कक्षा - 10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

भाग - 6

केशव शर्मा

रिवीजन

LAL / KKA

$$\sin\theta = \frac{L}{K} \qquad \operatorname{cosec}\theta = \frac{K}{L}$$

$$\cos\theta = \frac{A}{K} \qquad \sec\theta = \frac{K}{A}$$

$$\tan\theta = \frac{L}{A} \qquad \cot\theta = \frac{A}{L}$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

त्रिकोणमितीय सर्वसमिकाएँ

$1 = \sin^2\theta + \cos^2\theta$ ① $\sin^2\theta + \cos^2\theta = 1$

$K^2 = L^2 + A^2$

दोनो तरफ K^2 से भाग देनेपर

$$\frac{K^2}{K^2} = \frac{L^2}{K^2} + \frac{A^2}{K^2}$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

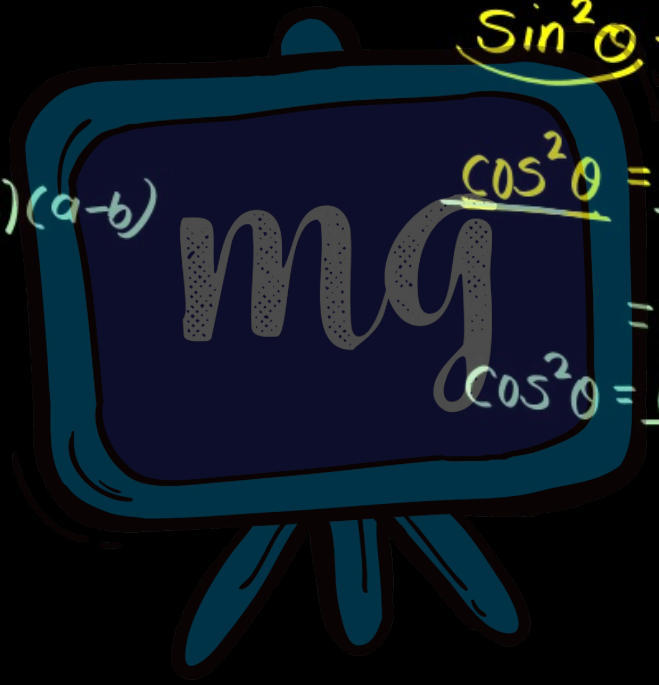
$\cos^2\theta = 1 - \sin^2\theta$

$a^2 - b^2$

$= (a+b)(a-b)$

$= 1^2 - \sin^2\theta$

$\cos^2\theta = (1+\sin\theta)(1-\sin\theta)$



सिद्ध कीजिए –

θ. $\dfrac{\cos^2\theta}{1-\sin\theta} = 1+\sin\theta$

LHS. $\dfrac{\cos^2\theta}{1-\sin\theta}$

$= \dfrac{1^2-\sin^2\theta}{1-\sin\theta} = \dfrac{(1+\sin\theta)(1-\sin\theta)}{1-\sin\theta}$

