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कक्षा-10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

(Introduction To Trigonometry)

भाग - 3

कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

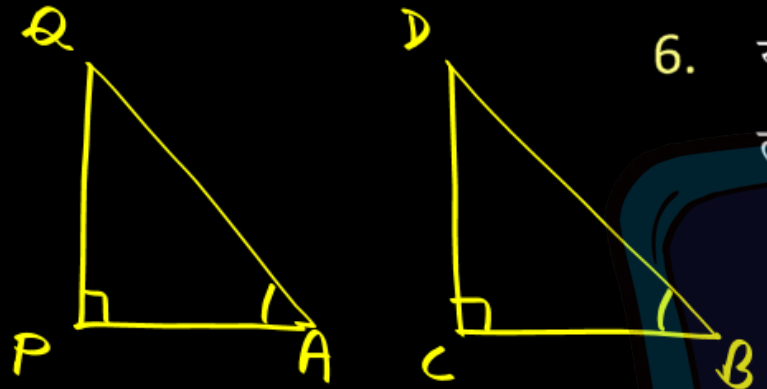
By : Rajendra Yadav Sir

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प्रश्नावली 8.1

6. यदि $\angle A$ और $\angle B$ न्यून कोण हो, जहाँ $\cos A = \cos B$, तो दिखाइए कि $\angle A = \angle B$



$\cos A = \cos B$

$\Rightarrow \frac{AP}{AQ} = \frac{BC}{BD}$

$\Rightarrow \frac{AP}{BC} = \frac{AQ}{BD} = K$

$\Rightarrow AP = K\,BC$ —— ①

$AQ = K\,BD$ —— ②

अब, $PQ^2 = AQ^2 - AP^2$

तथा $CD^2 = BD^2 - BC^2$

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अतः

$$PQ = \sqrt{AQ^2 - AP^2}$$

तथा $CD = \sqrt{BD^2 - BC^2}$

अब, $$\frac{PQ}{CD} = \frac{\sqrt{AQ^2 - AP^2}}{\sqrt{BD^2 - BC^2}}$$

$$\Rightarrow \frac{PQ}{CD} = \frac{\sqrt{K^2BD^2 - K^2BC^2}}{\sqrt{BD^2 - BC^2}} = \frac{K\sqrt{BD^2 - BC^2}}{\sqrt{BD^2 - BC^2}} = K$$

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अत: $\frac{AP}{BC} = \frac{AQ}{BD} = \frac{PQ}{CD} = k$

$\Rightarrow$ SSS समरूपता कसौटी से :

$\triangle APQ \sim \triangle BCD$

अत: संगत कोण बराबर होंगे।

$\Rightarrow \angle A = \angle B$

इतिसिद्धम्।

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कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

0° के लिये विभिन्न त्रिकोणमितीय अनुपात

$BC = 0$

$\Rightarrow \theta = 0$

अत: $\sin\theta = \frac{BC}{AC}$

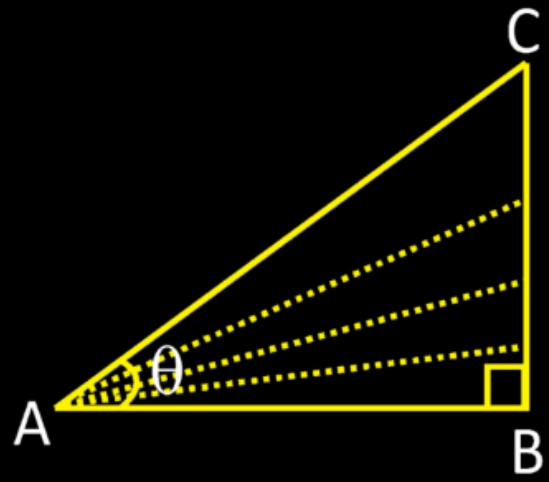
$\Rightarrow \sin 0^\circ = \frac{0}{AC} = 0$

$\Rightarrow \sin 0^\circ = 0$

$\cos\theta = \frac{AB}{AC}$

$\cos 0^\circ = \frac{AB}{AB}$

$\cos 0^\circ = 1$



(समकोण त्रिभुज)

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कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

