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RIICO GROW WITH RAJASTHAN

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कक्षा-10 गणित | हिंदी माध्यम

ARJUN BATCH

त्रिकोणमिति का परिचय

अध्याय-8, भाग-8



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आज क्या पढेंगे ?

1 प्रश्नावली 8.3



प्रश्नावली 8.3

Q.4 निम्नलिखित सर्वसमिकाएँ सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यून कोण है :

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

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sin/cos

tan/cot/sec/cosec

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Sol :-

LHS.

$$(\text{cosec}\,\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$\left\{\text{cosec}\,\theta = \frac{1}{\sin\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}\right\}$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

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$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$(a^2 - b^2) = (a+b)(a-b)$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

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(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

Col. – LHS

$= \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$= \dfrac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

$= \dfrac{\cos^2 A + 1^2 + \sin^2 A + 2\times 1\times \sin A}{(1+\sin A)\cos A}$

$= \dfrac{(\sin^2 A + \cos^2 A) + 1 + 2\sin A}{(1+\sin A)\cos A}$

$\{\sin^2\theta + \cos^2\theta = 1\}$

$= \dfrac{1+1+2\sin A}{(1+\sin A)\cos A}$

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$$= \frac{2 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\cos A}$$

$$= \frac{2}{\cos A} \qquad \left\{ \frac{1}{\cos A} = \sec A \right\}$$

$$= 2\sec A = RHS$$

$$\boxed{LHS = RHS}$$

Hence Proved

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(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$

हल :- LHS.

$$= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$\{a^3-b^3 = (a-b)(a^2+b^2+ab)\}$$

$$= \frac{1}{\sin\theta-\cos\theta}\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\frac{1-\frac{\cos\theta}{\sin\theta}}{1} \qquad \frac{1-\frac{\sin\theta}{\cos\theta}}{1}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

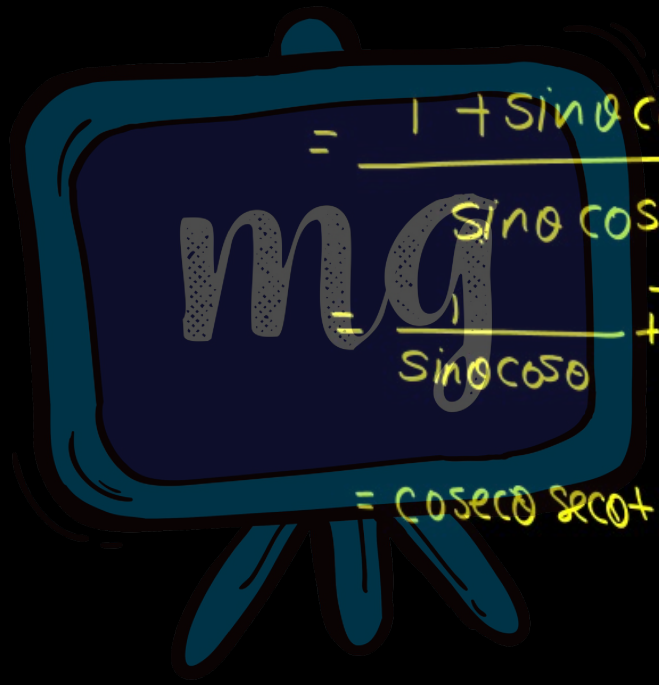
$$\frac{\sin\theta-\cos\theta}{\sin\theta} \qquad \frac{\cos\theta-\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta\times\sin\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos\theta\times\cos\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

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$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$$

$$= \operatorname{cosec}\theta \sec\theta + 1$$

$$\left\{ \frac{1}{\sin\theta} = \operatorname{cosec}\theta \quad \frac{1}{\cos\theta} = \sec\theta \right\}$$

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(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

हल :- LHS

$= \dfrac{1+\sec A}{\sec A}$

$= \dfrac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$ $\left\{\sec A = \dfrac{1}{\cos A}\right\}$

$= \dfrac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \dfrac{(1+\cos A)\cancel{\cos A}}{\cancel{\cos A}}$

$= 1+\cos A$

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RHS.

$$= \frac{\sin^2 A}{1-\cos A}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1^2 - \cos^2\theta$$

$$(a^2 - b^2)$$

$$(a+b)(a-b)$$

$$1^2 - \cos^2\theta = (1+\cos\theta)(1-\cos\theta)$$

$$= \frac{1-\cos^2 A}{1-\cos A}$$ ✓

$$= \frac{(1+\cos A)(1-\cos A)}{1-\cos A}$$

$$= 1+\cos A$$

LHS = RHS 'H.P.'