$= 1+\sin\theta$

(ii) $\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

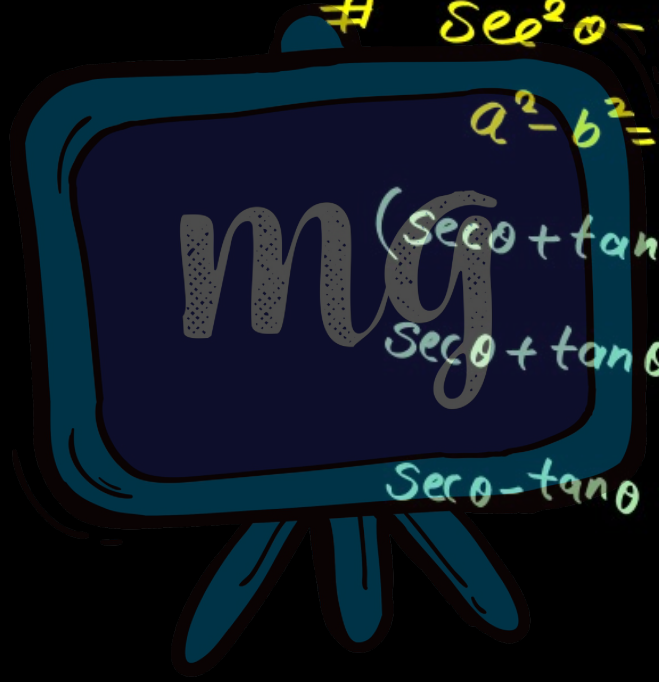
$K^2 = L^2 + A^2$

A^2 का भाग देनेपर

$$\frac{K^2}{A^2} = \frac{L^2}{A^2} + \frac{A^2}{A^2}$$

$\sec^2\theta = \tan^2\theta + 1$

$\sec^2\theta - \tan^2\theta = 1$



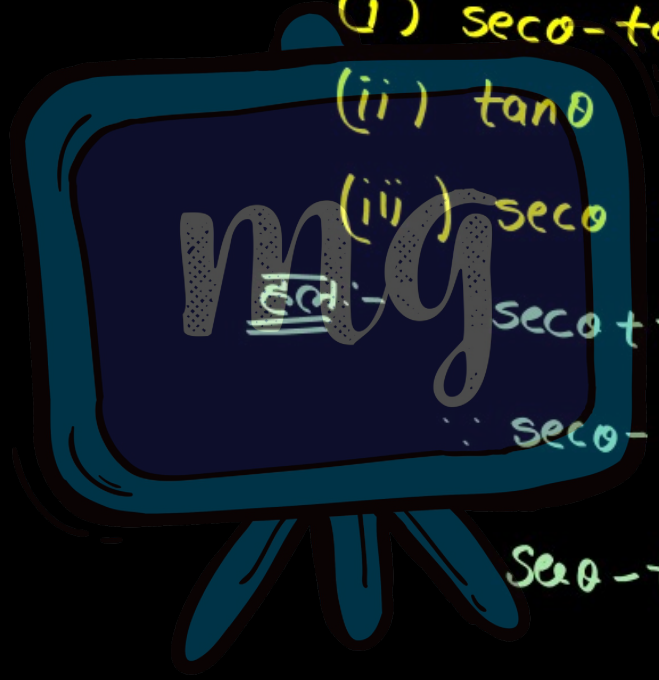
$\# \quad \sec^2\theta - \tan^2\theta = 1$

$a^2 - b^2 = (a+b)(a-b)$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$

$\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$



Q. यदि $\sec\theta + \tan\theta = 2p$ है, तो

(i) $\sec\theta - \tan\theta$

(ii) $\tan\theta$

(iii) $\sec\theta$ का मान ज्ञात कीजिए।

हल :- $\sec\theta + \tan\theta = 2p$ —(i)

$\because \sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$

$\sec\theta - \tan\theta = \dfrac{1}{2p}$ —(ii)

समी. (i) व (ii) को जोड़ने पर

$$\sec\theta + \tan\theta = 2p$$

$$\sec\theta - \tan\theta = \frac{1}{2p}$$

$$2\sec\theta = \frac{2p}{1} + \frac{1}{2p}$$

$$2\sec\theta = \frac{4p^2+1}{2p}$$

$$\sec\theta = \frac{4p^2+1}{4p}$$

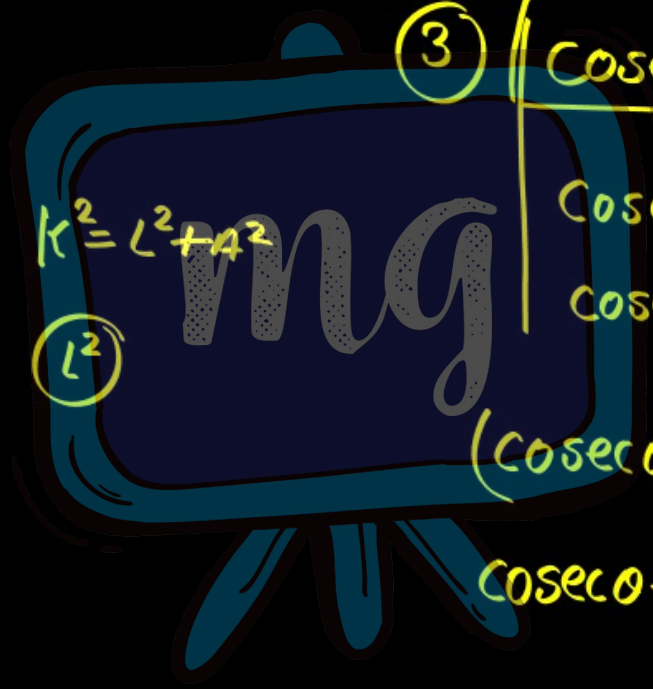
समी. (i) में से समी. (ii) को घटाने पर

$$\sec\theta + \tan\theta = 2p$$

$$-\sec\theta \mp \tan\theta = \frac{1}{-2p}$$

$$2\tan\theta = \frac{2p}{1} - \frac{1}{2p}$$

$$2\tan\theta = \frac{4p^2-1}{2p}$$



(3) $\boxed{\operatorname{cosec}^2\theta - \cot^2\theta = 1}$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

