Under CSR Support by
ipca | Foundation

missiongyan
LET'S LEARN TOGETHER

कक्षा - 12

गणित

अध्याय – 5

सांतत्य तथा अवकलनीयता

भाग – 8

केशव शर्मा

Under CSR Support by ipca | Foundation

missiongyan® LET'S LEARN TOGETHER

54. यदि $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$ तो दर्शाइए कि $\frac{dy}{dx} = -\frac{y}{x}$

$x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$

$x = (a^{\sin^{-1}t})^{\frac{1}{2}}$, $y = (a^{\cos^{-1}t})^{\frac{1}{2}}$

$x = a^{\frac{1}{2}\sin^{-1}t}$, $y = a^{\frac{1}{2}\cos^{-1}t}$

$\log x = \log a^{\frac{1}{2}\sin^{-1}t}$; $\log y = \log a^{\frac{1}{2}\cos^{-1}t}$

$\log x = \frac{1}{2}\sin^{-1}t \log a$; $\log y = \frac{1}{2}\cos^{-1}t \log a$

Under CSR Support by ipca Foundation

missiongyan LET'S LEARN TOGETHER

$$\frac{dy}{dt} = \frac{-y\log a}{2\sqrt{1-t^2}}$$

उत्तर :

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-y\log a}{2\sqrt{1-t^2}}}{\frac{x\log a}{2\sqrt{1-t^2}}}$$

$$\boxed{\frac{dy}{dx} = -\frac{y}{x}}$$

$\log x = \frac{1}{2}\sin^{-1}t\log a$

t के सापेक्ष अवकलन —

$$\frac{1}{x}\cdot\frac{dx}{dt} = \frac{1}{2}\log a\,\frac{1}{\sqrt{1-t^2}}$$

$$\frac{dx}{dt} = \frac{x\log a}{2\sqrt{1-t^2}}$$

$\log y = \frac{1}{2}\cos^{-1}t\log a$

t के सापेक्ष अवकलन —

$$\frac{1}{y}\cdot\frac{dy}{dt} = \frac{\log a}{2}\,\frac{-1}{\sqrt{1-t^2}}$$

$\therefore$

Under CSR Support by
ipca | Foundation

missiongyan
LET'S LEARN TOGETHER

द्वितीय कोटि का अवकलज

(Second Order Derivative)

$$y = 4x^2$$

$$y' = \frac{dy}{dx} = \frac{d}{dx} 4x^2 = 4\frac{d}{dx} x^2 = 4 \cdot 2x$$

$$y' = \frac{dy}{dx} = 8x$$

x के सापेक्ष अवकलन करने पर

$$y'' \text{ या } \frac{d^2y}{dx^2} = \frac{d}{dx} 8x = 8\left(\frac{dx}{dx}\right)^1 = 8$$

Under CSR Support by ipca | Foundation

missiongyan® LET'S LEARN TOGETHER

$$\frac{d^2y}{dx^2} = \frac{d}{dx}3x^2 + \frac{d}{dx}\sec^2 x$$

$$\frac{d^2y}{dx^2} = 6x + 2\sec x \cdot \frac{d}{dx}\sec x$$

$$\frac{d^2y}{dx^2} = 6x + 2\sec x \cdot \sec x \tan x$$

$$\boxed{\frac{d^2y}{dx^2} = 6x + 2\sec^2 x \tan x}$$

उदाहरण-14 : यदि $y = x^3 + \tan x$ है तो $\frac{d^2y}{dx^2}$, ज्ञात कीजिए।

हल :-

$y = x^3 + \tan x$

x के सापेक्ष अवकलन —

$$\frac{dy}{dx} = 3x^2 + \sec^2 x$$

पुनः x के सापेक्ष ——

LET'S LEARN TOGETHER

Under CSR Support by
ipca | Foundation

missiongyan
LET'S LEARN TOGETHER

$\frac{d^2y}{dx^2} = A(-\sin x) - B\cos x$

$\frac{d^2y}{dx^2} = -A\sin x - B\cos x$

$\frac{d^2y}{dx^2} = -(A\sin x + B\cos x)$

$\frac{d^2y}{dx^2} = -y$

$\boxed{\frac{d^2y}{dx^2} + y = 0}$

उदाहरण–15 : यदि $y = A\sin x + B\cos x$ है तो सिद्ध कीजिए कि $\frac{d^2y}{dx^2} + y = 0$ है।

हल :-

$y = A\sin x + B\cos x$

$\frac{dy}{dx} = A\cos x + B(-\sin x)$

$\frac{dy}{dx} = A\cos x - B\sin x$

Under CSR Support by
ipca | Foundation

missiongyan®
LET'S LEARN TOGETHER

उदाहरण–16 : यदि $y = \sin^{-1} x$ है तो दर्शाइए कि

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$ है।

LET'S LEARN TOGETHER

Under CSR Support by
ipca | Foundation

missiongyan®
LET'S LEARN TOGETHER

55. $e^{6x} \cos 3x$



Under CSR Support by
ipca | Foundation

missiongyan®
LET'S LEARN TOGETHER

56. यदि $y = 5\cos x - 3\sin x$ है तो सिद्ध कीजिए कि $\frac{d^2y}{dx^2} + y = 0$

LET'S LEARN TOGETHER

Under CSR Support by ipca | Foundation

missiongyan® LET'S LEARN TOGETHER

$y_1 \to y'$

$y_2 \to y''$

$\frac{1}{x} = x^{-1}$

$\frac{d}{dx} x^{-1} = -1\, x^{-1-1} = -x^{-2} = \frac{-1}{x^2}$

57. यदि $y = 3\cos(\log x) + 4\sin(\log x)$ है तो दर्शाइए कि $x^2 y_2 + xy_1 + y = 0$

