कक्षा – 12

गणित

समाकलन

अध्याय – 7

भाग - 11

प्रश्नावली 7.5

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प्रश्नावली 7.5

➤ प्रश्न 1 से 21 तक के प्रश्नों में परिमेय फलनों का समाकलन कीजिए।

Q6. $\dfrac{1-x^2}{x(1-2x)}$

$= \dfrac{1-x^2}{x-2x^2} = \dfrac{-x^2+1}{-2x^2+x}$

$= \dfrac{1}{2} + \dfrac{-\frac{x}{2}+1}{-2x^2+x}$

$= \dfrac{1}{2} + \dfrac{-x+2}{2(-2x^2+x)}$

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{-x+2}{2x(-2x+1)}$$

अब, माना $\frac{-x+2}{x(-2x+1)} = \frac{A}{x} + \frac{B}{-2x+1}$

$$\frac{-x+2}{x(-2x+1)} = \frac{A(-2x+1)+Bx}{x(-2x+1)}$$

$$\frac{-x+2}{x(-2x+1)} = \frac{-2Ax+A+Bx}{x(-2x+1)}$$

$$\frac{-x+2}{x(-2x+1)} = \frac{(-2A+B)x + A}{x(-2x+1)}$$

तुलना करने पर :

$A = 2$,

$-2A + B = -1$

$-2 \times 2 + B = -1$

$B = -1 + 4 = 3$

$B = 3$

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अब, $\int \frac{1-x^2}{x(1-2x)}\,dx = \int \left\{ \frac{1}{2} + \frac{1}{2}\left[\frac{-x+2}{x(-2x+1)} \right] \right\} dx$

$= \int \frac{1}{2}\,dx + \frac{1}{2}\left[\int \frac{A}{x}\,dx + \int \frac{B}{-2x+1}\,dx \right.$

$= \int \frac{1}{2}\,dx + \frac{1}{2}\left[\int \frac{2}{x}\,dx + \int \frac{3}{-2x+1}\,dx \right]$

$= \frac{x}{2} + \frac{1}{2}\left[2\log|x| + 3\log|-2x+1| \right] + C$

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$$\int \frac{x}{(x^2+1)(x-1)}\,dx$$

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Q7. $\dfrac{x}{(x^2+1)(x-1)}$

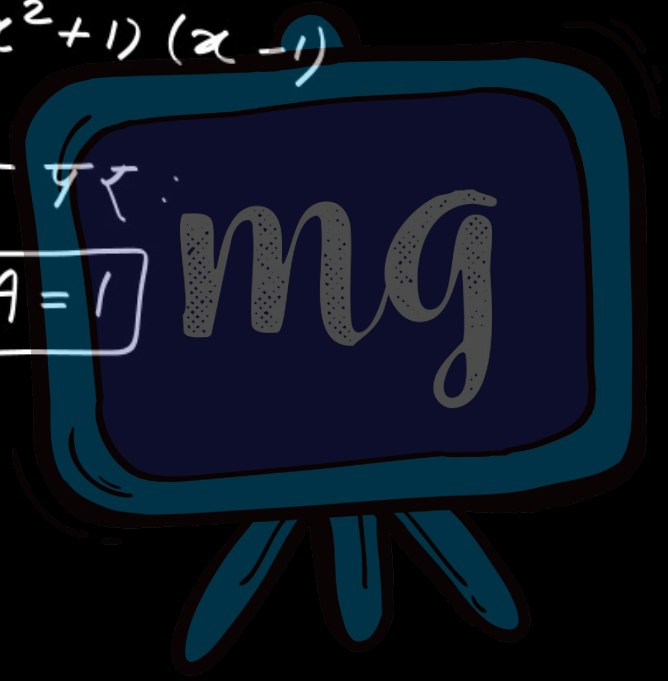
माना $\dfrac{x}{(x^2+1)(x-1)} = \dfrac{A}{x^2+1} + \dfrac{B}{x-1}$

$$\frac{x}{(x^2+1)(x-1)} = \frac{A(x-1)+B(x^2+1)}{(x^2+1)(x-1)} = \frac{Ax-A+Bx^2+B}{(x^2+1)(x-1)}$$

$$\Rightarrow \frac{x}{(x^2+1)(x-1)} = \frac{Bx^2 + Ax - A + B}{(x^2+1)(x-1)}$$

तुलना करने पर:

$\boxed{B = 0}$, $\boxed{A = 1}$



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$$\int \frac{x}{(x-1)^2(x+2)}\,dx$$

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Q8. $\dfrac{x}{(x-1)^2(x+2)}$

माना $\dfrac{x}{(x-1)^2(x+2)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x+2)}$

$$\frac{x}{(x-1)^2(x+2)} = \frac{A(x-1)(x+2) + B(x+2) + C(x-1)^2}{(x-1)^2(x+2)}$$

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$$\Rightarrow \quad x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

