Under CSR Support by ipca | Foundation

CLASS - 10

MATHEMATICS

CH–1 : Real Numbers

CBSE Board

Most Important Questions

Shubham Tiwari

1. If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where 'a' and 'b' are prime numbers, then find the LCM (p, q).

Soln.

$LCM(p, q)$

$p = ab^2$ | $q = a^2b$

$LCM(p, q) = a^2b^2$

2. Find the value of ‘a’, if HCF (a, 18) = 2 and LCM (a, 18) = 36.

1 2

Solu:

$$\text{LCM} \times \text{HCF} = a \times b$$

$$36 \times 2 = a \times 18$$

$$\frac{\cancel{36}^{2} \times 2}{\cancel{18}} = a$$

$$\boxed{4 = a}$$

3. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, then find the numbers.

Solu:

$$\frac{15}{11} = \frac{a}{b}$$

$a = 15 \times HCF$

$b = 11 \times HCF$

$$\boxed{\begin{aligned} a &= 15 \times 13 = 195 \\ b &= 11 \times 13 = 143 \end{aligned}}$$

$$\frac{\cancel{14}}{\cancel{21}} = \left.\frac{2}{3}\right\} \frac{HCF}{HCF}$$

$$\boxed{\left.\begin{array}{l} 14 = 2 \times ⑦ \\ 21 = 3 \times ⑦ \end{array}\right\}} \text{— HCF}$$

$$\frac{2 \times 7}{3 \times 7} \qquad \frac{14}{21}$$

57×11

1

5 2 7

$\boxed{6\ 2\ 7}$

$a = \overset{6}{1}5 \times 13$

$a = 195$

$b = 13 \times 11$

$\boxed{b = 143}$

$1 \quad 3 \times 11$

$\boxed{1\ \ 4\ \ 3}$

42×11

$\underline{4\ 6\ 2}$

$$\frac{\cancel{14}}{\cancel{21}} = \left[\frac{2}{3}\right] \frac{\times 7}{\times 7}$$

$$\frac{14}{21} = \left[\frac{2 \times \cancel{7}}{3 \times \cancel{7}}\right]$$

$$14 = 2 \times 7$$

$$21 = 3 \times 7$$

$$\boxed{HCF = 7}$$

4. If p and q are two distinct prime numbers, then find their HCF.

Soln

as p and q are distinct prime No. then the HCF of these two numbers is 1.

5. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.

Solu.

$LCM = 14(HCF)$ —①

$LCM + HCF = 600$ —②

$14(HCF) + HCF = 600$

$15(HCF) = 600$

$$15\,(HCF) = 600$$

$$HCF = \frac{\cancel{600}}{\cancel{15}}$$

$$\boxed{HCF = 40}\ ✓$$

$$LCM = 14\,(40)$$

$$\boxed{LCM = 560}$$

$$HCF \times LCM = a \times b$$

$$40 \times 560 = 280 \times b$$

$$\frac{40 \times \overset{2}{\cancel{56}}\cancel{0}}{\cancel{28}\cancel{0}} = b$$

$$\boxed{80 = b}$$

6. Show that 12^n cannot end with digit 0 or 5 for any natural number n.

7. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance and incomplete steps?

Soln

A 40, 40, 40, 40 ⑩ → 400

B 42 ⑧ × → 400

C 45 ⑥ × → 400

Soln LCM (40, 42, 45)

2 | 40, 42, 45

2 | 20, 21, 45

2 | 10, 21, 45

3 | 5, 21, 45

3 | 5, 7, 15

| 5, 7, 5

5 | 5, 7, 5

7 | 1, 7, 1

| 1, 1, 1

$$\text{Lcm}\ (40, 42, 45) = 2^3 \times 3^2 \times 5 \times 7$$

$$= 10 \times 2^2 \times 3^2 \times 7$$

$$= 10 \times 4 \times 9 \times 7$$

$$= 10 \times \overset{4}{3}6 \times 7$$

$$= 10 \times 252$$

$$\boxed{\text{Lcm} = 2520\ \text{cm}}$$

This is the minimum distace each will cover.

8. Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

<u>Soln:</u> $\textcircled{a} + \underline{17} = \underbrace{\text{lcm}(520, 468)}_{} \rightarrow$ ①

Now let's find out the lcm of 520 and 468

2	520
2	260
2	130
5	65
13	13
	1

2	468
2	234
3	117
3	39
13	13
	1

$520 = 2^3 \times \boxed{5^1} \times 13^1$

$468 = 2^2 \times \boxed{3^2} \times 13^1$

LCM:- $13^1 \times 2^3 \times 3^2 \times 5^1$

$$LCM:-\ 13 \times 2^3 \times 3^2 \times 5^1$$

$$= 13 \times 2^2 \times 3^2 \times 10$$

$$= 13 \times 4 \times 9 \times 10$$

$$= \overset{1}{5}2 \times 9 \times 10$$

$$\boxed{LCM = 4680}$$

Now by placing LCM in eq ①

$$a + 17 = 4680$$

$$\boxed{a} = 4680 - 17 = \boxed{4663}$$

9. In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

10. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.

Soln.

$\boxed{999999}$

Let's find out the LCM(24,15,36)

2	24, 15, 36
2	12, 15, 18
2	6, 15, 9
3	3, 15, 9
	1, 5, 3

3	1, 5, 3
5	1, 5, 1
	1, 1, 1

$$\left.\begin{matrix}5\\7\end{matrix}\right\} \quad \underline{35} \times 2 = \frac{\cancel{70}}{\cancel{5}} = 14$$

