KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

कक्षा - 10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

भाग - 3

केशव शर्मा

KOLORS
make life better

missiongyan®
LET'S LEARN TOGETHER



रिवीजन

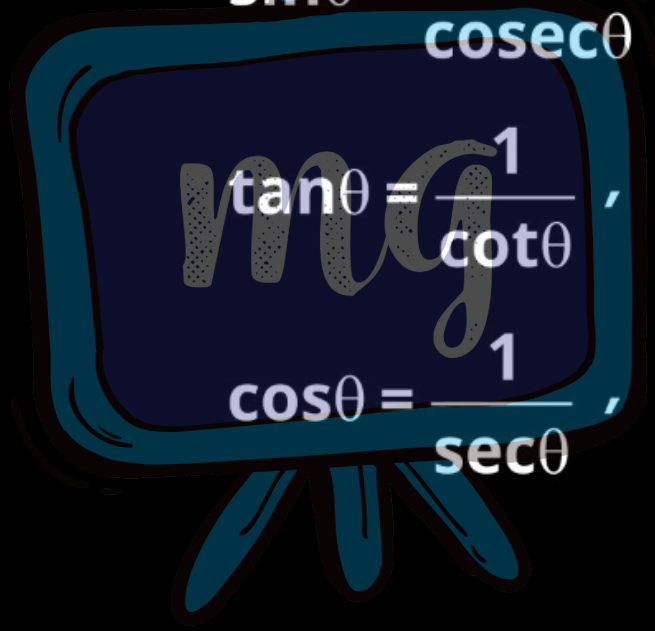
$$\sin\theta = \frac{L}{K} \qquad \operatorname{cosec}\theta = \frac{K}{L}$$

$$\cos\theta = \frac{A}{K} \qquad \sec\theta = \frac{K}{A}$$

$$\tan\theta = \frac{L}{A} \qquad \cot\theta = \frac{A}{L}$$

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER



$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \quad \sec\theta = \frac{1}{\cos\theta}$$

KOLORS
make life better

missiongyan®
LET'S LEARN TOGETHER

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad , \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$



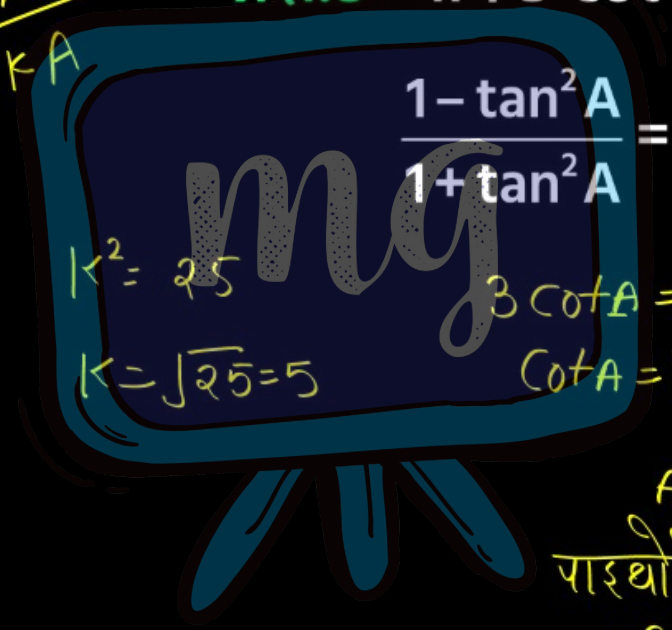
KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