यदि $x = 1$ हो:

$$1 = B(3)$$

$$\boxed{B = \frac{1}{3}}$$

यदि $x = -2$ हो:

$$-2 = C(-3)^2$$

$$-2 = 9C$$

$$\boxed{C = \frac{-2}{9}}$$

यदि $x = 2$ हो:

$$2 = A(4) + B(4) + C(1)$$

$$2 = 4A + 4 \times \frac{1}{3} - \frac{2}{9}$$

$$4A = 2 - \frac{4}{3} + \frac{2}{9}$$

$$4A = \frac{18 - 12 + 2}{9} = \frac{8}{9}$$

$$\boxed{A = \frac{2}{9}}$$

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हल :

$$\int \frac{x}{(x-1)^2(x+2)}\,dx = \int \frac{A}{x-1}\,dx + \int \frac{B}{(x-1)^2}\,dx + \int \frac{C}{x+2}\,dx$$

$$= \frac{2}{9}\int \frac{1}{x-1}\,dx + \frac{1}{3}\int \frac{1}{(x-1)^2}\,dx - \frac{2}{9}\int \frac{1}{x+2}\,dx$$

$$= \frac{2}{9}\log|x-1| - \frac{2}{9}\log|x+2| + \frac{1}{3}\int \frac{1}{(x-1)^2}\,dx$$

माना $x-1 = t$

$dx = dt$

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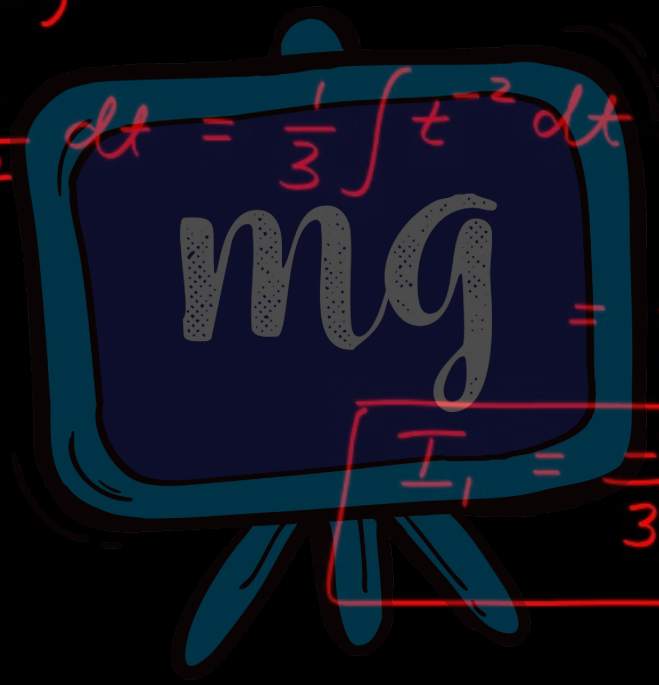
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$$I_1 = \frac{1}{3}\int \frac{1}{(x-1)^2}\,dx$$

$$= \frac{1}{3}\int \frac{1}{t^2}\,dt = \frac{1}{3}\int t^{-2}\,dt = \frac{1}{3}\left(\frac{t^{-2+1}}{-2+1}\right) + c$$

$$= \frac{1}{3}\left(-\frac{1}{t}\right) + c$$

$$\boxed{I_1 = \frac{-1}{3(x-1)} + c}$$



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अत :

$$\int \frac{x}{(x-1)^2(x+2)}\,dx = \frac{2}{9}\log|x-1| - \frac{2}{9}\log|x+2| - \frac{1}{3(x-1)} + C$$



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Q9. $\dfrac{3x+5}{x^3-x^2-x+1} = \dfrac{3x+5}{x^2(x-1)-1(x-1)}$

$= \dfrac{3x+5}{(x-1)(x^2-1)} = \dfrac{3x+5}{(x-1)(x+1)(x-1)}$

$= \dfrac{3x+5}{(x-1)^2(x+1)}$

$$\int \frac{3x+5}{x^3-x^2-x+1}\,dx = \int \frac{3x+5}{(x-1)^2(x+1)}\,dx$$

माना $\dfrac{3x+5}{(x-1)^2(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1}$

$$\frac{3x+5}{(x-1)^2(x+1)} = \frac{A(x-1)(x+1) + B(x+1) + C(x-1)^2}{(x-1)^2(x+1)}$$

$\Rightarrow 3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

यदि $x=1$ होः

$8 = B(2)$

$B = 4$

यदि $x=-1$ होः

$-3+5 = C(-2)^2$

$2 = C(4)$

$C = \frac{1}{2}$

यदि $x=2$ होः

$11 = A(1)(3) + B(3) + C(1)^2$

$11 = 3A + 4\times 3 + \frac{1}{2}$

$3A = 11 - 12 - \frac{1}{2} = -1 - \frac{1}{2} = \frac{-2-1}{2}$

$A = \frac{-3}{2} \times \frac{1}{3} \Rightarrow A = \frac{-1}{2}$

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हल :

$$\int \frac{3x+5}{(x-1)^2(x+1)}\,dx = \int \frac{A}{x-1}dx + \int \frac{B}{(x-1)^2}dx + \int \frac{C}{x+1}dx$$

$$= -\frac{1}{2}\int \frac{1}{x-1}dx + 4\int \frac{1}{(x-1)^2}dx + \frac{1}{2}\int \frac{1}{x+1}dx$$

$$= -\frac{1}{2}\log|x-1| + 4\int \frac{1}{(x-1)^2}dx + \frac{1}{2}\log|x+1|$$

माना $x-1=t$

$dx=dt$

$$I_1 = 4\int \frac{1}{(x-1)^2}dx = 4\int \frac{1}{t^2}dt = 4\left(\frac{-1}{t}\right)+c$$

$$I_1 = 4\left(\frac{-1}{x-1}\right)+c$$

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अत.

$$\int \frac{3x-5}{(x-1)^2(x+1)}\,dx = -\frac{1}{2}\log|x-1| - \frac{4}{x-1} + \frac{1}{2}\log|x+1| + c$$