30° के लिये विभिन्न त्रिकोणमितीय अनुपात

$$\sin 30^\circ = \frac{AD}{AC}$$

$$\sin 30^\circ = \frac{a}{2a} = \frac{1}{2}$$

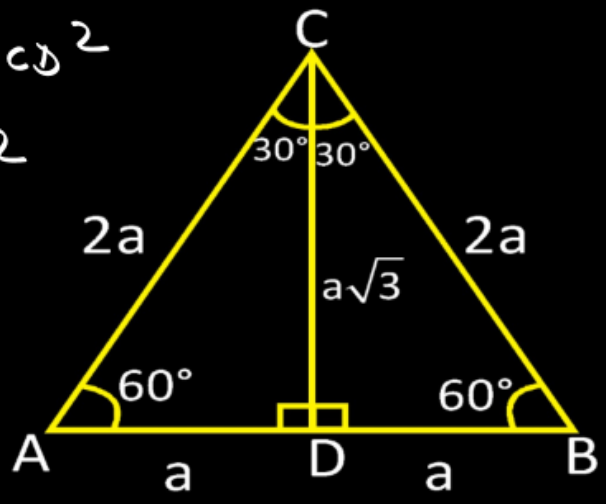
$$\cos 30^\circ = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$(2a)^2 = a^2 + CD^2$$

$$CD^2 = 4a^2 - a^2$$

$$CD^2 = 3a^2$$

$$CD = \sqrt{3}\,a$$



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45° के लिये विभिन्न त्रिकोणमितीय अनुपात

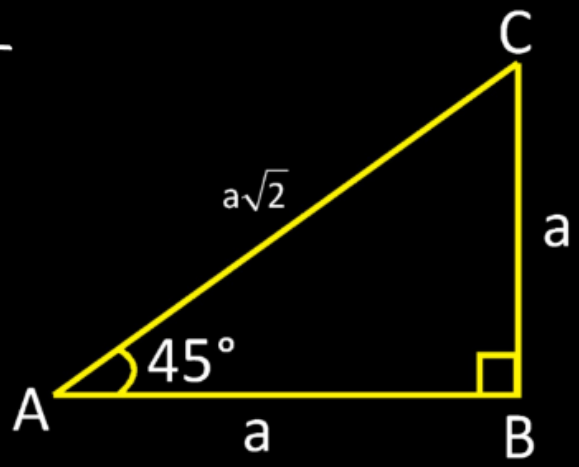
$$\sin 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$k^2 = a^2 + a^2$$

$$k^2 = 2a^2$$

$$k = \sqrt{2}\,a$$



(समकोण समद्विबाहु त्रिभुज)

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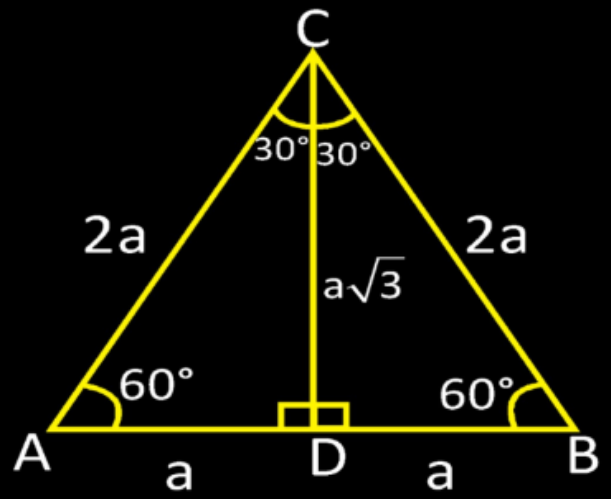
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कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

60° के लिये विभिन्न त्रिकोणमितीय अनुपात

$$\sin 60^\circ = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{a}{2a} = \frac{1}{2}$$



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कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

90° के लिये विभिन्न त्रिकोणमितीय अनुपात

$\sin 90^\circ = \frac{BC}{AC}$

$\sin 90^\circ = \frac{BC}{BC} = 1$

$\theta = 90^\circ$

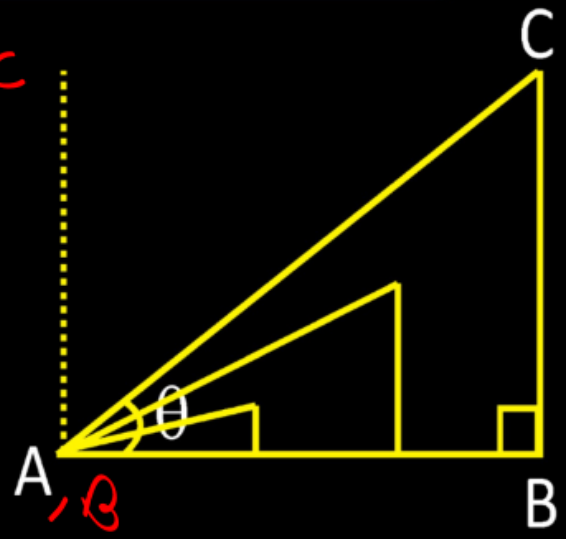
$\Rightarrow BC = AC$

$\sin 90^\circ = 1$

$\cos 90^\circ = \frac{AB}{AC}$

$\cos 90^\circ = \frac{0}{AC} = 0$

$\Rightarrow$ **$\cos 90^\circ = 0$**



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कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