प्रश्नावली 8.1

प्रश्न.8 यदि 3 cot A = 4, तो जाँच कीजिए कि

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$ है या नहीं।



$3\cot A = 4$

$\cot A = \frac{4}{3} = \frac{A}{L}$

$A = 4,\ L = 3,\ K = ?$

पाइथोगोरस प्रमेय

$K^2 = A^2 + L^2$

$= (4)^2 + (3)^2$

$K^2 = 16 + 9$

$K^2 = 25$

$K = \sqrt{25} = 5$

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

$\sin A = \frac{L}{K} = \frac{3}{5}$

$\cos A = \frac{A}{K} = \frac{4}{5}$

$\tan A = \frac{L}{A} = \frac{3}{4}$

LHS. $\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$

RHS. $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25}$

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

LHS = RHS

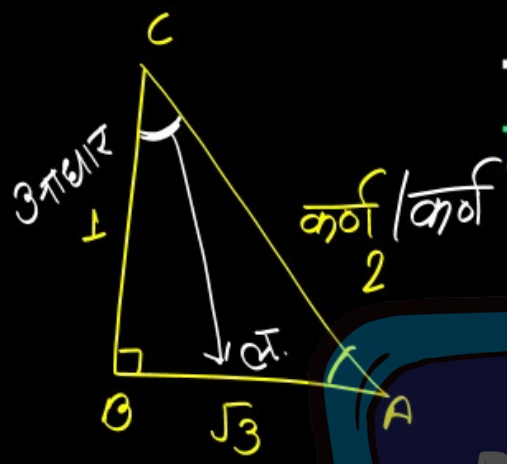
उदा:

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$



KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER



प्रश्न.9 त्रिभुज ABC में, जिसका कोण B समकोण है, यदि $\tan A = \frac{1}{\sqrt{3}}$, तो निम्नलिखित के मान ज्ञात कीजिए :

(i) $\sin A \cos C + \cos A \sin C$

$\tan A = \frac{L}{A} = \frac{1}{\sqrt{3}}$

पाइथोगोरस प्रमेय

$K^2 = A^2 + L^2$

$AC^2 = AB^2 + BC^2$

$AC^2 = (\sqrt{3})^2 + (1)^2 = 3 + 1 = 4$

$AC = \sqrt{4} = 2$

$\sin A = \frac{L}{K} = \frac{1}{2}$

$\cos A = \frac{A}{K} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{L}{K} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{A}{K} = \frac{1}{2}$

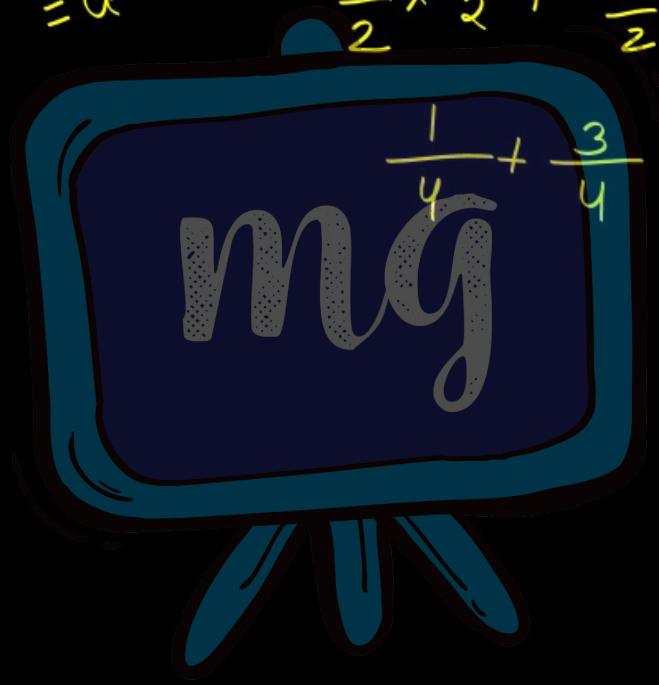
KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

