कक्षा - 10

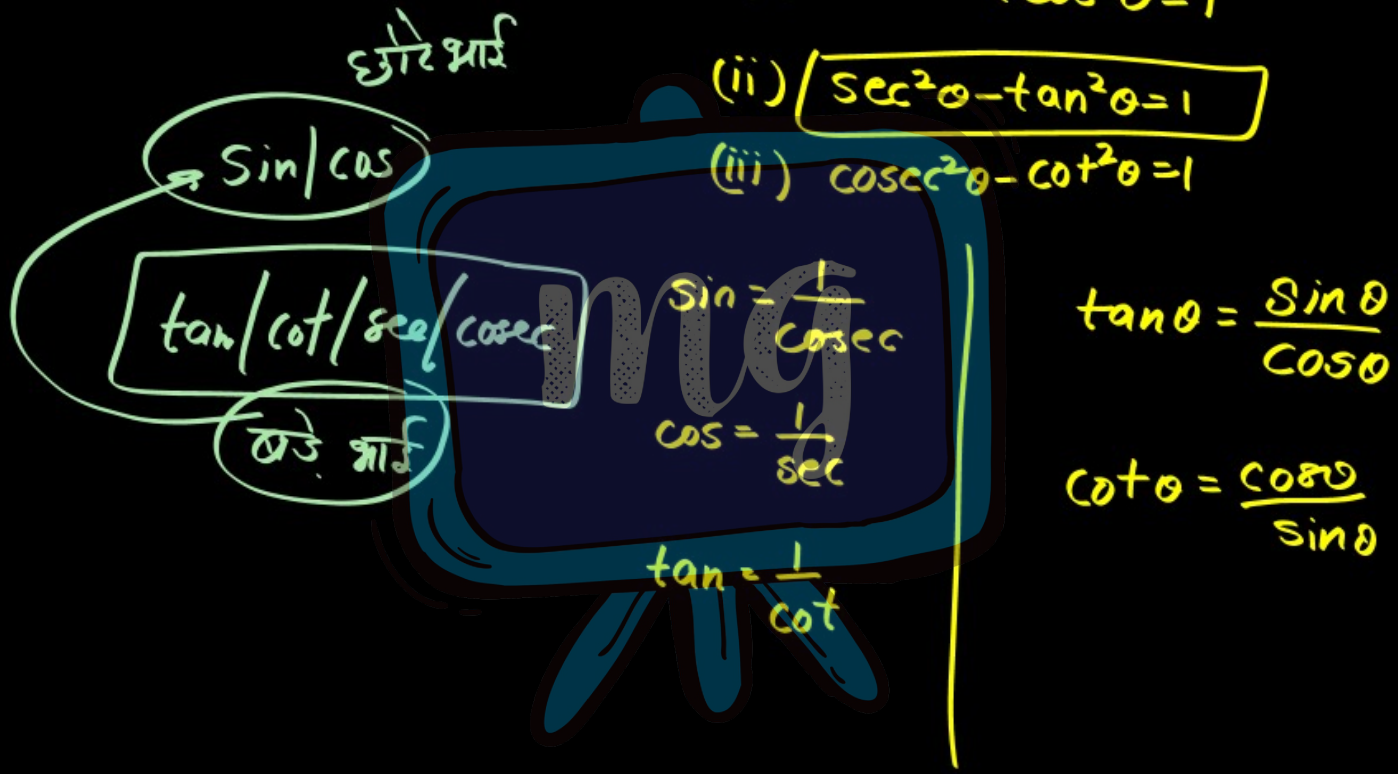
गणित

अध्याय - 8

त्रिकोणमिति का परिचय

भाग - 7

केशव शर्मा



(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\sec^2\theta - \tan^2\theta = 1$

(iii) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

प्रश्नावली 8.3

Q.3 सही विकल्प चुनिए और अपने विकल्प की पुष्टि कीजिए :

(i) $9 \sec^2 A - 9 \tan^2 A$ बराबर है :

