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कक्षा - 10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

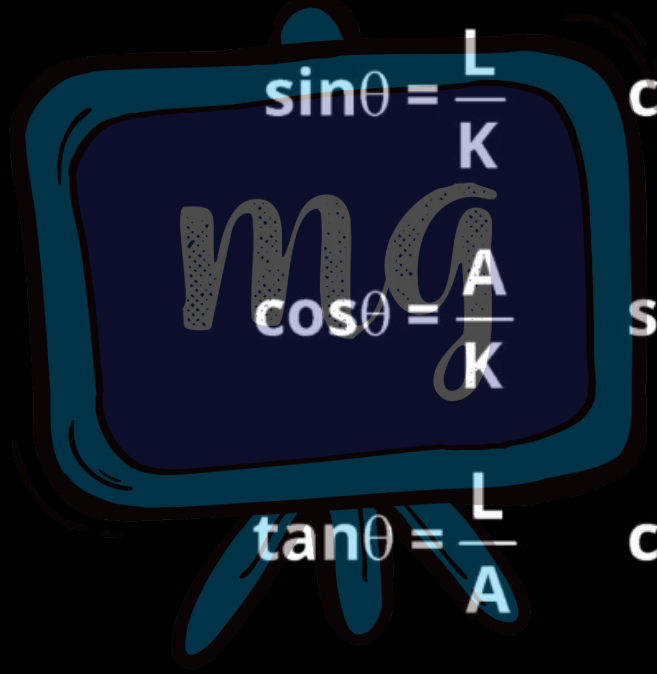
भाग - 9

केशव शर्मा

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रिवीजन



$$\sin\theta = \frac{L}{K} \qquad \operatorname{cosec}\theta = \frac{K}{L}$$

$$\cos\theta = \frac{A}{K} \qquad \sec\theta = \frac{K}{A}$$

$$\tan\theta = \frac{L}{A} \qquad \cot\theta = \frac{A}{L}$$

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$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

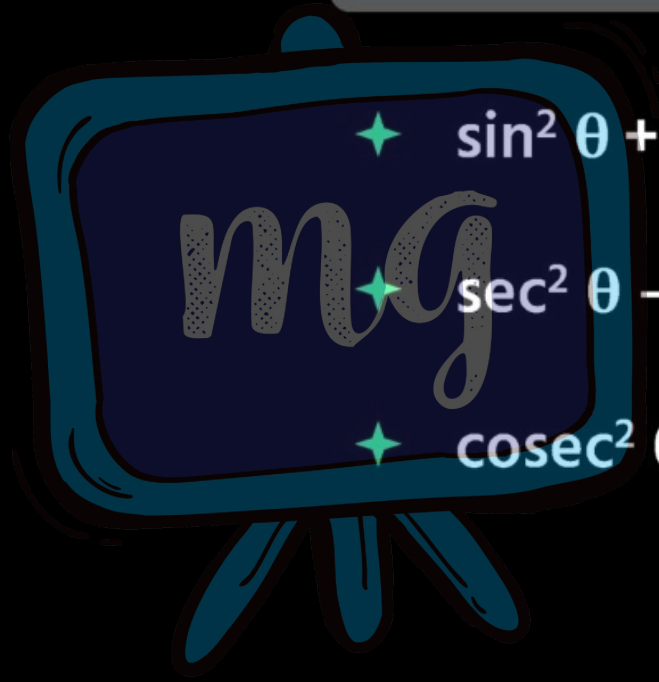
$$\cos\theta = \frac{1}{\sec\theta}, \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

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त्रिकोणमितीय सर्वसमिकाएँ



- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

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प्रश्नावली 8.3

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

LHS $= \sqrt{\frac{1+\sin A}{1-\sin A}}$

परिमेय करण करने पर

$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A}$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \sec A + \tan A$

$= \text{RHS}$

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$$= \sqrt{\frac{(1+\sin A)^2}{(1)^2-(\sin A)^2}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$\left\{ \begin{array}{l} \sin^2 A+\cos^2 A = 1 \\ \cos^2 A = 1-\sin^2 A \end{array} \right.$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

