RIICO
GROW WITH RAJASTHAN

Under CSR Support By Foundation missiongyan

कक्षा-10 गणित | हिंदी माध्यम

missiongyan
LET'S LEARN TOGETHER

ARJUN BATCH

त्रिकोणमिति का परिचय

अध्याय-8, भाग-6



RIICO
GROW WITH RAJASTHAN

missiongyan
LET'S LEARN TOGETHER

Aim-100

आज क्या पढेंगे ?

1 त्रिकोणमितीय सर्वसमिकाएँ



RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

CAL / KKA

$\sin\theta = \frac{\text{लं.}}{\text{कर्ण}}$

$\cos\theta = \frac{\text{आ.}}{\text{कर्ण}}$

$\tan\theta = \frac{\text{लं.}}{\text{आ.}}$

$\cot\theta = \frac{\text{आ.}}{\text{लं.}}$

$\sec\theta = \frac{\text{कर्ण}}{\text{आ.}}$

$\text{cosec}\theta = \frac{\text{कर्ण}}{\text{लम्ब}}$

$\sin\theta = \frac{1}{\text{cosec}\theta}$

$\text{cosec}\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

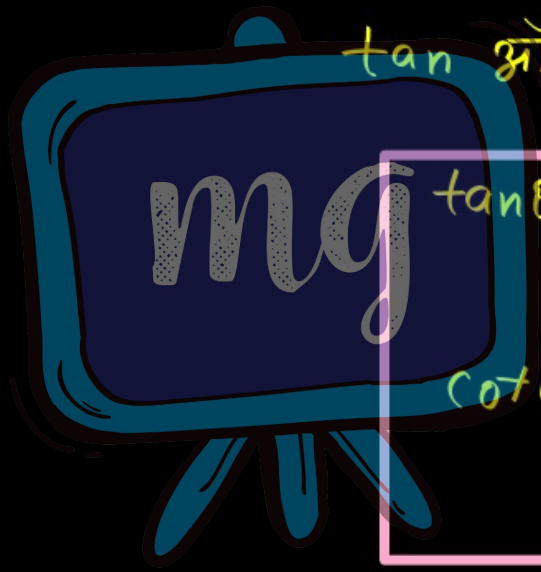
$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

tan और cot को sin/cos में बदलना

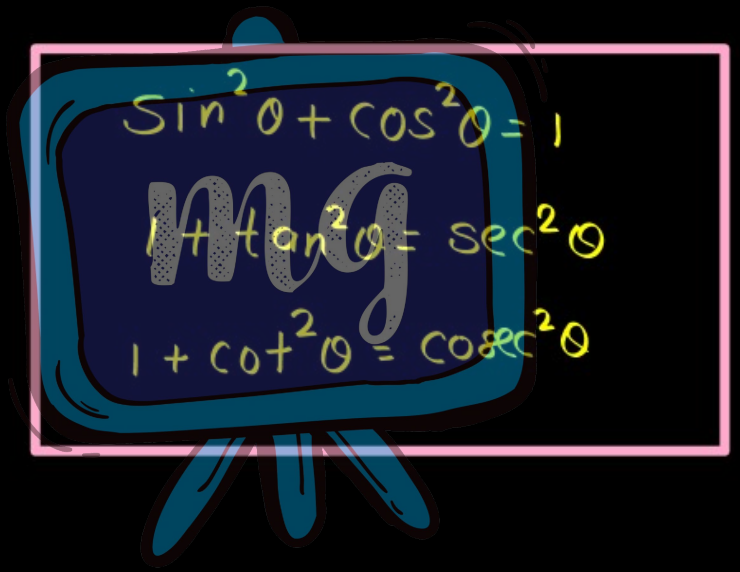


$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER




mg

त्रिकोणमितीय सर्वसमिकाएँ

$1 = \sin^2\theta + \cos^2\theta$

कोण Same होना चाहिए

कर्ण^2 = लम्ब^2 + आधार^2

दोनो तरफ कर्ण^2 का भाग देनेपर

$$\frac{\text{कर्ण}^2}{\text{कर्ण}^2} = \frac{\text{लम्ब}^2 + \text{आधार}^2}{\text{कर्ण}^2}$$

$$1 = \frac{\text{लम्ब}^2}{\text{कर्ण}^2} + \frac{\text{आधार}^2}{\text{कर्ण}^2}$$