(अ) 1

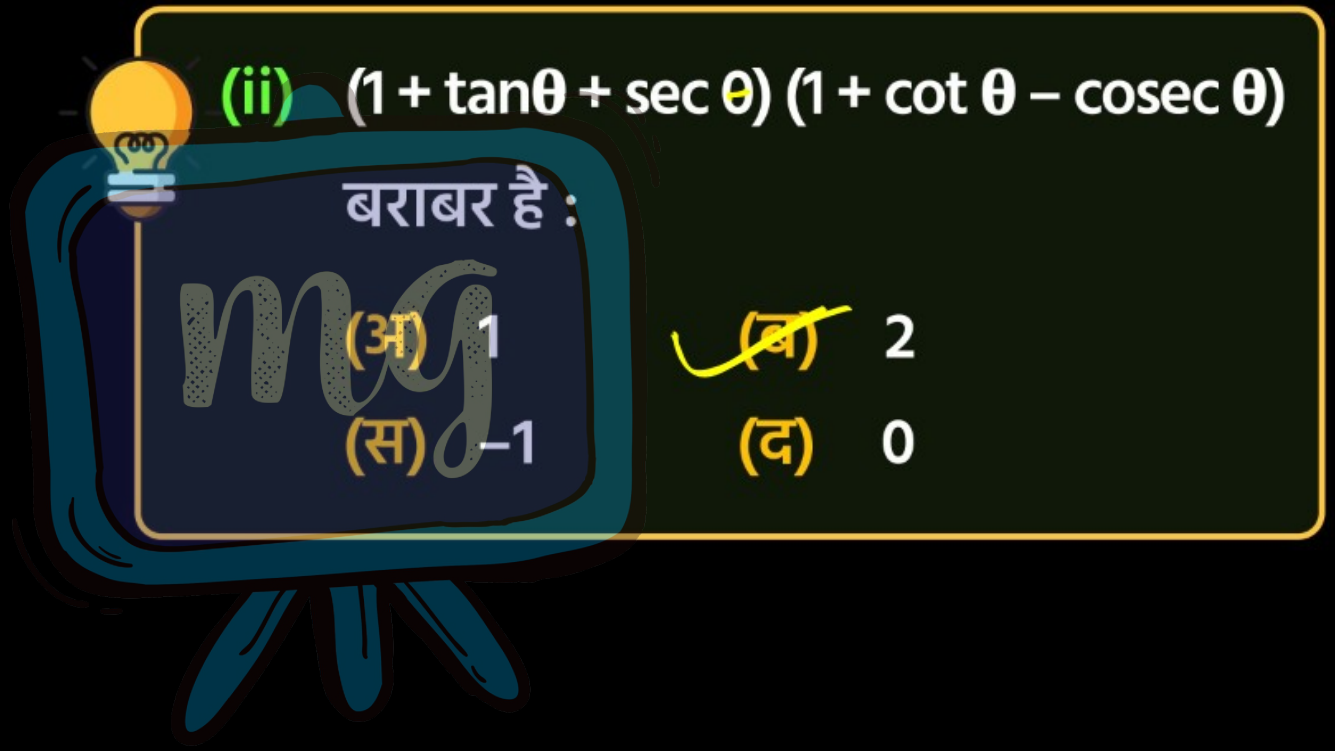
(ब) 9

(स) 8

(द) 10

$9(\sec^2 A - \tan^2 A)$

$9 \times 1 = 9$



$$(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$$

$$\left(\frac{1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}}{1}\right)\left(\frac{1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}}{1}\right)$$

$$\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$\frac{[(\sin\theta+\cos\theta)+1][(\sin\theta+\cos\theta)-1]}{\sin\theta\cos\theta}$$

$$\begin{matrix} a & + & b & \quad & a & - & b \end{matrix}$$

$$= a^2-b^2$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\sec\theta=\frac{1}{\cos\theta}$$

$$\cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$\operatorname{cosec}\theta=\frac{1}{\sin\theta}$$