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$= \sec A + \tan A$

$= RHS.$

$\left\{ \frac{1}{\cos A} = \sec A \right.$

$\left. \frac{\sin A}{\cos A} = \tan A \right\}$

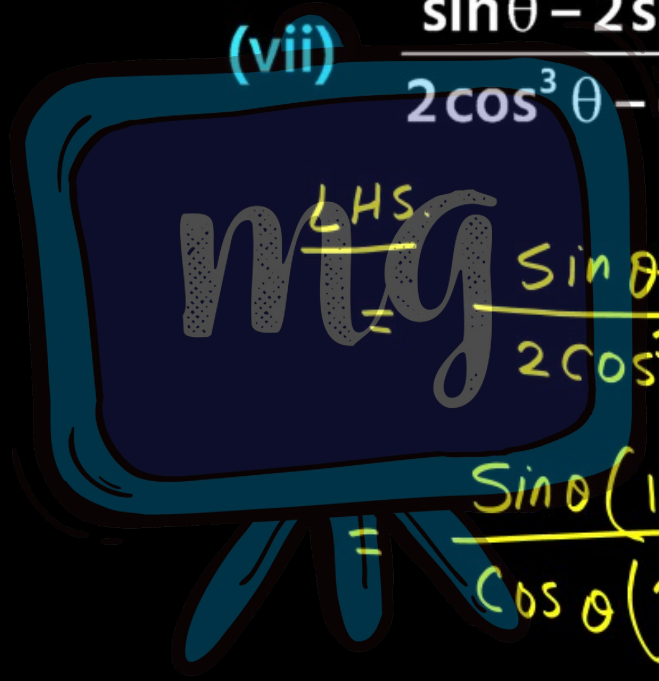


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प्रश्नावली 8.3

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$



LHS.

$= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

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$$= \frac{\sin\theta(1-2(1-\cos^2\theta))}{\cos\theta(2\cos^2\theta-1)} \quad \left\{ \begin{array}{l} \sin^2\theta+\cos^2\theta=1 \\ \sin^2\theta=1-\cos^2\theta \end{array} \right\}$$

$$= \frac{\tan\theta(1-2+2\cos^2\theta)}{(2\cos^2\theta-1)} \quad \left\{ \frac{\sin\theta}{\cos\theta}=\tan\theta \right\}$$

$$= \frac{\tan\theta(-1+2\cos^2\theta)}{(2\cos^2\theta-1)}$$

$$= \frac{\tan\theta(2\cos^2\theta-1)}{(2\cos^2\theta-1)} = \tan\theta = RHS$$

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प्रश्नावली 8.3

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$

LHS $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 \times \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \times \sec A \times \cos A$

$= (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2 \times \cancel{\sin A} \times \frac{1}{\cancel{\sin A}} + 2 \times \frac{1}{\cancel{\cos A}} \times \cancel{\cos A}$

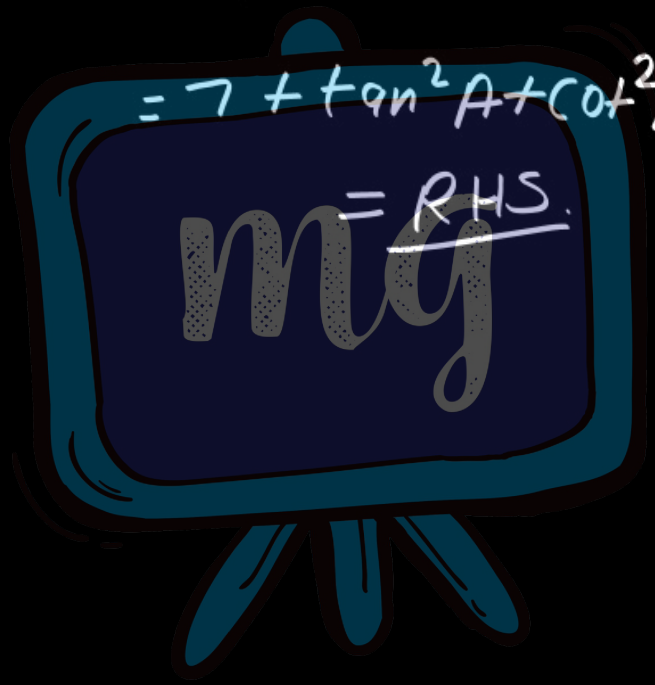
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$$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$$

