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ई-पाठशाला कार्यक्रम

Class 12th

MATHS

समाकलन

Integral

RAPID REVISION



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$t = x^5 + 1$

L.L. $x = -1$

$t = (-1)^5 + 1 = -1 + 1$

$t = 0$

प्रश्न $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\, dx$ का मान ज्ञात कीजिए।

(2022)

हल :- $I = \int_{-1}^{1} 5x^4 \sqrt{x^5+1}\, dx$

माना $x^5 + 1 = t$

$5x^4 dx = dt$

U.L. $x = 1$

$t = (1)^5 + 1$

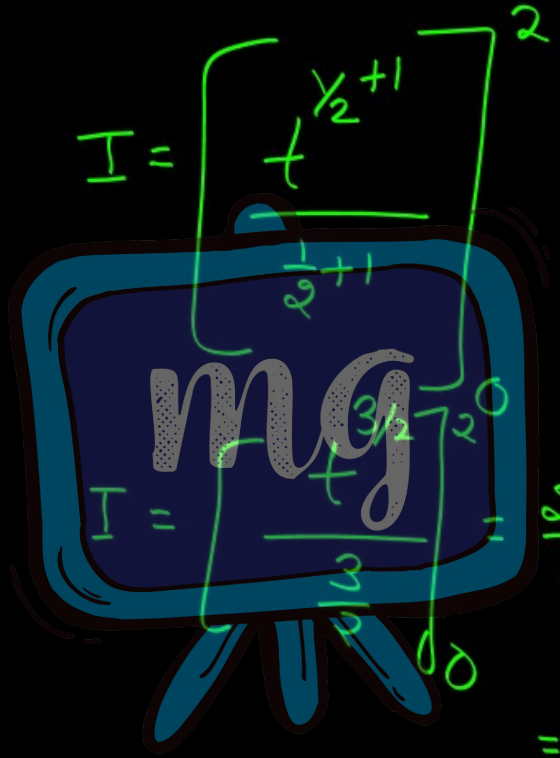
$t = 1 + 1$

$t = 2$

$I = \int_{0}^{2} \sqrt{t}\, dt$

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$$I = \left[\frac{t^{1/2+1}}{\frac{1}{2}+1} \right]_0^2$$

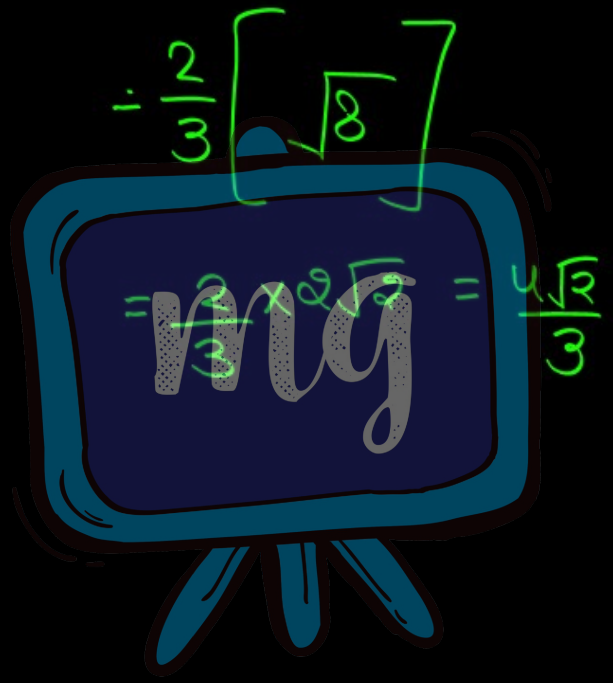
$$I = \left[\frac{t^{3/2}}{\frac{3}{2}} \right]_0^2$$