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(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

$\frac{1}{\sin}$ $\frac{\cos}{\sin}$

$\sec \frac{1}{\cos}$

$\tan \frac{\sin}{\cos}$

हल :- LHS

$= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

अंश और हर में $\sin A$ से भाग देने पर

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$$\{\operatorname{cosec}^2\theta - \cot^2\theta = 1\}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{1 + \cot A - \operatorname{cosec} A} \qquad a^2 - b^2$$

$$= \frac{(\cot A + \operatorname{cosec} A) - \left[\frac{(\operatorname{cosec} A + \cot A)}{(\operatorname{cosec} A - \cot A)}\right]}{1 + \cot A - \operatorname{cosec} A}$$

$$= \frac{(\cot A + \operatorname{cosec} A)\left[1 - \frac{(\operatorname{cosec} A - \cot A)}{}\right]}{1 + (\cot A - \operatorname{cosec} A}$$

$$= \frac{\frac{\cos A - \sin A + 1}{\sin A}}{\frac{\cos A + \sin A - 1}{\sin A}}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

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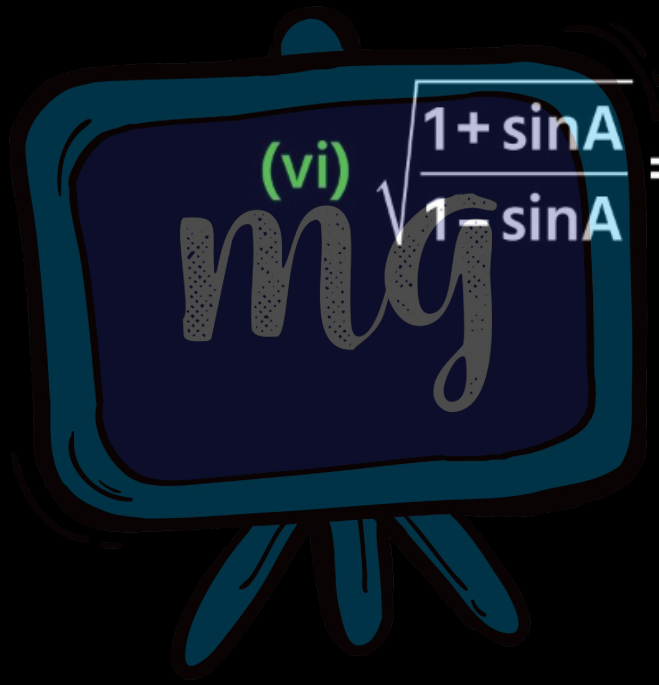
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$$= \frac{(\cot A + \operatorname{cosec} A)\,[1 - \operatorname{cosec} A + \cot A]}{1 + \cot A - \operatorname{cosec} A}$$

$$= \cot A + \operatorname{cosec} A = \text{RHS}$$

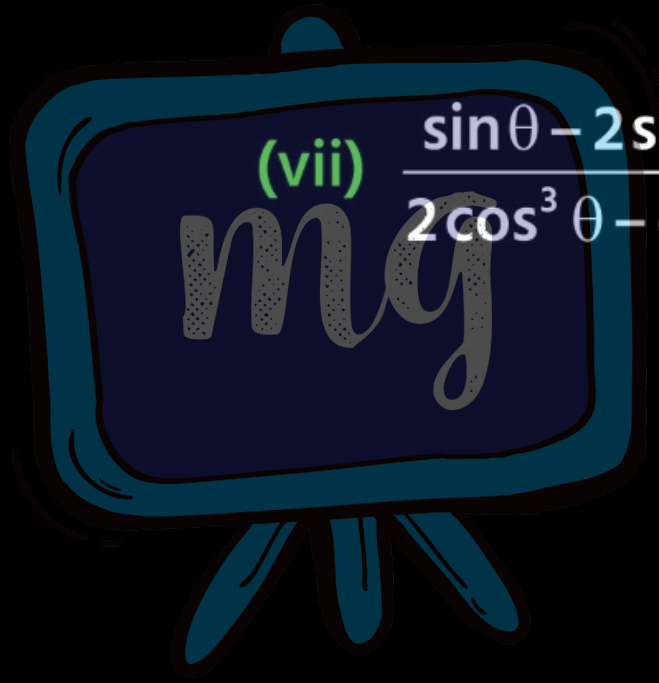
$$\text{LHS} = 1$$

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(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

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(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

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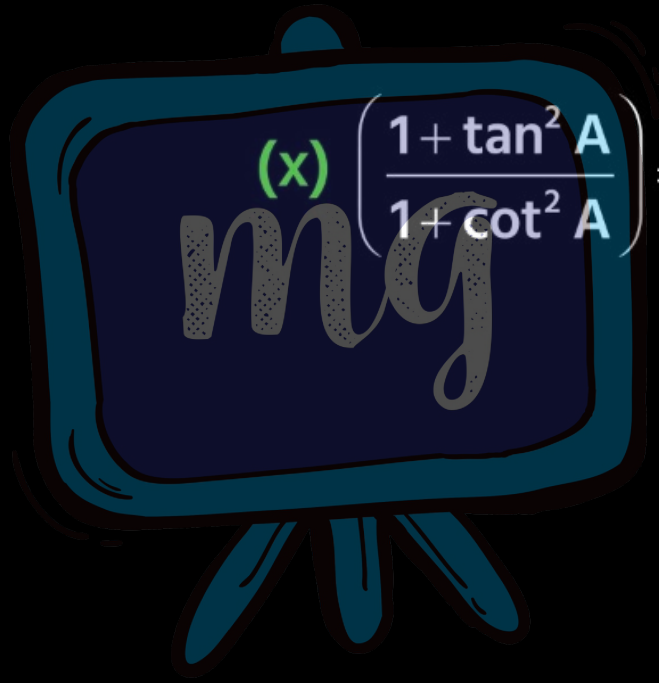
(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$

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(ix) $(\text{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

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(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$