$$\frac{\cancel{70}}{\cancel{7}} = 10$$

$$LCM = \underline{2} \times 2^2 \times 3^2 \times \underline{5}$$

$$= 10 \times 4 \times 9$$

$$\boxed{LCM = 360}$$

if we divide 999999 by 360 then we will get 279 as remainder,

Hence 999999 − 279 = 999720

$$5\,\overline{)\,27\,}\,(5$$

$$\begin{array}{r} 27 \\ 25 \\ \hline 2 \end{array}$$

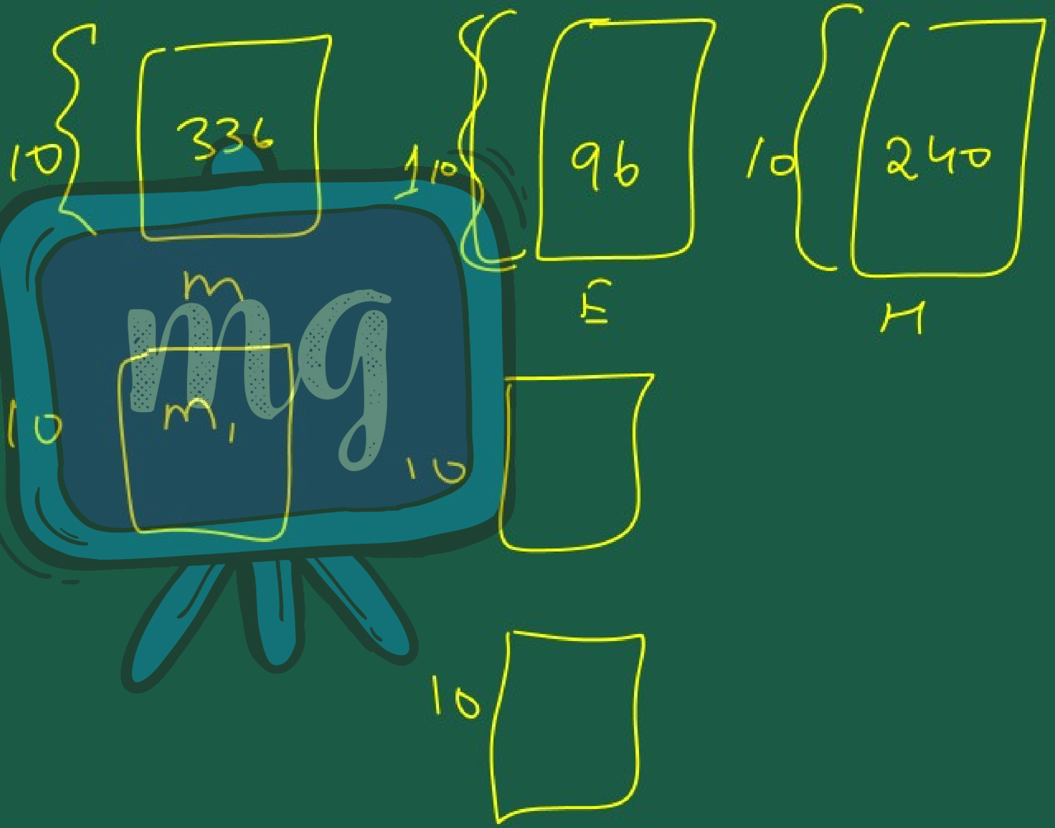
$$27 - 2 = \boxed{\frac{25}{5}}$$

$$\boxed{360}\,\overline{)\,999999\,}\,($$

$$\begin{array}{r} \\ \hline 279 \end{array}$$

$$\begin{array}{r} 999999 \\ -\ 279 \\ \hline 999720 \\ \hline \end{array}$$

11. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic wise and the height of each stack is the same. The number of English books is 96, the number of Hindi books is 240 and the number of Mathematics books is 336. Assuming that the books are of the same thickness, determine the number of stacks of English, Hindi and Mathematics books.


10
336
m
10
96
E
10
240
M
10
m,
10
10

HCF of 336, 240, 96

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	240
2	120
2	60
2	30
3	15
5	5
	1

2	336
2	168
2	84
2	42
3	21
7	7
	1

E $96 = 2^5 \times 3^1$

H $240 = \boxed{2^4 \times 3^1} \times 5^1$

m $336 = \boxed{2^4 \times 3^1} \times 7^1$

$HCF = 2^4 \times 3^1$

$= 16 \times 3$

$\boxed{HCF = 48}$

Stacks for Hindi

$\frac{240}{48} = 5$

No of stacks for maths $= \frac{336}{48} = 7$

Stacks required for English

$\frac{96}{48} = 2$

12. Given that $\sqrt{3}$ is irrational, prove that $5 + 2\sqrt{3}$ is irrational.

soln let to contrary $5 + 2\sqrt{3}$ is a rational No.

$$5 + 2\sqrt{3} = \frac{p}{q} \left[\begin{array}{l} p, q \text{ are} \\ \text{co-prime integers} \\ q \neq 0 \end{array} \right\}$$

$$2\sqrt{3} = \frac{p}{q} - 5$$

$$\boxed{\sqrt{3} = \frac{p - 5q}{2q}}$$

as 2, 5, p and q are integers then $\frac{p-5q}{2q}$ is a rational no.

This contradiction occurs due to our wrong assumption

Hence this proves $5+2\sqrt{3}$ is a irrational no.

13. Given that $\sqrt{5}$ is irrational, prove that $2+3\sqrt{5}$ is irrational.

solu:

$$2+3\sqrt{5} = \frac{p}{q}$$

$$3\sqrt{5} = \frac{p}{q} - 2$$

$$\boxed{\sqrt{5}} = \boxed{\frac{p-2q}{3q}}$$

14. Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number.

soln: Let to contrary $\sqrt{2}+\sqrt{3}$ is a rational no.

$$\sqrt{2}+\sqrt{3} = \frac{p}{q} \quad \{p, q \text{ are co prime integers}, q \neq 0\}$$

$$(\sqrt{3})^2 = \left(\frac{p}{q} - \sqrt{2}\right)^2 \quad \{\text{By squaring both side}\}$$

$$3 = \frac{p^2}{q^2} + 2 - 2\sqrt{2}\, p/q$$

$$3 = \frac{p^2}{q^2} + 2 - 2\sqrt{2}\frac{p}{q}$$

$$3 - 2 - \frac{p^2}{q^2} = -2\sqrt{2}\,\frac{p}{q}$$

$$1 - \frac{p^2}{q^2} = -2\sqrt{2}\,\frac{p}{q}$$

$$\frac{q^2 - p^2}{\cancel{q^2}} = -2\sqrt{2}\;p/\cancel{q}$$

$$\frac{q^2 - p^2}{-2p} = \sqrt{2}$$

$$\frac{p^2 - q^2}{2p} = \sqrt{2}$$

as p, q, and 2 are integers then $\frac{p^2-q^2}{2p}$ is a rational No.

This contradiction arises due to our wrong assumption

Hence $\sqrt{2}+\sqrt{3}$ is an irrational No.

15. If p, q are prime positive integers, prove that $\sqrt{p}+\sqrt{q}$ is an irrational number.

$\frac{a}{b}$