2 = 3,\ b_2 = 4,\ c_2 = 5$

$$\frac{a_1}{a_2} = \frac{2}{3}$$

$$\frac{b_1}{b_2} = \frac{3}{4}$$

$$\frac{c_1}{c_2} = \frac{8}{5}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence the required eq. which shows the intersecting line will be

$$\boxed{3x + 4y + 5 = 0}$$

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(ii) parallel lines

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$$

$2x + 3y - 8 = 0$

$a_1 = 2, \; b_1 = 3, \; c_1 = -8$

On taking $a_2 = 4, \; b_2 = 6, \; c_2 = 5$

$$\boxed{4x + 6y + 5 = 0}$$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{8}{5}$

(iii) coincident lines

$2x + 3y - 8 = 0$

$a_1 = 2, \quad b_1 = 3 \quad c_1 = -8$

on taking $a_2 = 4, \quad b_2 = 6, \quad c_2 = -16$

$\frac{a_1}{a_2} = \frac{2}{4}, \quad \frac{b_1}{b_2} = \frac{3}{6}, \quad \frac{c_1}{c_2} = \frac{-8}{-16}$

$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{1}{2}$

$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

hence the required eq is

$\boxed{4x + 6y - 16 = 0}$

7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x–axis, and shade the triangular region.

$x - y = 1$

$x = 1 + y$

x	1	2	3
y	0	1	2

$3x + 2y - 12 = 0$

$3(1+y) + 2y - 12 = 0$

$3 + 3y + 2y - 12 = 0$

$-9 + 5y = 0$

$y = 9/5$

$y = 1.8$

$x = 1 + 9/5$

$x = \frac{14}{5}$

$\boxed{x = 2.8}$

$3x + 2y - 12 = 0$

$2y = 12 - 3x$

$\boxed{y = \frac{12 - 3x}{2}}$

x	2	4	6
y	3	0	-3

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Hence the

ΔABC has the

co-ordinates as

$A(2.8, 1.8)$

$B(1, 0)$

$C(2, 0)$