$$w/\rho \left[t^{3/2} \right]_0^2$$

$$= w/\rho \left[\rho^{3/2} - 0^{3/2} \right]$$

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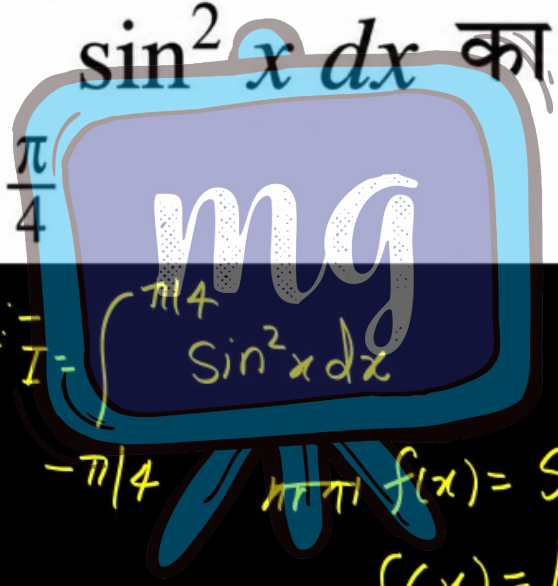


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प्रश्न

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$$ का मान ज्ञात कीजिए।



हल:-

$$I = \int_{-\pi/4}^{\pi/4} \sin^2 x \, dx$$

$f(x) = \sin^2 x$

$f(-x) = [\sin(-x)]^2 = [-\sin x]^2$

$f(-x) = \sin^2 x$

$\Rightarrow \boxed{f(x) = f(-x)}$

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$$I = 2\int_0^{\pi/4} \sin^2 x\, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$I = 2\int_0^{\pi/4} \frac{1}{2}[1 - \cos 2x]\, dx$$

$$I = \frac{2}{2}\int_0^{\pi/4} 1\, dx - \int_0^{\pi/4} \cos 2x\, dx$$

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$$I = [x]_0^{\pi/4} - \left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$$

$$= \left[\frac{\pi}{4} - 0\right] - \frac{1}{2}\left[\sin 2\left(\frac{\pi}{4}\right) - \sin 2\times 0\right]$$

$$= \frac{\pi}{4} - \frac{1}{2}[1 - 0]$$

$$I = \frac{\pi}{4} - \frac{1}{2}$$

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Q. $I = \int_{-\pi/2}^{\pi/2} \sin x \cos x \, dx$

$f(x) = \sin x \cos x \, dx$

$f(-x) = \sin(-x)\cos(-x)$

$= -\sin x \cos x$

$f(-x) = -f(x)$

$I = \int_{-\pi/2}^{\pi/2} \sin x \cos x \, dx = 0$

$$\int_{-a}^{a} f(x)\,dx = 0 \; ; \; f(x) \text{ विषम फलन } f(-x) = -f(x)$$

$$= 2\int_{0}^{a} f(x)\,dx \; , \; f(x) \text{ सम फलन } f(-x) = f(x)$$

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प्रश्न

$$\int_2^3 \frac{dx}{x^2-1}$$ का मान ज्ञात कीजिए ।

(2026)

हल :- $I = \int_2^3 \frac{dx}{x^2-1^2}$

$$\left\{ \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C \right\}$$

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$$I = \left[\frac{1}{2x}\log\left|\frac{x-1}{x+1}\right|\right]_2^3$$

$$I = \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right]$$

$$I = \frac{1}{2}\left[\log\left(\frac{2}{4}\right) - \log\left|\frac{1}{3}\right|\right]$$

$$I = \frac{1}{2}\left[\log\left(\frac{1}{2}\right) - \log\left(\frac{1}{3}\right)\right]$$

$$\{\log m - \log n = \log \frac{m}{n}\}$$

$$I = \frac{1}{2}\log\frac{\frac{1}{2}}{\frac{1}{3}}$$

$$I = \frac{1}{2}\log\left(\frac{3}{2}\right)$$

प्रश्न $\int_0^1 \left(\sin^{-1} x + \cos^{-1} x\right) dx$ का मान होगा –

अ) $\frac{\pi}{2}$ ब) π

स) $\frac{\pi^2}{4}$ द) π^2

(2025)

$$I = \int_0^1 (\sin^{-1}x + \cos^{-1}x)\, dx$$

$$I = \int_0^1 \frac{\pi}{2}\, dx = \frac{\pi}{2}\int_0^1 1 \cdot dx = \frac{\pi}{2}\left[x\right]_0^1 = \frac{\pi}{2}$$