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

$$\frac{\text{कर्ण}^2}{\text{लम्ब}^2} = \frac{\text{लम्ब}^2 + \text{आधार}^2}{\text{लम्ब}^2}$$

$$\text{cosec}^2\theta = \frac{\cancel{\text{लम्ब}^2}}{\cancel{\text{लम्ब}^2}} + \frac{\text{आधार}^2}{\text{लम्ब}^2}$$

$$\boxed{\text{cosec}^2\theta = 1 + \cot^2\theta}$$

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

$a^2 - b^2 = (a+b)(a-b)$

$1^2 - \cos^2\theta = \sin^2\theta$

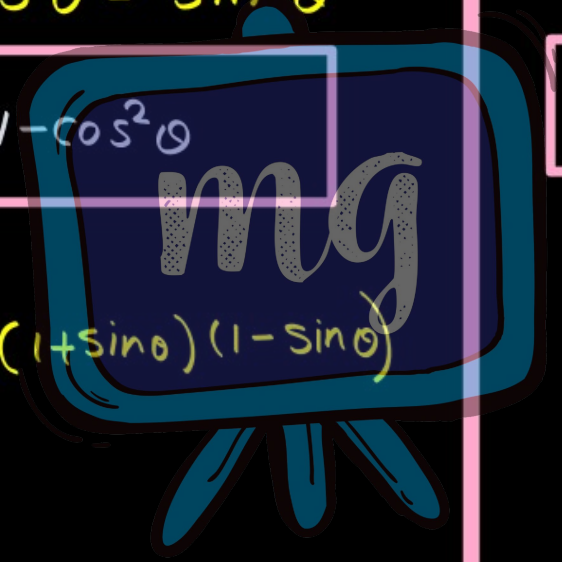
$(1+\cos\theta)(1-\cos\theta) = 1-\cos^2\theta$

$1 - \sin^2\theta = (1+\sin\theta)(1-\sin\theta)$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

$\cos^2\theta = 1-\sin^2\theta$



RIICO
GROW WITH RAJASTHAN

missiongyan
LET'S LEARN TOGETHER

Q. $2\left[\underset{(1+\sin\theta)(1-\sin\theta)}{a+b \quad a-b} + (1+\cos\theta)(1-\cos\theta)\right]$

Sol:- $2\left[1^2-\sin^2\theta + 1^2-\cos^2\theta\right]$

$2\left[1-\sin^2\theta+1-\cos^2\theta\right]$

$2\left[\cos^2\theta+\sin^2\theta\right]$

$2\times1=2$

mg

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\sec^2\theta - 1 = \tan^2\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$

$$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$$

RIICO
GROW WITH RAJASTHAN

missiongyan
LET'S LEARN TOGETHER

Q. यदि $\sec\theta + \tan\theta = 2$ है तो $\sec\theta$ व $\tan\theta$ का मान होगा?

हल :-

$$\sec\theta + \tan\theta = 2 \quad \text{— (i)}$$

$$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$$

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\sec\theta - \tan\theta = \frac{1}{2} \quad \text{-(ii)}$$

समी. (i) व (ii) को जोड़ने पर

$$\sec\theta + \tan\theta = 2$$
$$\sec\theta - \tan\theta = \frac{1}{2}$$

$$2\sec\theta = \frac{5}{2}$$

$$\sec\theta = \frac{5}{4}$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$$