(i) $\sin A \cos C + \cos A \sin C$

$\sqrt{a} \times \sqrt{a} = a$

$\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$



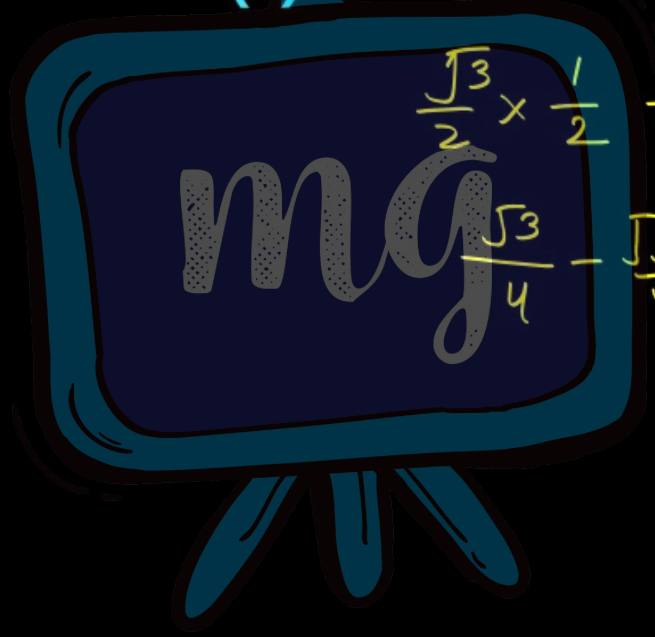
$\frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

प्रश्नावली 8.1

(ii) cos A cos C – sin A sin C



KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

प्रश्नावली 8.1



प्रश्न.10 ΔPQR में, जिसका कोण Q समकोण है, PR + QR = 25 cm और PQ = 5 cm है। sin P, cos P और tan P के मान ज्ञात कीजिए।

PR + QR = 25 cm PQ = 5cm

QR = 25 - PR ---(i)

पाइथागोरस प्रमेय

कर्ण² = लम्ब² + आधार²

(PR)² = (QR)² + (PQ)²

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

$(PR)^2 = (25-PR)^2 + (5)^2$ [समी. (i) से]