त्रिकोणमितीय अनुपातों की सारणी

कोण (डिग्री/रेडियन) → / त्रिकोणमितीय अनुपात ↓	$0°$ 0	$30°$ $\frac{\pi}{6}$	$45°$ $\frac{\pi}{4}$	$60°$ $\frac{\pi}{3}$	$90°$ $\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

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कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

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कुछ विशिष्ट कोणों के त्रिकोणमितीय अनुपात

T.R. ↓ / θ →	0°	30°	45°	60°	90°
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cotθ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

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$$\sec\theta = \frac{1}{\cos\theta}$$

$$0 = \frac{0}{1}$$

$$\Rightarrow \frac{1}{0} \rightarrow \infty$$



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$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\frac{0}{1} = 0$,

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

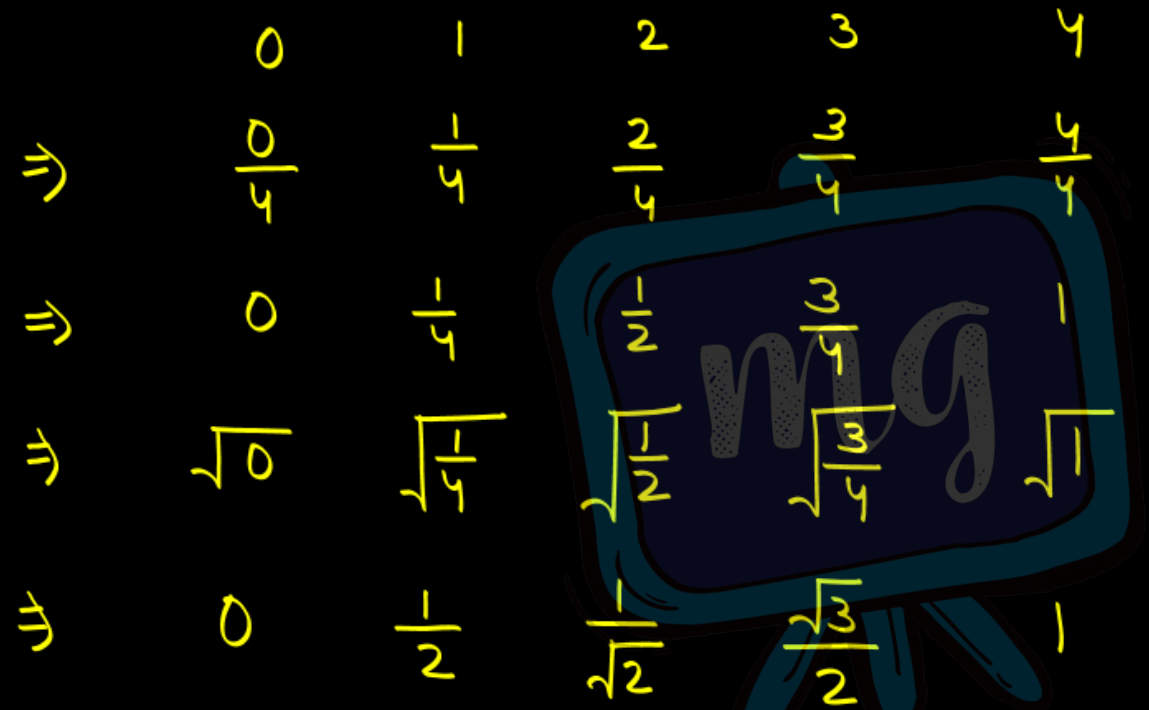
$\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\frac{1}{0} \to \infty$

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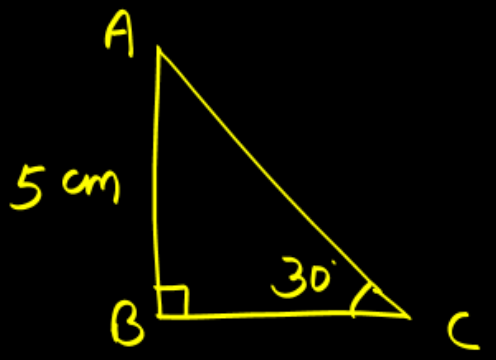
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उदाहरण 6. Δ ABC में जिसका कोण B समकोण है, AB = 5 cm और $\angle$ ACB = 30°। भुजाओं BC और AC की लंबाइयाँ ज्ञात करें।



$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{5}{BC}$$

$$\Rightarrow \boxed{BC = 5\sqrt{3}} \text{ cm}$$

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{5}{AC}$$

$$\Rightarrow \boxed{AC = 10} \text{ cm}$$