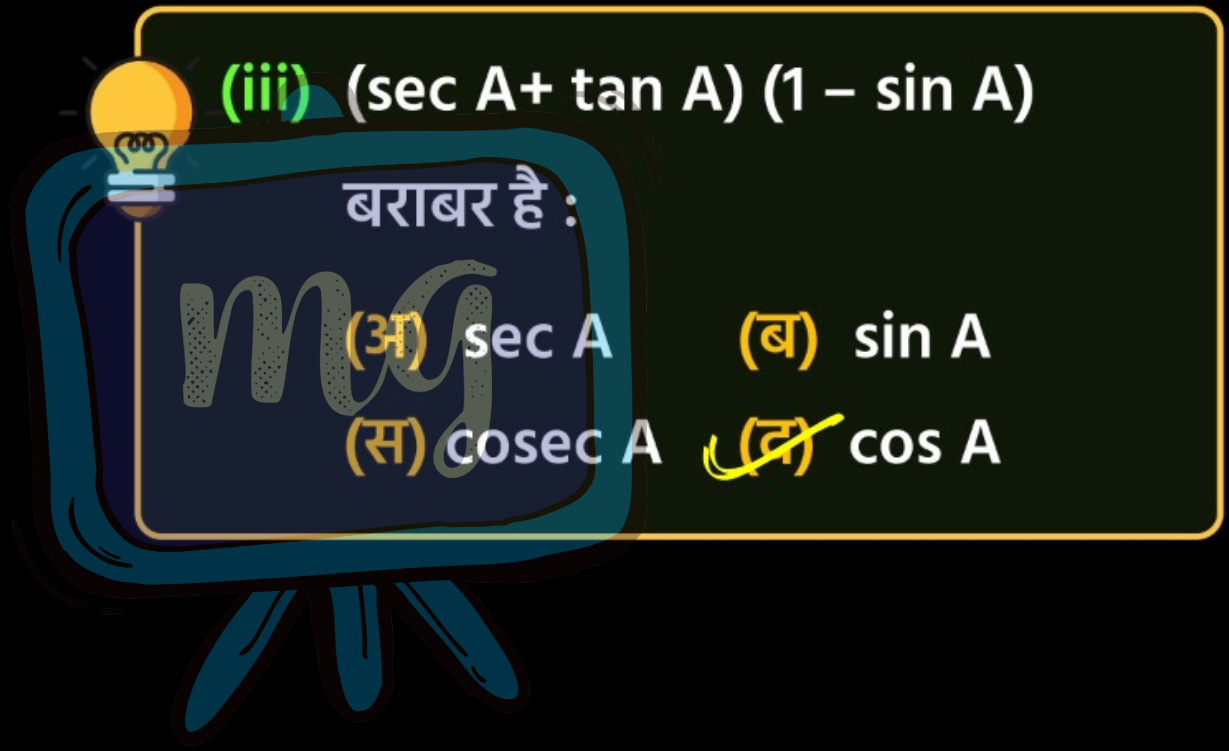
$$(\sin\theta+\cos\theta)^2-(\sin\theta+\cos\theta)+(\sin\theta+\cos\theta)-1$$

$$= \frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta} \quad \{\sin^2\theta+\cos^2\theta=1\}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$



$$\left\{\begin{array}{l}\sec A = \frac{1}{\cos A} \\ \tan A = \frac{\sin A}{\cos A}\end{array}\right\}$$