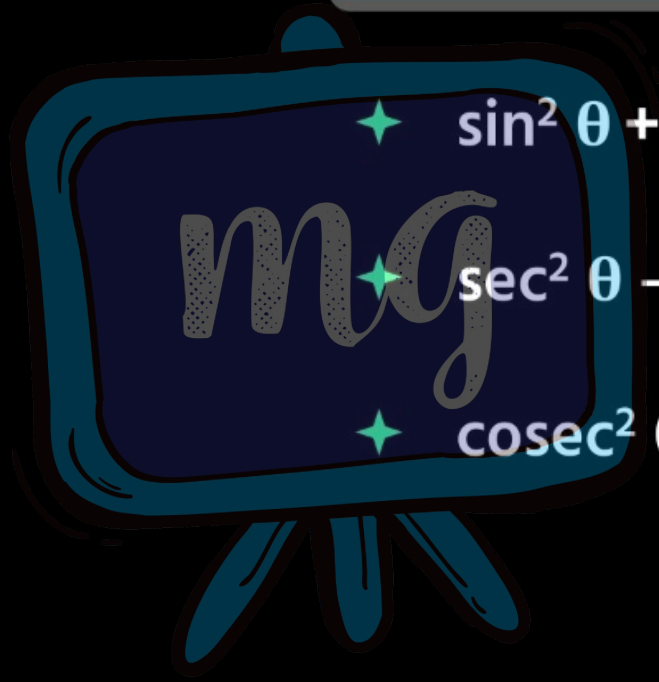
$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

$\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$

$K^2 = L^2 + A^2$

L^2

त्रिकोणमितीय सर्वसमिकाएँ



- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

प्रश्नावली 8.3

Q.1 त्रिकोणमितीय अनुपातों sin A, sec A और tan A को cot A के पदों में व्यक्त कीजिए।