प्रश्न $\int_1^3 \frac{dx}{x^2(x+1)}$ का मान ज्ञात कीजिए।

(2025)

$$I = \int_1^3 \frac{1}{x^2(x+1)}\,dx$$

$$I = \int_1^3 \frac{(x+1)-x}{x^2(x+1)}\,dx = \int_1^3 \frac{x+1}{x^2(x+1)}\,dx - \int_1^3 \frac{x}{x^2(x+1)}\,dx$$

$$= \int_1^3 \frac{1}{x^2}\,dx - \int_1^3 \frac{1}{x(x+1)}\,dx$$

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$$= \int_1^3 \frac{1}{x^2}\,dx - \int_1^3 \frac{(x+1)-x}{x(x+1)}\,dx$$

$$= \int_1^3 x^{-2}\,dx - \int_1^3 \frac{(x+1)}{x(x+1)}\,dx + \int_1^3 \frac{x}{x(x+1)}\,dx$$

$$= \left[\frac{x^{-3}}{-3}\right]_1^3 - \left[\log|x|\right]_1^3 + \left[\log|x+1|\right]_1^3$$

$$= -\frac{1}{3}\left[3^{-3} - 1^{-3}\right] - \left[\log 3 - \log 1\right] + \left[\log(3+1) - \log(1+1)\right]$$

$$= -\frac{1}{3}\left[\frac{1}{27} - \frac{1}{1}\right] - \log 3 + \log 4 - \log 2$$

$$= -\frac{1}{3}\left[\frac{1-27}{27}\right] - \left[\log 3 + \log 2\right] + \log 4$$

$$= -\frac{1}{3}\left[\frac{-26}{27}\right] - \log 6 + \log 4$$

$$= \frac{26}{81} + \log \frac{4}{6} = \frac{26}{81} + \log \frac{2}{3}$$

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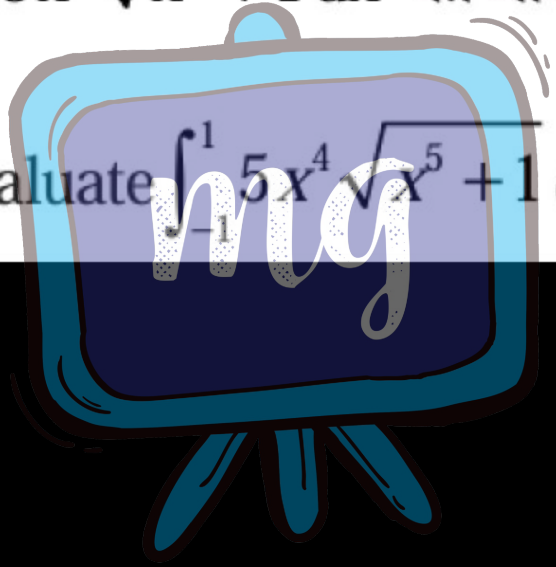
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प्रश्न

$\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx$ का मान ज्ञात कीजिए।

Evaluate $\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx$.

(2024)



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प्रश्न $\int_0^1 \frac{dx}{1+x^2}$ का मान $\pi/4$ है। (2022)

$\int \frac{1}{1+x^2} dx$

$= \tan^{-1}x + C$

$\int_0^1 \frac{1}{1+x^2} dx = [\tan^{-1}x]_0^1$

$= \tan^{-1}1 - \tan^{-1}0$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$

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$$\int_{1/3}^{1} \frac{\left[\frac{1}{x^2}-1\right]^{1/3}}{x^3}\,dx$$

माना $\frac{1}{x^2}-1 = t$

$\frac{-2}{x^3}\,dx = dt$

$\frac{1}{x^3}\,dx = -\frac{1}{2}\,dt$

Q. $\int_{1/3}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4}\,dx$

Ans:- $\int_{1/3}^{1} \frac{\left(x^3\left[\frac{1}{x^2}-1\right]\right)^{1/3}}{x^4}\,dx$