$$\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$$

$$\operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

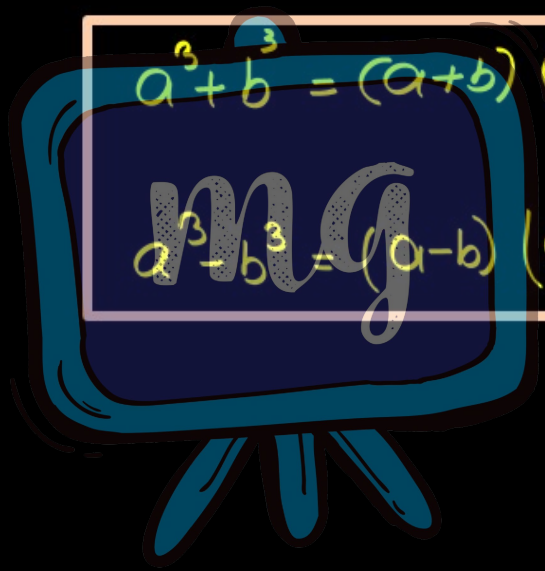
RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

$(a+b)^3$

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

$$a^3-b^3=(a-b)(a^2+b^2+ab)$$



RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER



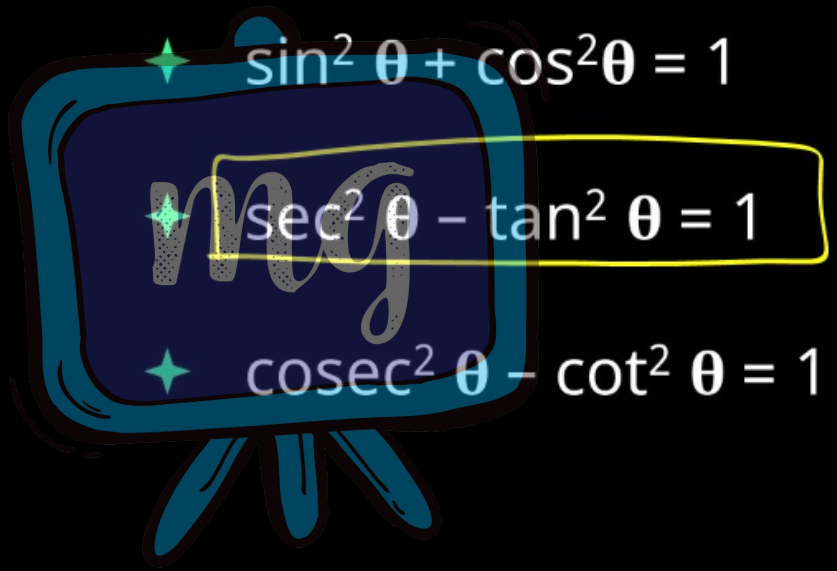
Seco का मान क्या मी. (i) के

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

त्रिकोणमितीय सर्वसमिकाएँ

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$



उदाहरण - 9

अनुपातों cos A, tan A और sec A को sin A के पदों में व्यक्त कीजिए।

$\tan A = \frac{\sin A}{\cos A}$

$\tan A = \frac{\sin A}{\sqrt{1-\sin^2 A}}$

हल :- $\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

$\cos A = \sqrt{1-\sin^2 A}$

$\sec A = \frac{1}{\cos A}$

$\sec A = \frac{1}{\sqrt{1-\sin^2 A}}$

RIICO
GROW WITH RAJASTHAN

missiongyan
LET'S LEARN TOGETHER

छोटे भाई

Sin / Cos

बड़े भाई

tan
cot
sec
cosec

$1-\sin^2A=\cos^2A$

उदाहरण - 10

सिद्ध कीजिए कि -

$\sec A\,(1-\sin A)\,(\sec A+\tan A)=1$

हल :- $\sec A\,(1-\sin A)\,(\sec A+\tan A)=1$

LHS

$\sec A\,(1-\sin A)\,(\sec A+\tan A)$

$\frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$

$\sec A=\frac{1}{\cos A}$

$\tan A=\frac{\sin A}{\cos A}$

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

$$= \frac{1}{\cos A}\left(1-\sin A\right)\left(\frac{1+\sin A}{\cos A}\right)$$

$$= \frac{1}{\cos A \times \cos A}\underbrace{(1-\sin A)}_{a-b}\underbrace{(1+\sin A)}_{a+b}$$

$$= \frac{1-\sin^2 A}{\cos^2 A} = \frac{1-\sin^2 A}{\cos^2 A}$$

$\{ 1-\sin^2 A = \cos^2 A \}$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1 = RHS$$