$y = 3\cos(\log x) + 4\sin(\log x)$

$\frac{dy}{dx} = 3(-\sin(\log x)) \cdot \frac{1}{x} + 4\cos(\log x) \cdot \frac{1}{x}$

$y_1 = -3\sin\log x \cdot \frac{1}{x} + 4\cos\log x \cdot \frac{1}{x}$

$y_1 x = -3\sin\log x + 4\cos\log x$

$y_2 = -3\left[\sin\log x \cdot \left(\frac{-1}{x^2}\right) + \frac{1}{x}\cos\log x \cdot \frac{1}{x}\right]$

$+4\left[\cos\log x \cdot \left(\frac{-1}{x^2}\right) + \frac{1}{x}(-\sin\log x) \cdot \frac{1}{x}\right]$

Under CSR Support by ipca | Foundation

missiongyan® LET'S LEARN TOGETHER

$$y_2 = \frac{3}{x^2}\sin\log x - \frac{3}{x^2}\cos\log x - \frac{4}{x^2}\cos\log x - \frac{4}{x^2}\sin\log x$$

$$x^2y_2 = 3\sin\log x - 3\cos\log x - 4\cos\log x - 4\sin\log x$$

$$= -(3\cos\log x + 4\sin\log x) + 3\sin\log x - 4\cos\log x$$

$$x^2y_2 = -y - (4\cos\log x - 3\sin\log x)$$

LET'S LEARN TOGETHER

$$x^2y_2 = -y - y_1x$$

$$\boxed{x^2y_2 + y_1x + y = 0}$$

Under CSR Support by
ipca | Foundation

missiongyan
LET'S LEARN TOGETHER

58. यदि $y = Ae^{mx} + Be^{nx}$ है तो दर्शाइए कि

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

$y = Ae^{mx} + Be^{nx}$

$\frac{dy}{dx} = Ae^{mx} \cdot m + Be^{nx} \cdot n$

$\frac{d^2y}{dx^2} = Am\, e^{mx} \cdot m + Bn \cdot e^{nx} \cdot n$

LET'S LEARN TOGETHER

Under CSR Support by
ipca | Foundation

missiongyan®
LET'S LEARN TOGETHER

LHS.

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$Am^2e^{mx} + Bn^2e^{nx} - (m+n)(Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$$

LET'S LEARN TOGETHER

Under CSR Support by ipca | Foundation

missiongyan® LET'S LEARN TOGETHER

1. यदि $2x + 8y = \sin x$ तो $\frac{dy}{dx}$

2024

(A) $\frac{\sin x - 2}{8}$ (B) $\frac{\cos x - 2}{8}$

(C) $\frac{\cos x + 2}{2}$ (D) $\frac{\cos x + 2}{3}$

$2x + 8y = \sin x$

x ——————

$\frac{d\,2x}{dx} + \frac{d\,8y}{dx} = \frac{d\,\sin x}{dx}$

$2.1 + 8\frac{dy}{dx} = \cos x$

$\frac{dy}{dx} = \frac{\cos x - 2}{8}$

mg LET'S LEARN TOGETHER

Under CSR Support by ipca | Foundation

missiongyan® LET'S LEARN TOGETHER

2. यदि $y = \cos\sqrt{x}$ हो, तो $\frac{dy}{dx}$ का मान ..... होगा।

2024

$y = \cos\sqrt{x}$

x ———

$\frac{dy}{dx} = \frac{d}{dx}\cos\sqrt{x}$

$\frac{dy}{dx} = -\sin\sqrt{x} \cdot \frac{d}{dx}\sqrt{x}$

$\frac{dy}{dx} = -\sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$\boxed{\frac{dy}{dx} = -\frac{\sin\sqrt{x}}{2\sqrt{x}}}$

LET'S LEARN TOGETHER

Under CSR Support by ipca | Foundation

missiongyan LET'S LEARN TOGETHER

3. यदि $\sin^2 x + \cos^2 y = 1$ हो, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

2024

हल :-

$\sin^2 x + \cos^2 y = 1$

$\times$ ——————

$\frac{d}{dx}\sin^2 x + \frac{d}{dx}\cos^2 y = \frac{d}{dx}1$

$2\sin x \cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0$

$\sin 2x - \sin 2y \frac{dy}{dx} = 0$

$\sin 2x = \sin 2y \frac{dy}{dx}$

$\frac{\sin 2x}{\sin 2y} = \frac{dy}{dx}$

LET'S LEARN TOGETHER

Under CSR Support by ipca | Foundation

missiongyan® LET'S LEARN TOGETHER

4. $\log(\cos.e^x)$ का x के सापेक्ष अवकलन कीजिए।

2024

$$y = \log((\cos e^x)$$

$$\frac{dy}{dx} = \frac{d}{dx}\log((\cos e^x)$$

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot \frac{d}{dx}\cos e^x$$

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot -\sin e^x \cdot \frac{d}{dx} e^x$$

$$\frac{dy}{dx} = \frac{-\sin e^x}{\cos e^x} \cdot e^x$$

LET'S LEARN TOGETHER