$$(\sec A + \tan A)(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

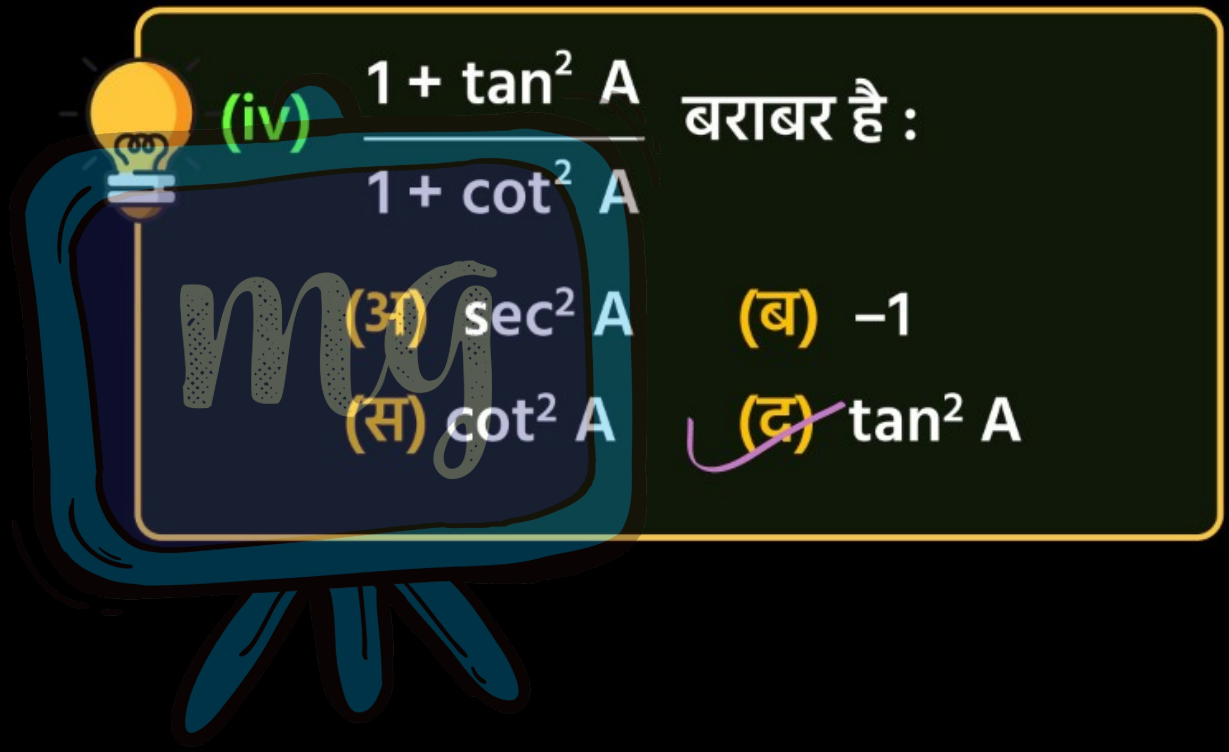
$$\left\{\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ \cos^2\theta = 1 - \sin^2\theta\end{array}\right\}$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$(a+b)(a-b)$

$$= \frac{1^2 - \sin^2 A}{\cos A} = \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$



(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ बराबर है :

(अ) $\sec^2 A$

(ब) -1

(स) $\cot^2 A$

(द) $\tan^2 A$

M-II

$$= \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{\frac{1 + \cos^2 A}{\sin^2 A}}$$

$$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A} \Big/ \frac{\sin^2 A + \cos^2 A}{\sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