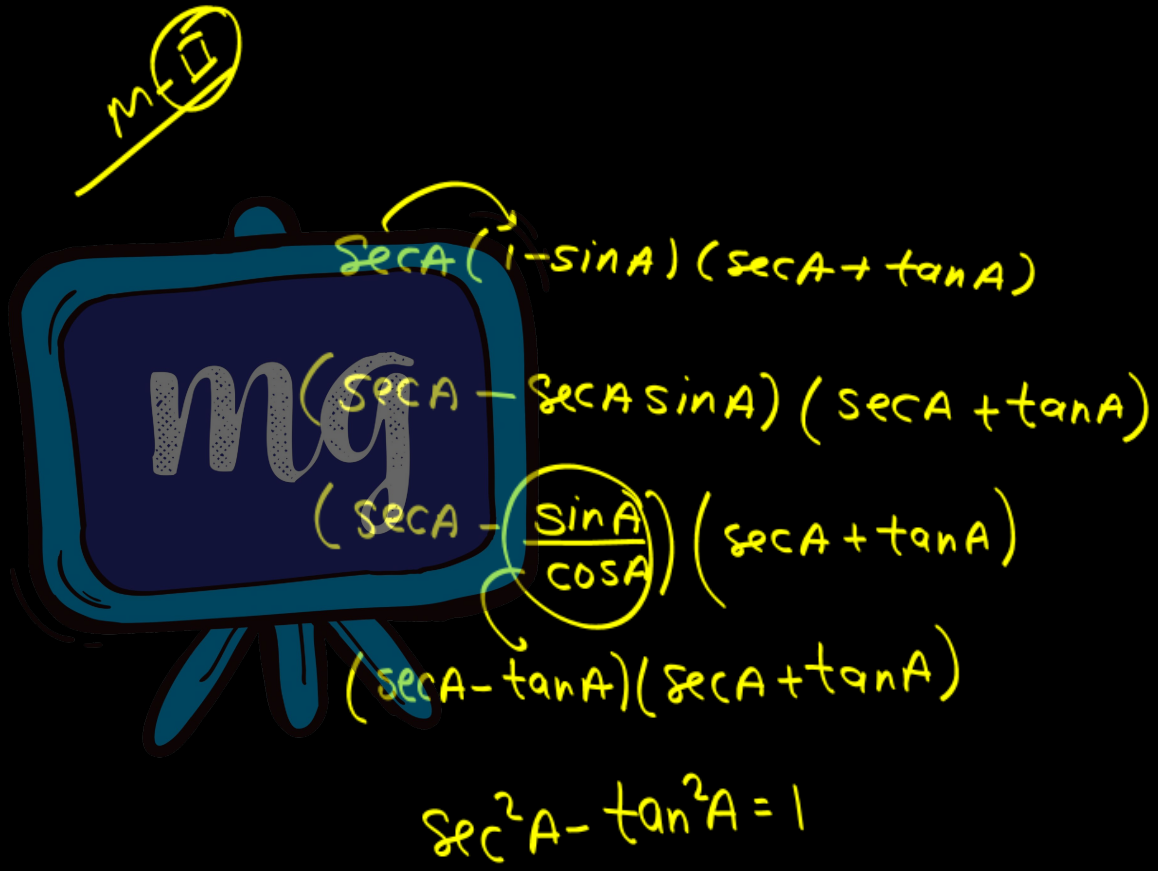
LHS = RHS

HP

mg

RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER



RIICO
GROW WITH RAJASTHAN

missiongyan®
LET'S LEARN TOGETHER

उदाहरण - 11

सिद्ध कीजिए कि -

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\text{cosec } A - 1}{\text{cosec } A + 1}$$

RIICO
GROW WITH RAJASTHAN

missiongyan
LET'S LEARN TOGETHER

उदाहरण - 12

सर्वसमिका $\sec^2\theta = 1 + \tan^2\theta$ का प्रयोग करके सिद्ध कीजिए कि

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$$