$PR^2 = (25)^2 + (PR)^2 - 2\times 25 \times PR + 25$

$PR^2 - PR^2 = 625 - 50PR + 25$

$0 = 650 - 50PR$

$50PR = 650$

$PR = 13$

$PR = 13$ cm

समी. (i) में मान रखने पर

$QR = 25 - PR$

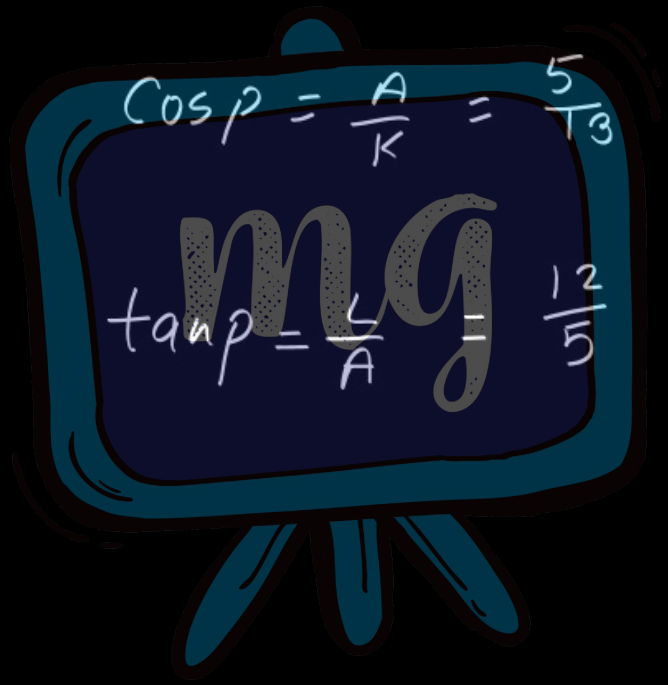
$= 25 - 13$

$QR = 12$ cm

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

$$\sin P = \frac{L}{K} = \frac{12}{13}$$

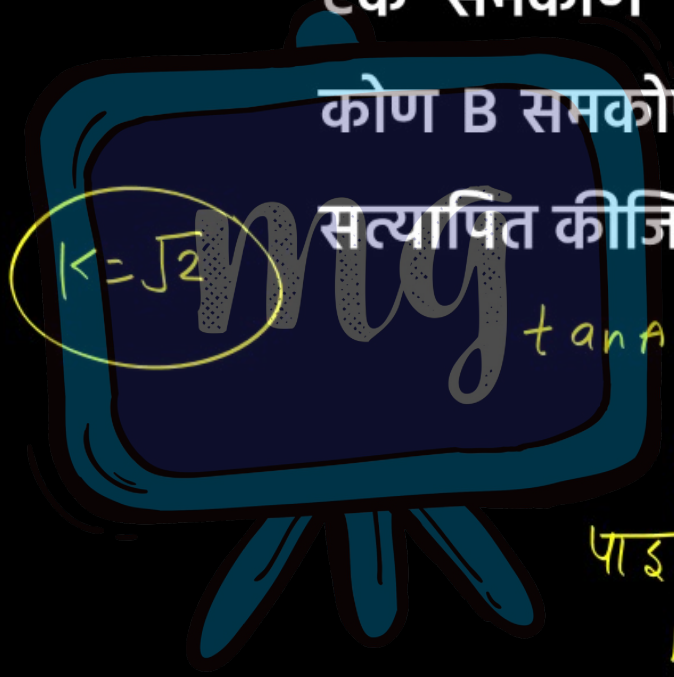


KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

उदाहरण 4.

एक समकोण त्रिभुज ABC में, जिसका कोण B समकोण है, यदि tan A = 1 तो सत्यापित कीजिए कि 2 sin A cos A = 1



$\tan A = \frac{\perp}{1} = \frac{L}{A}$

$L = 1,\ A = 1,\ K = ?$

पाइथोगोरस प्रमेय

$K^2 = A^2 + L^2$

$K^2 = (1)^2 + (1)^2$

$K^2 = 1 + 1 = 2$

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

LHS.

$2 \sin A \cos A$

$\sin A = \frac{L}{K} = \frac{1}{\sqrt{2}}$

$\cos A = \frac{A}{K} = \frac{1}{\sqrt{2}}$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

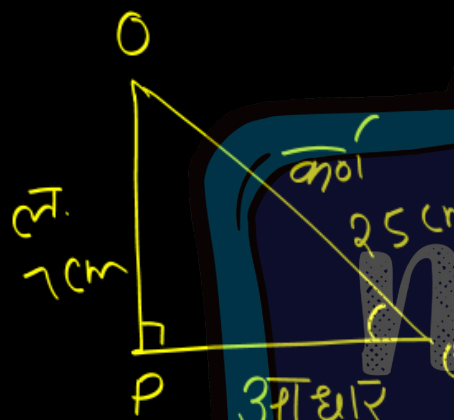
$= \frac{2}{2} = 1 = RHS$

Hence: $2 \sin A \cos A = 1$

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

उदाहरण 5.



$\triangle OPQ$ में, जिसका कोण P समकोण है, $OP = 7$ cm और $OQ - PQ = 1$ cm sin Q और cos Q के मान ज्ञात कीजिए।

$OP = 7$ cm

$OQ - PQ = 1$ cm

$OQ = 1 + PQ$ — (1)

पाइथागोरस प्रमेय

कर्ण^2 = लम्ब^2 + आधार^2

$(OQ)^2 = (OP)^2 + (PQ)^2$

KOLORS
make life better

missiongyan
LET'S LEARN TOGETHER

समी.-(i) से OQ का
मान रखने पर

$(1+PQ)^2 = (7)^2 + (PQ)^2$

$(1)^2 + (PQ)^2 + 2 \times 1 \times PQ = 49 + (PQ)^2$

$1 + 2PQ = 49$

$2PQ = 49 - 1$

$2PQ = 48$ 24

$PQ = 24$

अतः $OQ = 1 + PQ$

$OQ = 1 + 24 = 25$ cm