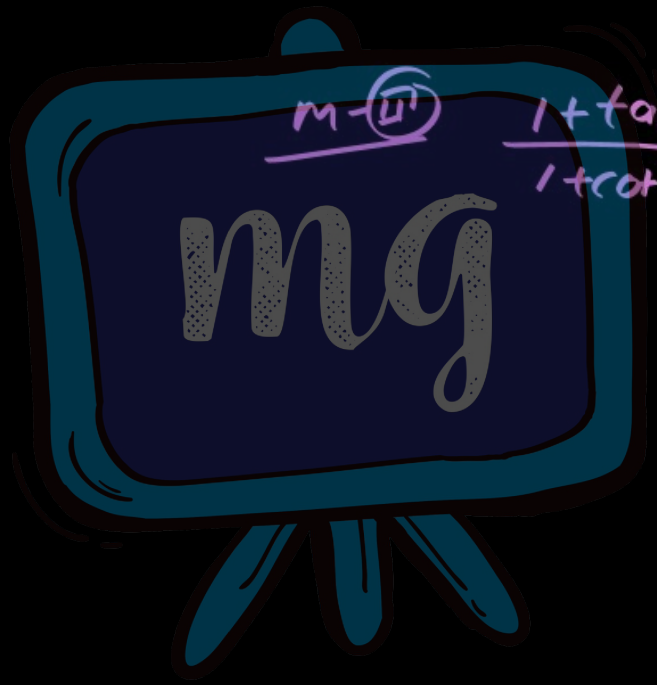
M-I

$$\frac{1 + \tan^2 A}{1 + \cot^2 A}$$

$$\frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}}$$

$$\frac{1 + \tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}}$$

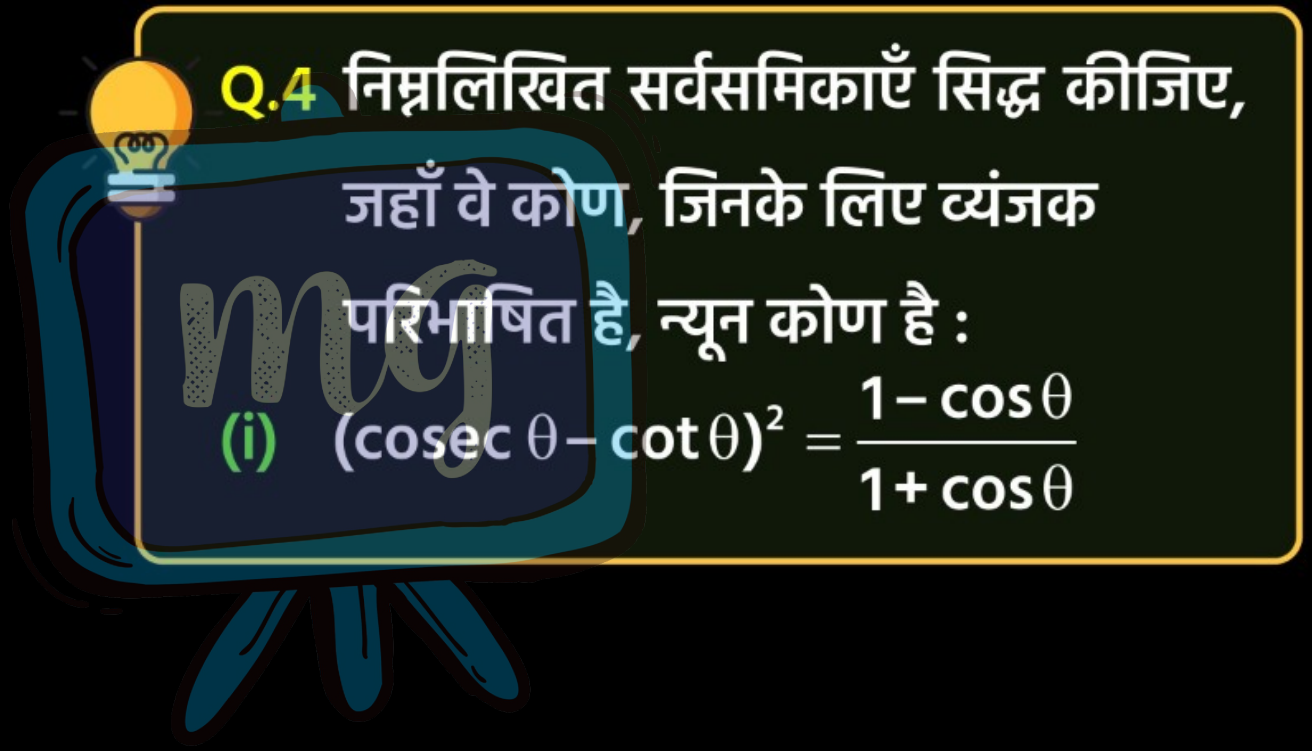
$$\tan^2 A$$



M-III

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1}{\frac{\cos^2 A}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A}$$



Q.4 निम्नलिखित सर्वसमिकाएँ सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यून कोण है :

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

$$\left.\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta &= 1 - \cos^2\theta \end{aligned}\right\}$$

LHS.

$$(\operatorname{cosec}\theta - \cot\theta)^2$$

$$\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}$$

$\{a^2-b^2=(a+b)(a-b)\}$

$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} = RHS$$

LHS = RHS

"H.P."

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

हल LHS.

$= \frac{\cos A}{(1+\sin A)} + \frac{1+\sin A}{\cos A}$

$\therefore \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

$$= \frac{\cos^2 A + 1^2 + \sin^2 A + 2 \times 1 \times \sin A}{(1+\sin A)\cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2\sec A \quad \left(\frac{1}{\cos A} = \sec A\right)$$

LHS = RHS

H.P.

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

Eq. - LHS.

$$= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1-\frac{\cos\theta}{\sin\theta}}{1}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1-\frac{\sin\theta}{\cos\theta}}{1}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left(\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right)$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left(\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right)$$

$\{a^3 - b^3 = (a-b)(a^2+b^2+ab)\}$

$$= \frac{1}{\sin\theta - \cos\theta} \left(\frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta} \right)$$

$\perp$

$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta} + 1$$

$= \text{cosec}\theta\,\sec\theta + 1 \quad = \text{RHS}$

LHS = RHS

HP