$\int_{1/3}^{1} \frac{(x^3)^{1/3}\left[\frac{1}{x^2}-1\right]^{1/3}}{x^4}\,dx$

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$$\frac{1}{x^2} - 1 = t$$

LL. $x = \frac{1}{3}$

$$\frac{1}{\left(\frac{1}{3}\right)^2} - 1$$

$$9 - 1 = \underline{8}$$

$x = 1$

$$\frac{1}{(1)^2} - 1$$

$$1 - 1 = 0$$

$$I = \frac{-1}{2} \int_{8}^{0} t^{1/3}\, dt$$

$$I = \frac{-1}{2} \int_{0}^{8} t^{1/3}\, dt$$

$$I = \frac{1}{2}\left[\frac{t^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_0^8$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$



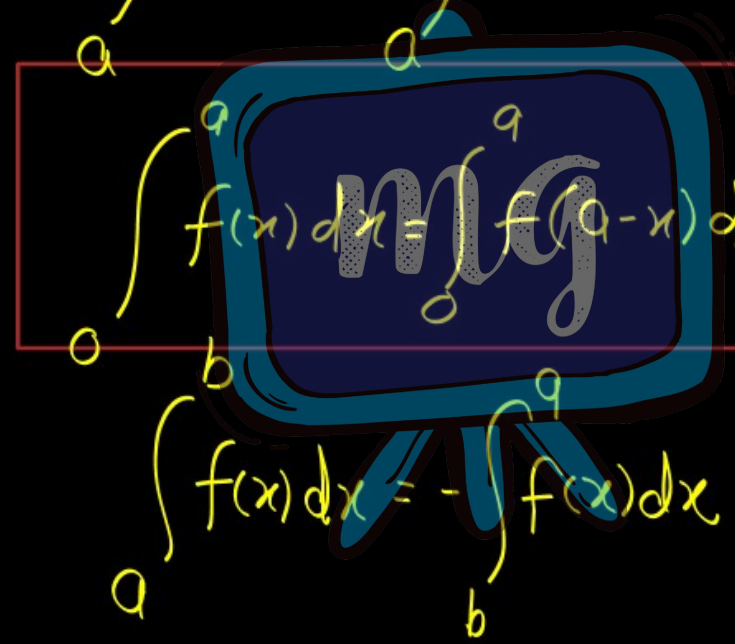
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$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$



$$\int_0^{2a} f(x)\,dx = 0 \quad ; \quad f(2a-x) = -f(x)$$

$$= 2\int_0^{a} f(x)\,dx \quad ; \quad f(2a-x) = f(x)$$



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$$2I = \int_0^{\pi/2} \frac{\sin^{2026} x + \cos^{2026} x}{\sin^{2026} x + \cos^{2026} x}\, dx$$

$$2I = [x]_0^{\pi/2}$$

$$I = \frac{1}{2}\cdot\frac{\pi}{2} = \boxed{\pi/4}$$

$$I = \int_0^{\pi/2} \frac{\sin^{2026} x}{\sin^{2026} x + \cos^{2026} x}\, dx \quad -(i)$$

$$I = \int_0^{\pi/2} \frac{\left[\sin\left(\frac{\pi}{2}-x\right)\right]^{2026}}{\left[\sin\left(\frac{\pi}{2}-x\right)\right]^{2026} + \left[\cos\left(\frac{\pi}{2}-x\right)\right]^{2026}}\, dx$$

$$I = \int_0^{\pi/2} \frac{\cos^{2026} x}{\cos^{2026} x + \sin^{2026} x}\, dx \quad -(ii)$$

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Q. $\int_{-1}^{1} \sin^{1005} x \cos^{1006} x \, dx = 0$



$f(x) = \sin^{1005} x \cos^{1006} x$

$f(-x) = \sin^{1005}(-x) \cos^{1006}(-x)$

$= [-\sin x]^{1005} \cos^{1006} x$

$= -\sin^{1005} x \cos^{1006} x$

$f(-x) = -f(x)$


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