$$= 7 + \tan^2 A + \cot^2 A$$

$$= \text{RHS}$$



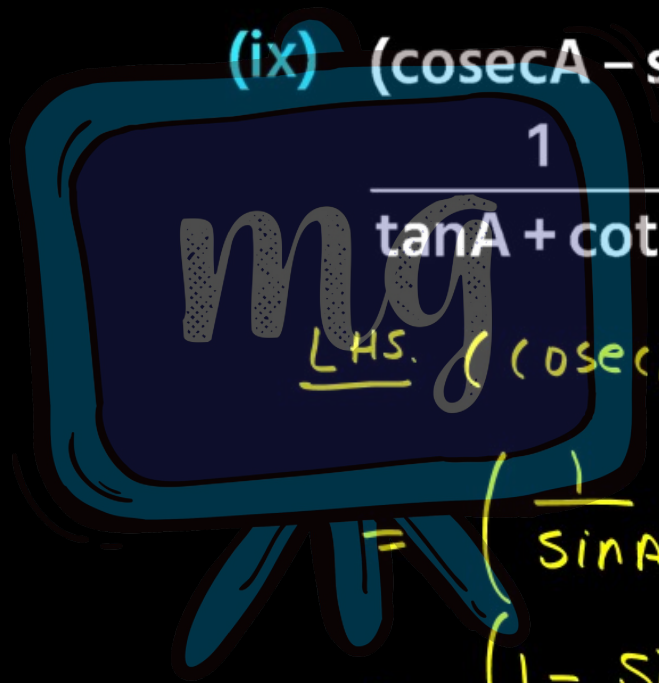
$$\left.\begin{aligned} &\operatorname{Cosec}^2 A - \cot^2 A = 1 \\ &\operatorname{Cosec}^2 A = 1 + \cot^2 A \\ &\sec^2 A = 1 + \tan^2 A \end{aligned}\right\}$$

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प्रश्नावली 8.3

(ix) $(\text{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$



LHS. $(\text{cosec}A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \dfrac{\sin A}{1}\right)\left(\dfrac{1}{\cos A} - \dfrac{\cos A}{1}\right)$

$= \left(\dfrac{1 - \sin^2 A}{\sin A}\right)\left(\dfrac{1 - \cos^2 A}{\cos A}\right)$

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$$= \frac{\cos^2 A \times \sin^2 A}{\sin A \times \cos A}$$

$$= \cos A \times \sin A$$

RHS.

$$= \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} =$$

$$\left\{ \begin{array}{l} \sin^2 A + \cos^2 A = 1 \\ \sin^2 A = 1 - \cos^2 A \\ \cos^2 A = 1 - \sin^2 A \end{array} \right.$$

$$\left\{ \begin{array}{l} \tan A = \frac{\sin A}{\cos A} \\ \cot A = \frac{\cos A}{\sin A} \end{array} \right.$$

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$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \times \cos A}}$$

$\{ \sin^2 A + \cos^2 A = 1 \}$

$$= \frac{\sin A \times \cos A}{1} = \sin A \times \cos A$$

RHS = LHS

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प्रश्नावली 8.3

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$= \frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{1+\left(\frac{\sin A}{\cos A}\right)^2}{1+\left(\frac{\cos A}{\sin A}\right)^2}$

$\left\{\tan A = \frac{\sin A}{\cos A}, \quad \cot A = \frac{\cos A}{\sin A}\right\}$

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$$= \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A} \Big/ \frac{\sin^2 A + \cos^2 A}{\sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

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$\sec^2 A = 1 + \tan^2 A$

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$= \left(\frac{1-\tan A}{1-\cot A}\right)^2$$

$$= \frac{(1-\tan A)^2}{(1-\cot A)^2}$$

$$= \frac{(1)^2 + (\tan A)^2 - 2 \times 1 \times \tan A}{(1)^2 + (\cot A)^2 - 2 \times 1 \times \cot A}$$

$$= \frac{1 + \tan^2 A - 2\tan A}{1 + \cot^2 A - 2\cot A} = \frac{\sec^2 A - 2\tan A}{\operatorname{cosec}^2 A - 2\cot A}$$

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$$= \frac{\sec^2 A - 2\tan A}{\operatorname{cosec}^2 A - 2\cot A}$$

$$= \frac{\frac{1}{\cos^2 A} - 2\frac{\sin A}{\cos A}}{\frac{1}{\sin^2 A} - 2\frac{\cos A}{\sin A}} = \frac{\frac{1 - 2\sin A\cos A}{\cos^2 A}}{\frac{1 - 2\sin A\cos A}{\sin^2 A}}$$

$$\frac{\cos^2 A}{\cos A}$$

$$= \frac{(1 - 2\sin A\cos A)(\sin^2 A)}{(\cos^2 A)(1 - 2\sin A\cos A)} = \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$