हल:-

$$\tan A = \frac{1}{\cot A}$$

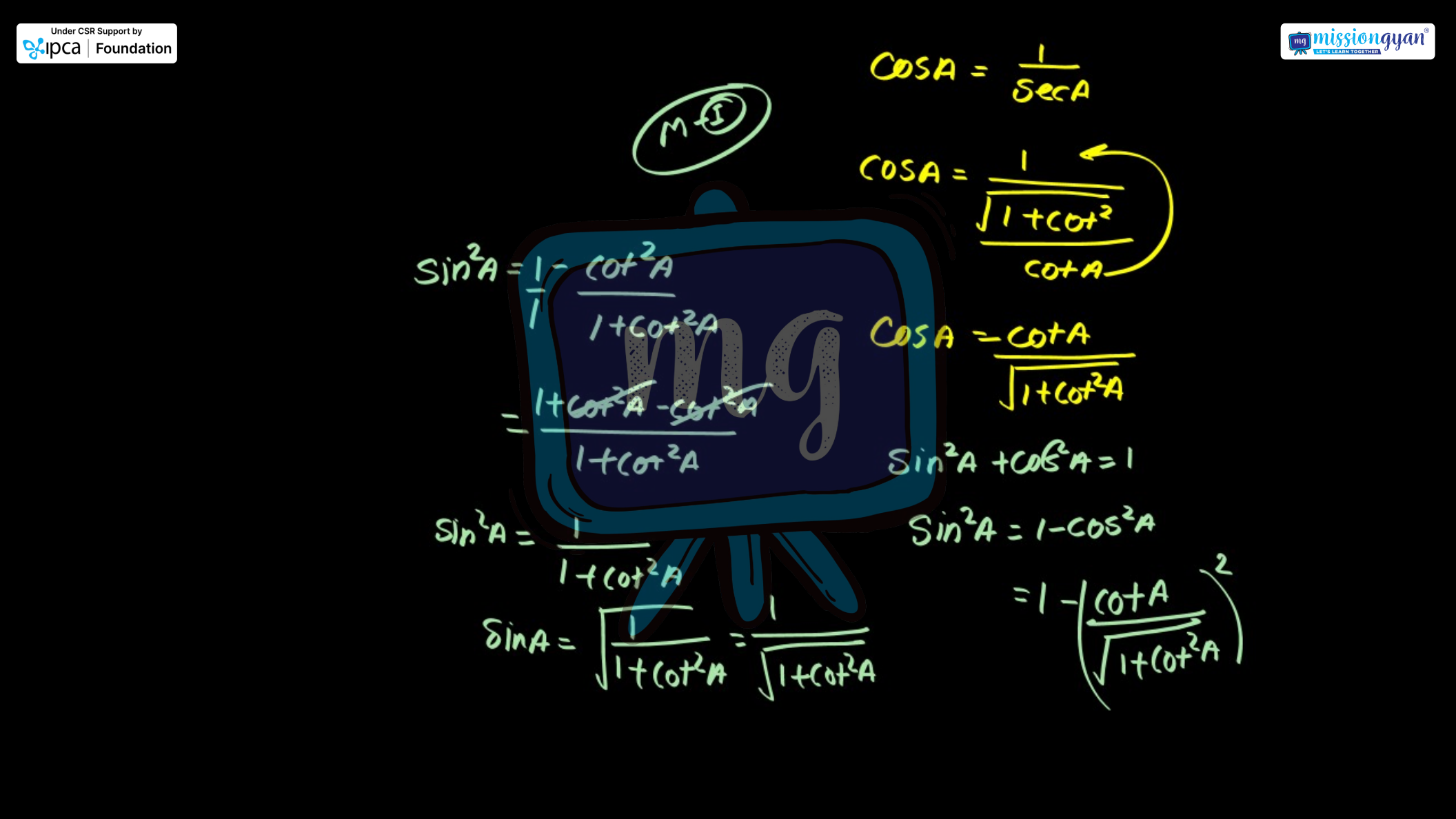
$$\sec^2 A = 1 + \tan^2 A$$

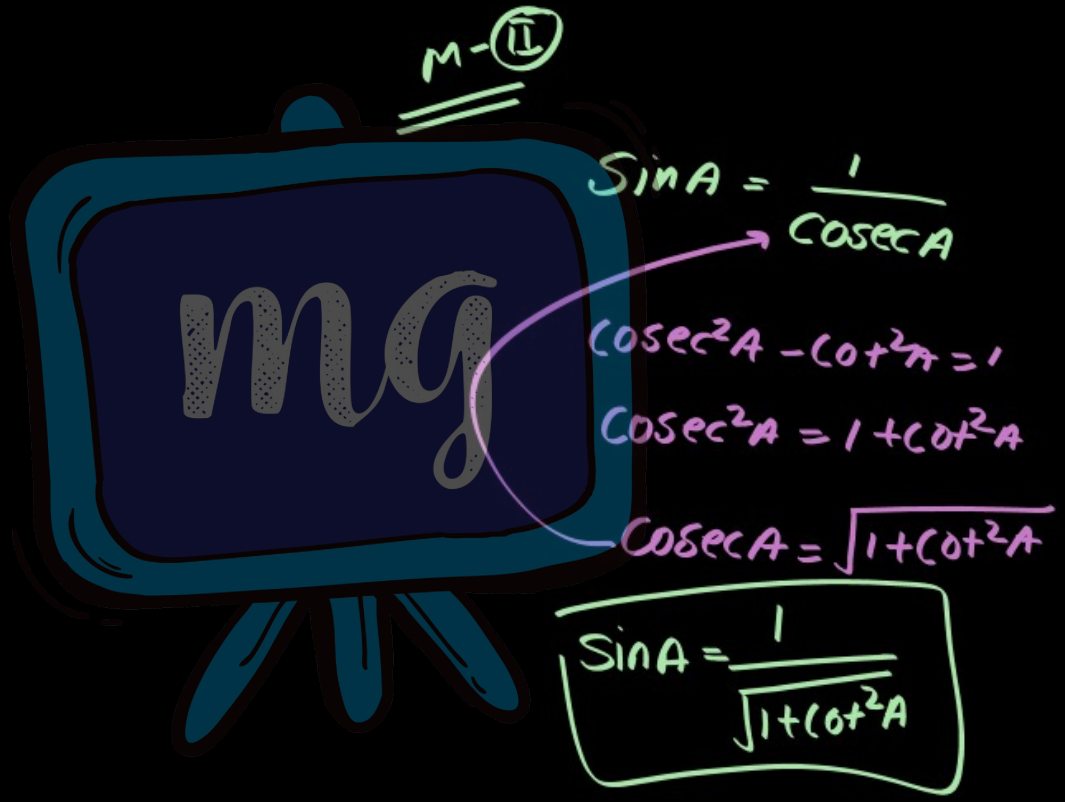
$$\sec^2 A = 1 + \frac{1}{\cot^2 A}$$

$$\sec^2 A = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \sqrt{\frac{1 + \cot^2 A}{\cot^2 A}}$$

$$\sec A = \frac{\sqrt{1 + \cot A}}{\cot A}$$





Q.2 $\angle A$ के अन्य सभी त्रिकोणमितीय अनुपातों को sec A के पदों में लिखिए।

हल :-

$$\cos A = \frac{1}{\sec A}$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\sec^2 A - 1 = \tan^2 A$$

$$\sqrt{\sec^2 A - 1} = \tan A$$

$$\cot A = \frac{1}{\tan A}$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\sin A = \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin^2 A = 1 - \left(\frac{1}{\sec A}\right)^2$$

$$= \frac{1}{1} - \frac{1}{\sec^2 A}$$

$$\sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$\operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Q1. $\angle B$ के लिए अन्य सभी त्रिकोणमितीय अनुपातों को $\cos B$ के रूप में लिखिए।

Q2. यदि $\sec A - \tan A = \frac{1}{3}$ तो

(i) $\sec A + \tan A$

(ii) $2 \sec A$

(iii) $3 \tan A$ के मान ज्ञात करो।